SOVIET PHYSICS JETP VOLUME 36, NUMBER 4 APRIL, 1973

INSTABILITY OF A CURRENT-CARRYING PLASMA AT CYCLOTRON HARMONICS AND THE ANOMALOUS RESISTANCE

D. G. LOMINADZE

Physics Institute, Georgian Academy of Sciences

Submitted May 22, 1971

Zh. Eksp. Teor. Fiz. **63**, 1300–1311 (October, 1972)

A theory is developed of instability of a current-carrying plasma at electron-cyclotron frequencies; the nonlinear stage of the process is investigated. The anomalous resistance that results from cyclotron harmonic build-up is determined. In the given case, the appearance of the anomalous resistance itself (anomalous scattering of electrons by vibrations) leads to stabilization of the instability. The idea of the approach to nonlinear saturation of instabilities of the kind considered here is that use is made of the analogy between the role of ordinary binary collisions and of particle scattering by vibrations created in an unstable plasma.

INTRODUCTION

THE anomalously large plasma resistance observed in many experiments was attributed to ion-sound instability. In the weak-turbulence theory, formulas were obtained for the electric-conductivity coefficient in the presence of ion-sound instability[1]. The ion-sound instability can occur, however, only in the case of strong non-isothermy, $T_e \gg T_i$. Yet an anomalous resistance is produced also when this condition is not satisfied, particularly in collisionless shock waves[2,3]. In these experiments, the electric current flows perpendicular to the magnetic field. This can produce in the plasma various instabilities of the two-stream type. Whenever the ion drift velocity v_d relative to the electrons is smaller than the thermal velocity of the electrons, these instabilities, which have a maximum increment of the order of the hybrid frequency $\gamma \sim (m_e/m_i)^{1/2}\omega_{He}$, can lead to an anomalous resistance of the plasma[4-8]. In some cases, however, for example in collisionless shock waves, these instabilities, which have a relatively small increment, do not have time to develop and can therefore not be responsible for the anomalous resistance. The anomalous resistance may then be caused by instabilities of the electron-cyclotron oscillations, which have been under lively discussion recently[9-18]. These oscillations propagate perpendicular or almost perpendicular to the magnetic field and have frequencies on the order of $n\omega_{He}$.

If current flows in the plasma, the electron-cyclotron oscillations can become unstable. When the drift velocity is high enough, a hydrodynamic instability of the two-stream type sets in, with a growth increment that is maximal in the region of intersection of the frequencies of the electron-cyclotron branch and of the beam branch[9], or else when the electron-cyclotron branch intersects the ion-sound branch[9-11,13,14]. A kinetic instability of the type observed in waves with negative energy sets in at low drift velocities[11-13,16].

The present paper deals with the nonlinear stage of instability at electron-cyclotron frequencies and their harmonics in a plasma with a transverse current, and with the determination of the value of the anomalous resistance. It is shown that the main mechanism leading to saturation of the oscillations in the case of kinetic instability, and also to hydrodynamic instability at $T_i \gtrsim T_e$, is the electronic nonlinearity (in contrast to the ion-sound instability, for which the mechanism of saturation of the oscillations as a result of induced scattering of the waves by ions is important[19]). The presence of unstable oscillations leads to the appearance of strong scattering of the electrons by the turbulent pulsations of the electric field. In this case the presence of an effective collision frequency leads to stabilization of the instability[1)].

A somewhat different picture arises at $T_e \gg T_i$. In this case the appearance of the effective collision frequency may not lead to saturation of the oscillations, but only to a decrease of the increment. Utilization of strong-turbulence theory[21] shows that in this case, at a sufficiently high noise level, the nonlinear stage of the electron-cyclotron instability goes over into an ion-sound instability.

It should be noted that the transition of the nonlinear stage of the electron-cyclotron instability into ion-sound instability of a plasma without a magnetic field was first pointed out in a numerical experiment[15].

1. LINEAR THEORY OF THE INSTABILITY OF A CURRENT-CARRYING PLASMA AT CYCLOTRON HARMONICS

The dispersion equation for potential electron-cyclotron oscillations in a plasma with a current flowing perpendicular to the constant magnetic field is of the form

$$1 + \delta\varepsilon_e + \delta\varepsilon_i = 0; \tag{1}$$

$$\delta\varepsilon_e = \frac{2\omega_{pe}^2}{k^2 v_{Te}^2}\Big[1 + i\sqrt{\pi}\, z_0 \sum_{n=-\infty}^{\infty} A_n(x) W(z_n)\Big], \tag{2}$$

$$\delta\varepsilon_i = \frac{2\omega_{pi}^2}{k^2 v_{Ti}^2}[1 + i\sqrt{\pi}\, z_i W(z_i)]; \tag{3}$$

$$v_{T\alpha} = (2T_\alpha / m_\alpha)^{1/2}, \quad x = {}^1/_2 k^2 \rho_e^2, \quad \rho_e = v_{Te} / \omega_{He}, \quad A_n(x) = e^{-x} I_n(x),$$

[1)]Investigations of the anomalous resistance due to kinetic instability at cyclotron harmonics were briefly reported by Galeev et al. [20]

$$z_n = \frac{\omega - n\omega_{He}}{k_z v_{Te}}, \quad z_i = \frac{\omega - \mathbf{k}\mathbf{v}_d}{k v_{Ti}}, \quad \omega_{p\alpha}^2 = \frac{4\pi n_0 e^2}{m_\alpha},$$

$$\omega_{He} = \frac{eH_0}{m_e c}, \quad W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2} dt\right).$$

We choose a reference frame in which the electrons are at rest. Expression (2) is valid for oscillations propagating almost perpendicular to the magnetic field, for which $|\omega - n\omega_{He}| \gg k_z v_{Te}$. We shall consider the case when the ion drift velocity relative to the electrons, v_d, is much smaller than the thermal velocity of the electrons. We then have for the unstable cyclotron oscillations $\omega \sim \mathbf{k}\cdot\mathbf{v}_d \sim n\omega_{He}$, so that the wavelength turns out to be much smaller than the Larmor radius, $k\rho_e \sim n v_{Te}/v_d \gg 1$. We can therefore assume in expression (2) that $A_n \approx 1/\sqrt{\pi} k\rho_e \ll 1$.

Since the growth increment of the cyclotron oscillations is much larger than the cyclotron frequency of the ions, we can assume that the ions are not magnetized and use expression (3) for $\delta\epsilon_i$.

A. Kinetic Instability

At $|z_n| \gg 1$, $k\rho_e \gg 1$, and $v_d \ll v_{Te}$ the dispersion equation (1) takes the form

$$1 + k^2\lambda_D^2 + \frac{T_e}{T_i}(1 + i\sqrt{\pi}\, z_i W(z_i)) - \sum_{n=-\infty}^{\infty} \frac{\omega A_n(x)}{\omega - n\omega_{He}} = 0, \tag{4}$$

where $\lambda_D^2 = T_e/4\pi n_0 e^2$. If the drift velocity is much lower than a certain value $v_d = v_0$, where

$$v_0^2 = \sqrt{\pi}\, v_{Te} v_{Ti} \frac{1 + k^2\lambda_D^2}{n}, \tag{5}$$

then oscillations can be excited by Cerenkov interaction between the oscillations and the resonant ions.

Recognizing that $A_n \ll 1$, we seek a solution of equation (4) in the form $\omega = n\omega_{He} + \Delta\omega$, where $|\Delta\omega| \ll n\omega_{He}$ and $|\Delta\omega| \ll k v_{Ti}$. We then obtain

$$\Delta\omega = \frac{n\omega_{He} A_n}{1 + k^2\lambda_D^2 + (T_e/T_i)\,[1 + i\sqrt{\pi}\, z_i W(z_i)]}, \quad z_i = (n\omega_{He} - \mathbf{k}\mathbf{v}_d)/k v_{Ti}. \tag{6}$$

The growth increment $\gamma = \operatorname{Im}\omega$ is in this case equal to

$$\gamma = -\frac{n\omega_{He}}{k\rho_e}\frac{T_i}{T_e}\frac{z_i e^{-z_i^2}}{[(1 + k^2\lambda_D^2)(T_i/T_e) + \psi(z_i)]^2 + [\sqrt{\pi}\, z_i e^{-z_i^2}]^2},$$

$$\psi(z) = 1 - 2z e^{-z^2}\int_0^z e^{t^2} dt. \tag{7}$$

In order of magnitude at $T_i \gtrsim T_e$, the maximum growth increment is reached at $z_i = -2^{-1/2}$ [11,12,16]:

$$\gamma_{max} \approx 0{,}4 \frac{v_d}{v_{Te}}\frac{T_e}{T_i}\omega_{He}(1 + k^2\lambda_D^2)^{-2}, \tag{8}$$

where $k\lambda_D = n\xi$, and $\xi \equiv (\omega_{He}/\omega_{pe})(v_{Te}/v_D)$. Obviously, the growth increment (8) decreases with increasing n.

The frequency shift is

$$\text{fr} \quad \operatorname{Re}\Delta\omega \sim \gamma_m (1 + k^2\lambda_D^2)\frac{T_i}{T_e} \gtrsim \gamma_m.$$

Using the same estimate for $\operatorname{Re}\omega$, we find that the condition $\Delta\omega \ll k v_{Ti}$ for the applicability of expressions (6)–(8) is satisfied at $T_i \gtrsim T_e$ if $v_d \ll v_0$. Expression (8) can be used in order of magnitude also when $v_d \lesssim v_0$.

If $T_e \gg T_i$, the growth increment (7) can increase abruptly under resonance conditions, when

$$\psi(z_i) + (T_i/T_e)(1 + k^2\lambda_D^2) = 0.$$

In this case $z_i \gg 1$, $\psi(z_i) \approx -1/2\, z_i^2$, and the difference $n\omega_H - \mathbf{k}\cdot\mathbf{v}_d$ coincides with the ion-sound frequency ω_s:

$$n\omega_{He} - \mathbf{k}\mathbf{v}_d = -\omega_s = -k v_s / (1 + k^2\lambda_D^2)^{1/2}, \tag{9}$$

where $v_s = (T_e/m_i)^{1/2}$. Consequently, the considered resonance corresponds to values of $\mathbf{k}$ at which the branches of the cyclotron and ion-acoustic oscillations cross. The growth increment (7) for such a resonance is equal to

$$\gamma \approx -\frac{1}{\pi}\frac{v_d}{v_{Te}}\frac{T_i}{T_e}\frac{1}{z_i} e^{z_i^2}\omega_{He}, \tag{10}$$

where $z_i^2 = (T_e/2T_i)(1 + k^2\lambda_D^2)^{-1}$.

In the case of the resonance (9), only a small group of ions, located in the tail of the Maxwellian distribution, takes part in the buildup of the oscillations. We note that formula (10) is valid only when $\gamma \ll k v_{Ti}$. This inequality takes the form

$$v_d^2 \ll v_1^2 \equiv n v_{Te} v_{Ti}(T_e/T_i)\,|z_i|\, e^{-z_i^2}, \quad |z_i| \gg 1. \tag{11}$$

We note that the inequality (11) can be satisfied when $z_i \gg 1$ only in a narrow region of the parameters, since v_d enters in the right-hand side via z_i.

It must be emphasized that since the frequency shift $\operatorname{Re}\Delta\omega$ vanishes in the case of the resonance (9) and $\operatorname{Re}\Delta\omega > \gamma$ at $T_i > T_e$, it follows that the applicability criteria $v_d \ll v_0$ and $v_d \ll v_1$ do not coincide when $T_e \sim T_i$. The "joining" of expressions (8) and (10) for the increments and of expressions (5) and (11) for the characteristic velocities is possible only when $k^2\lambda_D^2 \lesssim 1$ and $n \sim 1$.

B. Hydrodynamic Instability

If the drift velocity is high, then coherent excitation of electron-cyclotron oscillations by the ion beam is possible, such that $\gamma \gg k v_{Ti}$. In this case $z_i \gg 1$ and the dispersion equation (4) takes the form

$$1 + \frac{2\omega_{pe}^2}{k^2 v_{Te}^2}\left(1 - \sum_{n=-\infty}^{\infty}\frac{\omega A_n}{\omega - n\omega_{He}}\right) - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\mathbf{v}_d)^2} = 0. \tag{12}$$

The oscillations most strongly excited are those in the resonant region, where the branch of the cyclotron oscillations with frequency

$$\omega(k) = n\omega_{He}[1 + A_n / (1 + k^2\lambda_D^2)] \tag{13}$$

intersects the "beam" branch with frequency $\omega = \mathbf{k}\cdot\mathbf{v}_d$. In this case [9]

$$\gamma = \sqrt{3}\left(\frac{1}{16\sqrt{\pi}}\frac{m_e}{m_i}\frac{v_{Te}}{v_d}\right)^{1/3}\left(\frac{n}{1 + k^2\lambda_D^2}\right)^{2/3}\omega_{He}. \tag{14}$$

Expression (14) was obtained from (12) under the condition

$$v_d^2 \gg v_2^2 \equiv n v_{Te} v_s (1 + k^2\lambda_D^2)^{1/2}. \tag{15}$$

In addition, it is necessary to stipulate $\gamma \gg k v_{Ti}$, which leads to the condition

$$v_d^2 \gg v_3^2 \equiv n v_{Te} v_{Ti}(T_i/T_e)(1 + k^2\lambda_D^2)^2. \tag{16}$$

If $T_i \gtrsim T_e$, then the condition (16) is more stringent. On the other hand, if $T_e > T_i$, then condition (15) is more stringent if $T_e/T_i > 1 + k^2\lambda_D^2$, and (16) is more stringent if $T_e/T_i < 1 + k^2\lambda_D^2$ (this is obviously possible only if $k^2\lambda_D^2 > 1$).

If $\xi = k\lambda_D/n > 1$, then the growth increment (14) decreases like $n^{-2/3}$ with increasing n. If $\xi < 1$, then the increment increases with increasing number like $n^{2/3}$, reaches a maximum value $n = n^* \sim 1/\xi$ at $n < 1/\xi$, and then decreases like $n^{-2/3}$ when $n > 1/\xi$. It should be noted that expressions (8) and (14) can be joined at $v_d^2 \sim v_{Te}v_{Ti}$ only if $\xi < 1$ and $n \sim 1$.

If an equality opposite to (15) holds, then (12) yields, when the resonance condition (9) is satisfied, the following expression for the increment[9,10,13,14]:

$$\gamma = \left(\frac{1}{8\pi}\frac{m_e}{m_i}\right)^{1/4}\frac{n^{1/2}\omega_{He}}{(1+k^2\lambda_D^2)^{3/4}} \tag{17}$$

(here $\mathrm{Re}\,\omega = n\omega_{He}$). If $\xi > 1$, then the increment (17) decreases like $1/n$ with increasing n. If $\xi < 1$, then the increment increases like $n^{1/2}$ when $n < 1/\xi$, reaches a minimum at $n \sim n^*$, and decreases like $1/n$ at $n > 1/\xi$. In addition to the condition $v_d < v_2$, it is necessary also to stipulate $|\omega - \mathbf{k}\cdot\mathbf{v}_d| \sim \omega_s \gg kv_{Ti}$, i.e., $T_e > 2T_i(1 + k^2\lambda_D^2)$.

In concluding this section we describe the characteristic behavior of the increments as functions of the drift velocity. The dependence of the growth increment on the drift velocity v_d at $T_i > T_e$ and $\xi < 1$ is shown in Fig. 1. The growth increment in the region

$$1 \leqslant v_d/(v_{Te}v_{Ti})^{1/2} \lesssim (T_i/T_e)^{1/2}$$

is shown dashed in Fig. 1. In this region, the value of γ can be obtained by numerically solving the dispersion equation (4).

The dependence of the growth increments on the drift velocity v_d at $T_e \gg T_i$ and $\xi < 1$ in the hydrodynamic region is shown in Fig. 2. We note that the condition $\xi < 1$ signifies $v_d > v_{Te}(\omega_{He}/\omega_{pe})$. If $\xi > 1$, then, according to formulas (7), (8), (10), (14), and (17), the growth increments decrease sharply in absolute magnitude. The dashed curve in Fig. 2 shows the dependence of γ on v_d in the region of small v_d, where $\xi > 1$ $(\gamma \sim v_d^{3/2})$.

2. CLASSICAL COLLISIONS

We consider the linearized kinetic equation for the correction to the electron distribution function, with the collision integral in the Landau form[22]

$$-i(\omega - kv_\perp\cos\theta)f - \omega_{He}\frac{\partial f}{\partial\vartheta} + \frac{e}{m_e}E\frac{\partial f_0}{\partial v_x} = \mathrm{St}\,f, \tag{18}$$

where the linearized Fokker-Planck collision term can be written in the form

$$\mathrm{St} = \frac{3}{16}\nu_{ei}\left\{\left(b + \frac{2\sqrt{\pi}}{x}\right)\frac{1}{x_\perp^2}\frac{\partial^2}{\partial\theta^2} + \left[b + \frac{2\sqrt{\pi}}{x} + \frac{x_\perp^2}{x^2}\left(a - \frac{2\sqrt{\pi}}{x}\right)\right]\frac{\partial^2}{\partial x_\perp^2}\right\}; \tag{19}$$

Here $\nu_{ei} = 4\sqrt{2}\pi n_0 e^4 L/3m_e^{1/2}T_e^{3/2}$ is the frequency of the electron-ion collisions

$$a = -\frac{2}{x^2}\left[(2x^2 - 3)\frac{1}{x}\Phi(x) + 3e^{-x^2}\right],$$

$$b = \frac{2}{x^2}\left[(2x^2 - 1)\frac{1}{x}\Phi(x) + e^{-x^2}\right].$$

$$\Phi(x) = \int_0^x e^{-t^2}dt, \quad x = \frac{v}{v_{Te}}, \quad x_\perp = \frac{v_\perp}{v_{Te}}.$$

We seek the solution in the form of an expansion in two parameters: the amplitude of the oscillations and the collision frequency (the latter is assumed small in comparison with the difference between the natural oscillation frequencies and the cyclotron frequency):

$$f = f_M + f_{1,0} + f_{1,1}, \tag{20}$$

where the distribution is assumed Maxwellian in the zeroth approximation in the oscillation amplitude and in the collision frequency:

$$f_M = \pi^{-3/2}v_{Te}^{-3}\exp\left[-v^2/v_{Te}^2\right]. \tag{21}$$

The correction $f_{1,0}$ to the distribution function in a collisionless plasma is linear in the oscillation amplitude and is well known from the theory of linear plasma oscillations:

$$f_{1,0} = i\frac{2ekv_\perp\varphi}{m_e\omega_{He}v_{Te}^2}f_M\exp\left\{-i\frac{\omega}{\omega_{He}}\theta + ik\rho_\perp\sin\theta\int^{\theta}\cos\theta'\exp\left[i\frac{\omega}{\omega_{He}}\theta' - ik\rho_\perp\sin\theta'\right]d\theta'\right\},$$
$$\rho_\perp = v_\perp/\omega_{He}, \quad \mathbf{E} = -\nabla\varphi. \tag{22}$$

Finally, for the function $f_{1,1}$ we obtain the equation

$$-i(\omega - kv_\perp\cos\theta)f_{1,1} - \omega_{He}\frac{\partial f_{1,1}}{\partial\theta} = \mathrm{St}\,f_{1,0}. \tag{23}$$

Calculating the perturbations of the charge density and substituting in the Poisson equation, we obtain an additional term in the dispersion equation (1):

$$\delta\varepsilon_e' = i\cdot 8.8\nu_{eff}\frac{\omega_{pe}^2}{k^2v_{Te}^2}\frac{\omega k\rho_e}{\sqrt{\pi}(\omega - n\omega_{He})^2}, \quad \nu_{eff} = 0.51\nu_{ei}. \tag{24}$$

Let us illustrate the influence of the collisions on the stability of the plasma in the limiting case $v_d^2 < v_0^2$, when the role of the collisions is particularly clear. For the imaginary part of the frequency we obtain in this case in place of (8) the increment

$$\gamma_{max} \approx 0.4\omega_{He}\frac{v_d}{v_{Te}}\frac{T_e}{T_i}(1 + k^2\lambda_D^2)^{-2} - 4.4\nu_{eff}\left(\frac{v_{Te}}{v_d}\right)^2 n^2, \tag{25}$$

which contains the large factor $(k\rho_e)^2$. It results from the fact that the oscillating part of the distribution function contains a factor in the form $\exp(ik_\perp v_\perp/\omega_{He})$. Then, when the collision operator acts on the distribution function, the principal term is the one with the derivative of the phase shift.

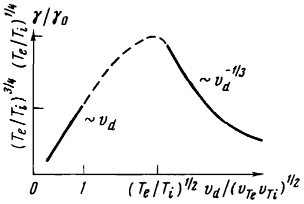

FIG. 1

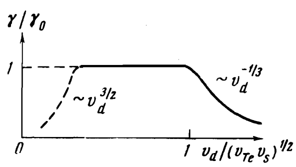

FIG. 2

FIG. 1. Growth increment (formulas (8) and (14)) vs drift velocity at $T_i > T_e$ and $\xi < 1$; $\gamma_0 \sim (m_e/m_i)^{1/4}\omega_{He}$.

FIG. 2. Growth increment (formulas (17) and (14) vs drift velocity at $T_e \gg T_i$ and $\xi < 1$.

3. EFFECTIVE COLLISION FREQUENCY IN A TURBULENT PLASMA

It follows from (25) that the collisions play a stabilizing role that is further enhanced by the additional factor $k^2\rho_e^2$. The idea of the approach to the problem of nonlinear saturation of an instability of this type lies in the analogy between the role of the ordinary pair collisions and scattering of particles by oscillations that appear in an unstable plasma[20]. As the instability develops, an anomalous plasma resistance appears, and consequently also an anomalous collision frequency. The conductivity is connected with the collision frequency by the relation

$$\sigma = ne^2 / m\nu_{eff}. \tag{26}$$

The turbulent frequency ν_{eff} of the collisions can be estimated from the following considerations. As they cause the oscillations to build up, the electrons lose momentum. The average momentum lost by the electrons per unit time is

$$\dot{P} = \sum \gamma_{\bar{k}} W_k \frac{k}{\omega}, \qquad W_k = \frac{k^2|\varphi_k|^2}{8\pi}\frac{\partial\varepsilon(\omega)\omega}{\partial\omega}. \tag{27}$$

With the aid of (27) we can determine ν_{eff} in the following manner:

$$m_e n_0 \nu_{eff} v_d = \sum \gamma_{\bar{k}} W_k \frac{k}{\omega}, \tag{28}$$

i.e.,

$$\nu_{eff} = \frac{1}{m_e n_0 v_d}\sum \gamma_{\bar{k}} W_k \frac{k}{\omega}. \tag{29}$$

When this effective collision frequency is taken into account in the dispersion equation of the linear theory of stability, we obtain an additional term in the form (24). The effective frequency ν_{eff} enters with the "Pitaevskiĭ factor," since the reaction of the oscillations on the plasma distribution should be described by the quasilinear diffusion equation in velocity space. In such an approach, the entire influence of the nonlinear effects is reduced by us to an introduction of a certain effective collision frequency that leads ultimately to stabilization of the plasma.

A. Plasma with Hot Ions (Kinetic Instability)

If $v_d < v_0$, then the instability is connected with the buildup of electron-cyclotron oscillations by the resonant ions. We have calculated above the increment of the linear instability with allowance for pair collisions (see Eq. (25)). We now use the already noted analogy between the role of ordinary pair collisions and scattering by the appearing field fluctuations. We can then say that the instability development leads to an effective collision frequency sufficient to stabilize the instability, i.e., (see[20])

$$\nu_{eff} \approx 0{,}1\left(\frac{v_d}{v_{Te}}\right)^3 \omega_{He}\frac{T_e}{T_i}\left[1+\left(\frac{\omega_{He}}{\omega_{pe}}\right)^2\left(\frac{v_{Te}}{v_d}\right)^2\right]^{-2}. \tag{30}$$

The harmonics $\omega = n\omega_{He}$ become stabilized when ν_{eff} takes on a value of the order

$$\nu_{eff} \sim \frac{1}{n^2}\left(\frac{v_d}{v_{Te}}\right)^3\frac{T_e}{T_i}\omega_{He},$$

i.e., the higher-order harmonics are stabilized first. The most "dangerous" modes are those with $k = n\omega_{He}/v_d$ and $n = 1$. When ν_{eff} reaches the limiting value (30), the oscillations at all modes attain saturation.

The connection obtained above between ν_{eff} and the turbulence level enables us to estimate the amplitude of the turbulent pulsations produced in the kinetic-instability regime:

$$W / n_0 T_e \sim (v_d / v_{Te})^4, \tag{31}$$

where W is the energy density of the plasma oscillations per unit volume. From this we can also obtain an expression for the energy density of the electric field of the oscillations

$$\frac{W_E}{n_0 T_e} \sim \left(\frac{v_d}{v_{Te}}\right)^3\left(\frac{\omega_{He}}{\omega_{pe}}\right)^2, \quad W_E = \frac{1}{16}\sum_k k^2|\varphi_k|^2. \tag{32}$$

B. Hydrodynamic Instability (Large Drift Velocity)

If the current velocity exceeds the value v_3 (see (16)), then hydrodynamic instability of the two-stream type sets in at $T_i > T_e$. The frequency and the linear growth increment are determined in this case by the expressions (13) and (14).

Nonlinear stabilization of the instability, as before, is due to the appearance of effective collisions. Unlike the kinetic instability, however, collisions with frequencies $k^2\rho_e^2\nu_{eff}$ of the order of the linear increment are incapable of completely stabilizing the instability, and can only decrease the increment. With the aid of an analysis similar to that in Sec. 2, we can find the instability increment during the nonlinear stage.

Using expression (24), we obtain from the dispersion equation at

$$\omega - n\omega_{He} = n\omega_{He}A_n / (1 + k^2\lambda_D^2) \gg k^2\rho_e^2\nu_{eff} > \gamma$$

the following expression for the increment:

$$\gamma = \gamma_L[4\gamma_L / 3\sqrt{3}\nu_{eff}k^2\rho_e^2]^{1/3}, \tag{33}$$

where γ_L is the linear increment defined by (14).

When the increment is decreased with increasing field fluctuations (and this growth continues, albeit at a slower rate), the ions can enter into resonance with the oscillations if $\gamma < kv_{Ti}$. The latter condition constitutes the condition for the stabilization of the oscillations and enables us to estimate the effective collision frequency at which this stabilization sets in. It turns out that the expression for the effective collision frequency literally coincides with that calculated for the case of the kinetic instability (see Eq. (30)). It must be emphasized here that at such a collision frequency we still have not gone beyond the limits of applicability of Eq. (33) (i.e., $k^2\rho_e^2\nu_{eff} < (\omega - n\omega_{He})$). Just as in the case of the kinetic instability, in the present case the modes with the large values of n are the first to reach saturation.

C. Hydrodynamic Instability (Low Drift Velocity)

In the case when $T_e \gg T_i$, the hydrodynamic instability develops with an even larger increment if the drift velocity is less than v_2. The presence of collisions, as will be shown, does not stabilize the instability, and only decreases the increment. Therefore the instability continues to grow also when $k^2\rho_e^2\nu_{eff} > \gamma_L$. In other words, the characteristic time of particle scattering by the turbulent pulsations is much shorter than the oscillation frequency $(\omega - n\omega_{He})$.

We can attempt to construct a theory of nonlinear stability by taking this fact into account from the very beginning. To this end, the kinetic equation for the particles must be integrated not along the perturbed trajectory in the magnetic field, but by taking into account the scattering by the turbulent pulsations. For cyclotron oscillations of an anisotropic plasma this was done in[21] and the following dispersion equation was obtained:

$$1 - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\mathbf{v}_d)^2} + \frac{\omega_{pe}^2}{k^2}\sum_{n=-\infty}^{\infty}\int dv_\perp \frac{\partial f_{0e}}{\partial v_\perp}\frac{J_n^2(k_\perp v_\perp/\omega_{He})}{\delta\omega_{\bar{k}} + ik^2D}$$
$$\times\left[n\omega_{He} + ik^2D\left(\sum_{p,q} D_q \frac{p\omega_{He}}{\delta\omega_q + i(\gamma_q + q^2D)}\right)\Big/\left(\sum_{p,q} D_q\frac{\delta\omega_q + i\gamma_q}{\delta\omega_q + i(\gamma_q + q^2D)}\right)\right] = 0,$$
$$D_q = \frac{c^2}{H^2}|\varphi_q|^2 J_p^2\left(\frac{q_\perp v_\perp}{\omega_{He}}\right)\Big/D, \quad D = \sum_q D_q, \tag{34}$$

where $\delta\omega_{\bar{k}} = \omega_k^R - n\omega_{He}$, and J_n is a Bessel function.

If $k^2D \gg \gamma_k$, then

$$\varepsilon(\omega, k, |\varphi_k|^2) = 1 - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\mathbf{v}_d)^2} + \frac{\omega_{pe}^2}{k^2}\Big[f_M(0) + \int \frac{\omega_{\bar{k}}(\delta\omega_{\bar{k}} - i\gamma_k)}{\gamma_k k^2 D + \delta\omega_{\bar{k}}^2}\frac{\partial f_M}{\partial v_\perp}J_n^2 dv_\perp\Big] = 0. \tag{35}$$

We see from this equation that the instability increment decreases in inverse proportion to the oscillation amplitude

$$\gamma = \mp\left(\frac{m_e}{m_i}\right)^{1/2}\frac{\sqrt{2}}{(1 + k^2\lambda_D^2)^{3/2}}\frac{\Gamma(3/4)\, n\omega_{He}^2}{\pi^2 k^2\bar{D}} \quad (\delta\omega_k = 0). \tag{36}$$

The nonlinear instability increment can be expressed in terms of the linear one as follows:

$$\gamma = 4\frac{\Gamma(3/4)}{\pi^{3/2}}\frac{1}{k^2\bar{D}}(\gamma_L)^2, \quad \bar{D} = \frac{c}{H}\left[\frac{2}{\pi}\frac{1}{k\rho_e}\sum_k |\varphi_k|^2\right]^{1/2}, \tag{37}$$

where γ_L is determined by Eq. (17).

The meaning of relation (37) can be explained in the following manner. The cyclotron resonance of the electrons with the wave is destroyed after a time on the order of the time required for the particles to diffuse a distance on the order of the wavelength. Therefore the time of effective interaction with the wave decreases with increasing amplitude, and consequently the work of the particles in the field of the wave also decreases (it is this work which determines the rate of development of the instability, i.e., the increment). The instability increment thus depends on the amplitude, and the growth of the noise with time is not exponential proportional to a certain power of the time.

We shall show now that allowance for the diffusion trajectories of the particles in turbulent fields is equivalent, within the framework of the instability theory developed by us, to introducing a certain effective collision frequency into the linear instability theory. Let us explain how the effective collision frequency is connected with the diffusion coefficient. Just as before, the loss of electron momentum due to the emission of the oscillations can be written in the form (27), where the oscillation energy is expressed in terms of the derivative of the dielectric constant defined by (35):

$$W_k = \frac{\partial\varepsilon(\omega, k, |\varphi_k|^2)}{\partial\omega}\frac{k^2|\varphi_k|^2}{8\pi}. \tag{38}$$

Expressing the momentum loss in terms of the effective collision frequency, we obtain for the latter the expression

$$\nu_{eff} = \frac{m_e}{T_e}\omega_{He}^2\frac{\Gamma(3/4)}{\pi}D. \tag{39}$$

We see that the diffusion coefficient, as expected, is connected with the collision frequency in classical fashion:

$$D = \frac{\pi}{2}\frac{1}{\Gamma(3/4)}\nu_{eff}\rho_e^2, \tag{40}$$

and consequently a formula for the order of magnitude of the increment can be obtained also by introducing the effective collision frequency, as was done earlier.

In the entire analysis of the case of hot electrons, we did not take into consideration the known ion-sound instability of a plasma without a magnetic field. The approximation of a weak magnetic field is valid for oscillations with wavelength on the order of the Debye radius, propagating not strictly perpendicular to the magnetic field. It must be borne in mind, however, that it has a much larger increment $\gamma \sim (v_d/v_{Te})\omega_{pi}$, if

$$\frac{v_d}{v_{Te}} \geqslant \frac{\omega_{He}}{\omega_{pi}}\left(\frac{m_e}{m_i}\right)^{1/4}\sigma^{1/4}, \tag{41}$$

where $\sigma = \max(1, \xi)$. In this case we can neglect the instability on the Bernstein modes. In addition, the weakening of the instability during the nonlinear stage also makes it important to take into account the anomalous resistance as the result of the development of ion-sound instability.

CONCLUSION AND DEDUCTIONS

The results of the calculations are conveniently represented in the form of plots of the instability increments for the harmonic n = 1 against the effective collision frequency. These plots have the simplest form in the case of kinetic instability. The instability saturates at the value of ν_{eff} given by (30) (see Fig. 3).

In the case of hydrodynamic instability at large drift velocity, there is a region of collision frequencies where the instability becomes partially weakened, $\gamma \sim (1/\nu_{eff})^{1/2}$, and final stabilization occurs at the same value of ν_{eff} as in the case of kinetic instability (see Fig. 4).

Finally, in a plasma with hot electrons, at a low drift velocity, the anomalous resistance is due to development

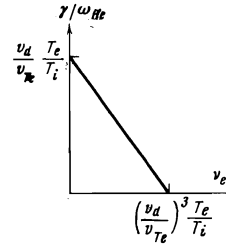

FIG. 3

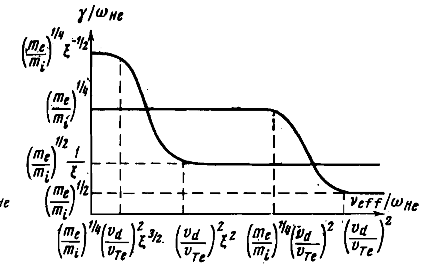

FIG. 4

FIG. 3. Growth increment vs effective frequency in the case of kinetic instability at $T_i \gtrsim T_e$ and $\xi < 1$.

FIG. 4. Growth increment vs effective frequency in the case of hydrodynamic instability at $v_d > v_3, v_2$, $\xi < 1$, and $T_i \gtrsim T_e$.

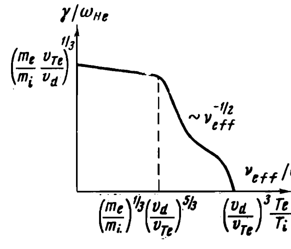

FIG. 5

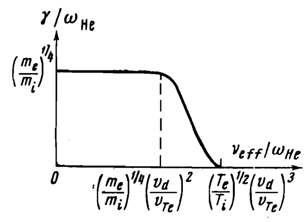

FIG. 5

FIG. 5. Growth increment vs effective frequency in the case of hydrodynamic instability at $T_e \gg T_i$ and $v_4 < v_d < v_2$.

FIG. 6. Growth increment vs effective frequency in the case of hydrodynamic instability at $T_e \gg T_i$ and $v_d < v_2, v_4$.

of hydrodynamic instability, and goes over in the limit $k^2\rho_e^2\nu_{eff} > \omega_{He}$ into the known ion-sound instability.

In the presence of collisions, the modes $n = 1$, $k = \omega_{He}/v_d$ are more difficult to stabilize (see Fig. 5), but in the limit of developed turbulence, when $\nu_{eff} \gg \omega_{He}(v_d/v_{Te})^2$, these modes have a small instability increment. A more important role in the production of the anomalous resistance is therefore played by short-wave oscillations[2)] with $k = \lambda_D^{-1}$, $n^* = (\omega_{pe}/\omega_{He})v_d/v_{Te}$. For these oscillations, the transition from the regime of Bernstein modes into ion-sound oscillations take place within a shorter time, since the transition occurs with a smaller value of ν_{eff}, and consequently with a lower turbulence level.

It should be noted, however, that the picture in Fig. 5 is valid only if the smallest nonlinear increment $\gamma \sim (m_e/m_i)^{1/2}\omega_{He}$ is larger than kv_{Ti}. In the opposite case, the oscillations become stabilized when the nonlinear growth increment (36) becomes of the order of kv_{Ti} (see Fig. 6). This occurs when $\nu_{eff} \sim (T_e/T_i)^{1/2}(v_d/v_{Te})^3\omega_{He}$. The picture shown in Fig. 6 takes place if $v_d < v_4$, where $v_4 \equiv v_{Te}(T_i/T_e)^{1/2} \times (1 + k^2\lambda_D^2)^{3/2}$.

The author is deeply grateful to R. Z. Sagdeev, A. A. Galeev, and K. N. Stepanov for numerous discussions of the considered questions and for valuable advice during the course of writing the paper, and also to A. D. Pataraya for a discussion.

[2)]If $v_d/v_{Te} < \omega_{He}/\omega_{pe}$, then the buildup of the resonant electron-cyclotron oscillations ($\bar{k} = n\omega_H/v_d$), which we consider here, becomes much weaker because $\bar{k}\lambda_D \gg 1$.

[1]R. Z. Sagdeev, Proc. Symp. Appl. Math., N.Y., 1965; AMS Provid., **18**, 281, 1967.

[2]Plasma Phys. and Contr. Nucl. Fus. Research Conf. Proceed. Novosibirsk, 1–7 August 1968, IAEA, Vienna, 1969, v. 1.

[3]M. Keilhacker and K. H. Steuer, Phys. Rev. Lett. **26**, 694, 1971.

[4]K. N. Stepanov, Zh. Tekh. Fiz. **34**, 2146 (1964) [Sov. Phys.-Tech. Phys. **9**, 1653 (1965)].

[5]M. V. Babykin, P. P. Gavrin, E. K. Zavoĭskiĭ, L. I. Rudakov, V. A. Skoryupin, and G. V. Sholin, Zh. Eksp. Teor. Fiz. **45**, 511 (1964) [Sov. Phys.-JETP **18**, 351 (1965)].

[6]V. L. Sizonenko and K. N. Stepanov, Nuclear Fusion **7**, 131 (1967).

[7]V. I. Aref'ev, I. A. Kovan, and L. I. Rudakov, ZhETF Pis. Red. **8**, 286 (1968) [JETP Lett. **8**, 176 (1968)].

[8]L. I. Grigor'eva, A. V. Longinov, A. I. Pyatak, V. L. Sizonenko, B. I. Smerdov, K. N. Stepanov, and V. V. Chechkin, Plasma Phys. and Contr. Nucl. Fus. Research Conf., Madison, 17–23 June 1971, IAEA, Vienna, v. III, 1971, p. 573.

[9]V. I. Kurilko and V. I. Miroshnichenko, in: Fizika plazmy i problemy upravlyaemogo termoyadernogo sinteza (Plasma Physics and Problems of Controlled Nuclear Fusion), No. 3, Ukr. Acad. Sci., Kiev, 1963, p. 161.

[10]H. V. Wong, Phys. Fl., **13**, 757, 1970.

[11]C. N. Lashmore-Davies, J. Physics, **A3**, L40, 1970; Phys. Fl., **14**, 1481, 1971.

[12]D. W. Forslund, R. L. Morse and C. W. Nielson, Phys. Rev. Lett. **25**, 1266, 1970; **27**, 1424, 1971.

[13]S. P. Gary and J. J. Sanderson, J. Plasma Phys. **4**, 739, 1970; S. P. Gary, J. Plasma Phys. **4**, 753, 1970; **6**, 561, 1971.

[14]M. Lampe, J. B. McBride, J. H. Orens and R. N. Sudan, Phys. Lett. **35A**, 131, 1971.

[15]M. Lampe, W. M. Manheimer, J. B. McBride, J. H. Orens, R. Shanny and R. N. Sudan, Phys. Rev. Lett. **26**, 1221, 1971.

[16]S. P. Gary and D. Biskamp, J. Phys. **A4**, 27, 1971.

[17]J. J. Sanderson and E. R. Priest, J. Phys. **A4**, L65, 1971.

[18]M. Lampe, W. M. Manheimer, J. B. McBride, J. H. Orens, K. Papudopoulos, R. Shanny and R. N. Sudan, Phys. Fl., **15**, 662, 1972.

[19]B. B. Kadomtsev, Voprosy teorii plazmy (Problems of Plasma Theory), No. 4, Atomizdat, 1964, p. 188.

[20]A. A. Galeev, D. G. Lominadze, A. D. Pataraya, R. Z. Sagdeev, and K. N. Stepanov, ZhETF Pis. Red. 15, 417 (1972) [JETP Lett. 15, 294 (1972)].

[21]A. A. Galeev, Zh. Eksp. Teor. Fiz. **57**, 1361 (1969) [Sov. Phys.-JETP **30**, 737 (1970)].

[22]L. D. Landau, Zh. Eksp. Teor. Fiz. **7**, 103 (1937).

[23]L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. **44**, 969 (1963) [Sov. Phys.-JETP **17**, 658 (1963)].

Translated by J. G. Adashko
138