

## COLLECTIVE RELAXATION OF NONEQUILIBRIUM PHOTOELECTRONS IN QUANTIZED MAGNETIC FIELDS

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The effect of collective relaxation of nonequilibrium photoelectrons on the conductivity of semiconductors in quantized magnetic fields is studied. It is assumed that photoelectron production occurs in the lower Landau band and that the photoelectron concentration considerably exceeds that of thermal electrons. It is shown that for  $\omega_p \gg \tau_R^{-1}$ ,  $\tau_\epsilon^{-1}$  ( $\omega_p$  is the photoelectron plasma frequency,  $\tau_R$  the photoelectron lifetime in the conduction band, and  $\tau_\epsilon$  the energy relaxation time for acoustic phonons), the photoelectron distribution function is mainly determined by collective processes. The latter results in considerable broadening of the energy range of the photoelectrons and this in turn leads to a significant increase of the conductivity in weak electric fields and to decrease of size of the region in which the conductivity is negative.

### 1. INTRODUCTION

COLLECTIVE effects, as is well known, are decisive in the dynamics of relaxation processes in a rarefied plasma (see, e.g., [1-3]). In a solid-state plasma these effects usually have little effect on the relaxation of the non-equilibrium electrons, this being due to their intensive interaction with the impurities, lattice vibrations, and pair collisions. In a number of cases, however, the collective relaxation, as will be shown in the present paper, can be significant at short lifetimes of the electrons and in an electron plasma of a semiconductor. We note that if the radiation source is monochromatic and generates in the conduction band strongly non-equilibrium photoelectrons, the aggregate of such photoelectrons can form in quantizing magnetic fields two (or more) streams moving opposite to each other. Indeed, in sufficiently strong magnetic fields, such that

$$\tau\omega_c \gg 1, \quad (1)$$

where  $\omega_c$  is the cyclotron frequency and  $\tau$  is the electron-momentum relaxation time, the energy spectrum of the electrons, in the case of quadratic and isotropic dispersion, is given by

$$\epsilon_{N,p} = (N + 1/2)\hbar\omega_c + p^2/2m. \quad (2)$$

Here  $N$  is the magnetic quantum number,  $p$  is the electron momentum along the magnetic field, and  $m$  is the effective mass. If the energy  $\epsilon_g$  of the produced photoelectrons satisfies the relation  $\hbar\omega_c/2 < \epsilon_g < 3\hbar\omega_c/2$ , then the photoelectrons are concentrated in the lower Landau subband ( $N = 0$ ). The produced photoelectrons then have a longitudinal momentum  $\pm p_0 = \pm [2m(\epsilon_g - 1/2\hbar\omega_c)]^{1/2}$ , i.e., they form two equal-density beams, that moving opposite each other with velocities  $\pm q_0 = \pm p_0/m$ . If these beams are strong enough, their presence can lead to two-stream instability. It is natural to expect the instability development to cause a considerable transformation of the electron distribution function, not connected with the interaction with the phonon or with pair collisions with the thermal

electrons. This in turn affects the conductivity due to the non-equilibrium photoelectrons in quantizing magnetic fields.

We consider in this paper the collective relaxation of strongly nonequilibrium photoelectrons and its influence on the conductivity in quantizing magnetic fields (condition (1) is satisfied). We confine ourselves to the case  $n \gg n_T$ , where  $n$  is the concentration of the non-equilibrium photoelectrons and  $n_T$  is a concentration of the thermal electrons. Under such conditions, as shown by Elesin<sup>[4]</sup>, a number of interesting effects appear, particularly an absolute negative conductivity; the possibility of collective relaxation resulting from the two-stream instability is disregarded.

### 2. TWO-STREAM INSTABILITY

Assume that the photoelectrons are produced under the influence of a monochromatic source only in the lower Landau subband. The time  $\tau_\epsilon$  of relaxation of the electron energy by the acoustic phonons, as shown in<sup>[5]</sup>, greatly exceeds the lifetime  $\tau_R$ , i.e., if we disregard instability, the produced photoelectron beams remain monoenergetic during the lifetime of the electron in the conduction band. It should be noted that, unlike the usual situation in a plasma, collisions with impurities and phonons, as well as pair collisions, transfer the electrons from one beam to another by virtue of the energy conservation law and the quasielasticity of the scattering. This process occurs within a momentum relaxation time  $\tau \ll \tau_R, \tau_\epsilon$ . Thus, an effective equalization of the concentrations of the two beams takes place within a time  $\tau$ .

Since  $n \gg n_T$  by assumption, and the concentration of the thermal electrons in real semiconductors is not too low ( $n_T \gtrsim 10^{11} \text{ cm}^{-3}$ ), it follows that the plasma frequency of the photoelectrons  $\omega_p = (4\pi e^2 n/m\kappa)^{1/2}$  ( $\kappa$  is the dielectric constant of the lattice) can greatly exceed the recombination frequency  $\nu_R = \tau_R^{-1}$  ( $\nu_R \sim 10^{10} \text{ sec}^{-1}$ <sup>[5]</sup>).

Taking into account the smallness of the parameter  $\nu_R \tau \ll 1$ , we obtain a dispersion relation for the quasi-neutral system, the electron distribution function for

which represents two beams moving opposite to each other with velocity  $\pm u_0$ . Since the electrons move along the magnetic field in classical fashion, assuming that the characteristic length of the perturbations are much larger than the magnetic radius  $L = (c\hbar/eH)^{1/2}$  (this is in fact equivalent to the assumption  $\hbar\omega_p \ll \epsilon_g \sim \hbar\omega_c$ ), we use the equation for the electron distribution function with respect to the longitudinal momenta or velocities in the lower Landau subband (the equation for the density matrix  $\langle N = 0 | \rho | N = 0 \rangle \equiv f(p) = f_+(p) + f_-(p)$  in the Landau representation) and the Poisson equation:

$$\frac{\partial f_{\pm}}{\partial t} + v \frac{\partial f_{\pm}}{\partial z} - \frac{e}{m} E_z \frac{\partial f_{\pm}}{\partial v} = -v [f_{\pm}(v) - f_{\mp}(-v)] - \nu n f_{\pm} + G, \quad (3)$$

$$\operatorname{div} E = \frac{4\pi e}{\epsilon} \left[ n_0 - \int_{-\infty}^{\infty} (f_+ + f_-) dv \right]. \quad (4)$$

Here  $\nu = \tau^{-1}$ ,  $f_{\pm}(v)$  are the distribution functions of the photoelectrons in the beams,  $v = p/m$ , the term  $G$  describes the photoelectron generation, and  $n_0$  is the density of the immobile compensating charge; the  $z$  axis is directed along the external magnetic field.

We consider first processes occurring within a time much shorter than the characteristic recombination and generation times. Consequently, when determining the behavior of a specified initial distribution function we can neglect the last two terms of (3). Taking into account the  $\delta$ -like character of the distribution function  $f_{\pm}(v)$ , we obtain from the zeroth and first moments of (3) the hydrodynamic equations for the system of two beams:

$$\frac{\partial n_{\pm}}{\partial t} + \frac{\partial}{\partial z} (n_{\pm} u_{\pm}) = -\nu (n_{\pm} - n_{\mp}), \quad (5)$$

$$\frac{\partial u_{\pm}}{\partial t} + u_{\pm} \frac{\partial u_{\pm}}{\partial z} = -\frac{e}{m} E_z, \quad (6)$$

where

$$n_{\pm} = \int_{-\infty}^{\infty} f_{\pm} dv, \quad u_{\pm} = \int_{-\infty}^{\infty} v f_{\pm} dv / \int_{-\infty}^{\infty} f_{\pm} dv.$$

In the derivation of (5) and (6) we took into account the conservation of the absolute value of the electron momentum in the collisions, which has led in natural fashion to the appearance of a term describing the equalization of the electron concentrations in the beams (the term in the right-hand side of the continuity equation) and to the absence of friction force in the equation of motion.

Linearizing the system (4)–(6), we obtain dispersion relations for the potential perturbations in which the electric field is directed along the external magnetic field, since such perturbations have the largest increment

$$\omega_p^2 [\omega^2 + k^2 u_0^2 + 2i\nu\omega] [\omega^2 - k^2 u_0^2 + 2i\nu\omega]^{-2} = 1. \quad (7)$$

In the approximation considered, this equation can be derived also directly from (3) and (4).

If  $\nu = 0$ , the last equation goes over into the ordinary dispersion relation for the instability of two identical beams moving opposite to each other:

$$\omega_p^2 / 2(\omega - ku_0)^2 + \omega_p^2 / 2(\omega + ku_0)^2 = 1. \quad (8)$$

The solution of Eq. (8) takes the following form:

$$\omega^2 = k^2 u_0^2 + \frac{\omega_p^2}{2} \pm \left[ 2k^2 u_0^2 \omega_p^2 + \frac{\omega_p^4}{4} \right]^{1/2}. \quad (9)$$

When  $k^2 u_0^2 < \omega_p^2$  we have  $\omega^2 < 0$ . Therefore, since the perturbations were chosen to be proportional to  $\exp(ikz - i\omega t)$ , instability can build up in the system. The maximum increment is  $\gamma = \omega_p / \sqrt{8}$  and is reached at  $k = \sqrt{3/8} \omega_p / u_0$ .

We consider now the influence of collisions on this instability. To this end, we analyze Eq. (8). When  $\gamma \sim \omega_p \gg \nu$ , the collisions cannot stabilize the instability. We call attention to the fact that the instability remains regardless of the ratio of  $\omega_p$  to  $\nu$ . Indeed, at the stability limit ( $\operatorname{Im} \omega = 0$ ), equating the real and imaginary parts of (7) separately to zero, we have

$$(\omega - ku_0)(\omega + ku_0)^2 + (\omega + ku_0)(\omega - ku_0)^2 = 2\omega\omega_p^2, \quad (10)$$

$$(\omega^2 - k^2 u_0^2)^2 - 4\nu^2 \omega^2 = \omega_p^2 (\omega^2 + k^2 u_0^2). \quad (11)$$

It is interesting that the collision frequency does not influence the stability limit. By determining the frequency at the stability limit from (10) and substituting it in (11), we obtain the same instability condition as in the case when there are no collisions.

The development of an instability of this type leads, as usual, to the appearance of a scatter of the photoelectron velocity in the region  $|v| < u_0$ . If the instability increment is much less than the frequency, a plateau is formed on the distribution function in velocity space. We can therefore expect the broadening of the beam distribution functions in velocity space to lead to stabilization of the instability when the collisions are taken into account.

Allowance for the broadening of the distribution functions changes the dispersion relation (7) into the following:

$$\begin{aligned} & \omega_p^2 [\omega^2 + k^2 u_0^2 - k^2 \Delta^2 + 2i\nu\omega] \\ & = [(\omega - ku_0)^2 - k^2 \Delta^2][(\omega + ku_0)^2 - k^2 \Delta^2] + i\nu(\omega - ku_0)[(\omega \\ & + ku_0)^2 - k^2 \Delta^2] + i\nu(\omega + ku_0)[(\omega - ku_0)^2 - k^2 \Delta^2] - 4\nu^2 \omega^2, \end{aligned} \quad (12)$$

where  $\Delta$  is a quantity characterizing the velocity scatter.

At the instability limit we have from (12)

$$k^2 = \omega_p^2 / (u_0^2 - \Delta^2). \quad (13)$$

Thus, the instability takes place at  $u_0^2 > \Delta^2$  ( $k^2 > 0$ ), and the limit is independent of the collision frequency, as before. Therefore, in such a formulation, the instability ceases at  $|\Delta| \geq u_0$ , and the stable distribution function of the photoelectrons takes the form of a plateau at  $|v| \leq u_0$ ; the height of the plateau, owing to the slowness of the generation and recombination processes, is determined by the initial concentration. The characteristic time required for the establishment of such a plateau is of the order of  $1/\omega_p \ll \tau_R$ , i.e., the assumptions made are valid.

It is interesting to note that an instability of this kind was numerically simulated by Dowson<sup>[6]</sup>, who has shown that within a time  $\sim 1/\omega_p$  the two beams produce, as a result of the instability, a plateau in the velocity interval from  $-u_0$  to  $u_0$ . The height of the plateau was determined by the law of electron-number conservation, and the energy of the noise was equal to the difference between the energies of the final and initial states.

When recombination is taken into account, the broad-

ening of the distribution function should stop somewhat sooner, namely at

$$u_0^2 - \Delta^2 \sim v_R^2 u_0^2 / \omega_p^2.$$

Such effects were taken into account by Ryutov<sup>[7]</sup>, who has shown that the broadening of the distribution function stops when the velocity becomes of the order of  $\tilde{\nu}/k$ , where  $\tilde{\nu}$  is the frequency of the momentum or concentration relaxation.

Thus, allowance for two-stream instability and for the associated collective relaxation within a time  $\sim 1/\omega_p$  leads to an effective broadening of the distribution function. Therefore, when finding the stationary state it is necessary to take this effect, i.e., it is necessary to include in the balance equation terms that describe the collective relaxation, analogous to the diffusion terms in the quasilinear theory<sup>[1-3]</sup>.

For an exact determination of the form of the distribution function, it is proposed to use the method of one-dimensional numerical simulation, which in the present situation can describe the phenomenon fully, unlike the usual cases in a plasma, when the problem is as a rule not one-dimensional.

### 3. THE PHOTOCURRENT

Let us calculate now the density of the static conduction current perpendicular to the magnetic field, taking into account the arguments advanced above concerning the character of the photoelectron distribution function in the presence of intensive collective relaxation. We assume that the electrons are scattered by impurities and acoustic phonons. Recognizing that the conduction current perpendicular to the quantizing magnetic field is due to migrations of the centers of the Larmor orbits of the electrons upon scattering, and that the photoelectrons populate only the lower Landau subband, we can write

$$j_x = e \sum_{k,p,k',p'} (X_k - X_{k'}) f(p) W_{kp k' p'} \quad (14)$$

where  $X_k$  is the coordinate of the center of the electron orbit (the  $x$  axis is directed along the static electric field),  $W_{kp k' p'}$  is the probability of transition of the electron from the state ( $N = 0, X_k, p$ ) to the state ( $N = 0, X_{k'}, p'$ ) when scattered by an impurity or by a phonon. If we neglect the small inelasticity of the scattering, the transition probability is

$$W_{kp k' p'} = \frac{2\pi V^2}{\hbar} \sum_q \exp\left\{-\frac{1}{2} L^2 q_{\perp}^2\right\} \delta\left(\frac{p'^2}{2m} - \frac{p^2}{2m} - FLq_y\right) \delta_{k',k+q_y} \delta_{p',p+q_z} \quad (15)$$

Here  $V$  is the amplitude of the scattering potential,  $F = eE_0L$ ,  $E_0$  is the intensity of the static electric field,  $q = (q_x, q_y, q_z)$ , and  $q_{\perp} = (q_x, q_y)$ . The appearance of the exponential in (15) is due to the characteristic of the wave functions on the electrons in a quantizing magnetic field.

Substituting (15) in (14) we obtain, following standard calculations and transformations,

$$j_x \sim I = \int_{-\infty}^{\infty} y e^{-y^2/2} \Phi(F, y) dy, \quad (16)$$

$$\Phi(F, y) = \begin{cases} \int_0^{\infty} \frac{d\varepsilon \mathcal{F}(\varepsilon)}{[\varepsilon(\varepsilon + Fy)]^{1/2}}, & y > 0 \\ \int_{-Fy}^{\infty} \frac{d\varepsilon \mathcal{F}(\varepsilon)}{[\varepsilon(\varepsilon + Fy)]^{1/2}}, & y < 0 \end{cases}, \quad (17)$$

where  $\mathcal{F}(\varepsilon(p)) = f(p)$ ,  $\varepsilon = p^2/2m$ ,  $y = Lq_y$ .

To determine the qualitative picture of the influence of the collective relaxation on the conductivity, with allowance for the results of the preceding section, we can assume that

$$\begin{aligned} \mathcal{F}(\varepsilon) &= \overline{\mathcal{F}} = \text{const} & \text{at } \varepsilon_1 \leq \varepsilon \leq \varepsilon_0, \\ \mathcal{F}(\varepsilon) &= 0 & \text{at } \varepsilon < \varepsilon_1, \varepsilon > \varepsilon_0; \\ \varepsilon_0 &= \varepsilon_s - 1/2\hbar\omega_c, & \varepsilon_1 \sim \varepsilon_0 (v_R/\omega_p)^4. \end{aligned}$$

Taking this into consideration, we obtain after calculating the function  $\Phi(F)$ ,  $y$  the following expression:

$$I = 2\overline{\mathcal{F}} \left[ \int_{-F/F}^{\infty} y e^{-y^2/2} \ln \frac{\sqrt{\varepsilon_0 + \sqrt{\varepsilon_0 + Fy}}}{\sqrt{\varepsilon_1 + \sqrt{\varepsilon_1 + Fy}}} dy + \int_{-\varepsilon_1/F}^{-F/F} y e^{-y^2/2} \ln \frac{\sqrt{\varepsilon_0 + \sqrt{\varepsilon_0 + Fy}}}{\sqrt{-Fy}} dy \right]. \quad (18)$$

We consider next two ranges of static electric fields: 1)  $F \ll \varepsilon_1$  and 2)  $\varepsilon_1 \ll F \ll \varepsilon_0$ .

In the first range, recognizing that the significant interval for the integration in (18) is  $|y| \lesssim 1$ , we can expand the integrand in powers of  $F/\varepsilon_0$  and  $F/\varepsilon_1$ . After expanding and calculating the integral we get

$$I = -\sqrt{2\pi} \overline{\mathcal{F}} F / \varepsilon_1. \quad (19)$$

We have put  $v_R \ll \omega_p$ , and have therefore neglected terms of order  $\varepsilon_1/\varepsilon_0$ . It is seen directly from (19) that since the right-hand side is negative the conduction current is directed opposite to the static electric field, i.e., absolute negative conductivity is obtained ( $I \sim -E_0$ ).

In the second range, the expression can be granted in powers of  $F/\varepsilon_0$  and  $\varepsilon_1/F$ . In this case, integration yields

$$I = \sqrt{2\pi} \overline{\mathcal{F}} F / \varepsilon_0. \quad (20)$$

Thus, the conductivity becomes positive in relatively strong electric fields.

It follows from the results that the intensive collective relaxation at  $\omega_p \gg v_R$ , by appreciably transforming the distribution function, changes significantly the shape of the current-voltage characteristic. First, the absolute value of the conductivity in weak electric fields much larger in the case of collective relaxation than the conductivity for a  $\delta$ -like distribution function (see<sup>[4]</sup>). Comparing the conductivities for equal concentrations of the photoelectrons, we obtain for their ratio

$$|I_{\text{coll}}|/|I| \approx \varepsilon_0/\varepsilon_1 \sim (\omega_p/v_R)^4 \gg 1. \quad (21)$$

Physically this result is quite understandable. The collective relaxation leads to population of the energy region  $\varepsilon \ll \varepsilon_0$  by photoelectrons. The density of states in this region is much higher than in the region where the photoelectrons are produced, this being due in final analysis to the one-dimensional character of the electron motion in the quantizing magnetic field. Their interaction with the impurities and with the phonons therefore becomes more intense. It is this which leads to the increase of the absolute value of the conductivity.

Second, when  $\omega_p \gg v_R$  the region of absolute nega-

tive conductivity becomes much narrower. Indeed, according to (20), the conductivity reverses sign in electric fields  $E_0 \sim \epsilon_1/eL$ . At the same time, the change of the sign of the conductivity for a  $\delta$ -like distribution function occurs at  $E_0 \sim \epsilon_0/eL$  (see<sup>[4]</sup>), i.e., in much stronger fields.

<sup>1</sup>A. A. Vedenov, in: *Voprosy teorii plazmy (Problems of Plasma Theory)*, Vol. 3, Gosatomizdat, 1963, p. 203.

<sup>2</sup>B. B. Kadomtsev, *ibid.*, Vol. 4, Gosatomizdat, 1964, p. 188.

<sup>3</sup>A. A. Ivanov and L. I. Rudakov, *Zh. Eksp. Teor. Fiz.* **51**, 1522 (1966) [*Sov. Phys.-JETP* **24**, 1027 (1967)].

<sup>4</sup>V. F. Elesin, *ibid.* **55**, 792 (1968) [**28**, 410 (1969)].

<sup>5</sup>M. A. Habbeger and H. Y. Fan, *Phys. Rev. Lett.* **12**, 99, 1964.

<sup>6</sup>J. Dowson, *Nucl. Fusion, Suppl.*, **P3**, 1033, 1962.

<sup>7</sup>D. D. Ryutov, *Zh. Eksp. Teor. Fiz.* **52**, 1378 (1967) [*Sov. Phys.-JETP* **25**, 916 (1967)].

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166