SECOND-HARMONIC GENERATION IN AN INHOMOGENEOUS LASER PLASMA

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Second harmonic coherent radiation emitted following incidence of an electromagnetic wave on a layer of inhomogeneous plasma with a density exceeding the critical value is considered. It is assumed that the plasma temperature is so high that Coulomb collisions are negligible and the structure of the electromagnetic field near the critical point at the fundamental frequency is due to thermal motion of the electrons. Such conditions are encountered in a laser plasma with an electron temperature $T_e \gtrsim 1$ keV and a characteristic inhomogeneity dimension 10^{-2}–10^{-3} cm. In this case the coefficient for transformation of laser radiation into the second harmonic may reach 1% for optimal angles of incidence and for a power of $\sim 10^{13}$ W/cm^2.

1. INTRODUCTION

NONLINEAR interactions of electromagnetic waves with a plasma have attracted renewed attention in connection with the use of powerful lasers for plasma heating[1]. These processes are the cause of the anomalous absorption of laser radiation[2] and the deformation of the spectrum of the radiation reflected from the plasma. Anomalous absorption in a laser plasma has not yet been observed directly in experiment.[1)] On the other hand, the reflected radiation spectrum has been studied in considerable detail. In particular, deformation of the spectra near the laser frequency [4-7] and second harmonic generation[8,9] were observed in a number of studies. The present paper is devoted to a theoretical determination of the coefficient of conversion of a plane electromagnetic wave into the second harmonic in an inhomogeneous plasma. Measurement of this quantity yields information on the electric field intensity, and also on the temperature and the electron density gradient in a laser plasma. The most important thing proves to be the fact that the characteristic dimension of the flare formed in the laser focus is much larger than the wavelength, i.e., the plasma we are discussing is weakly inhomogeneous.

In an homogeneous isotropic plasma, the process $t + t \to t$ of coalescence of two (nonpotential) transverse waves t with frequencies ω' and ω'' and wave vectors $\mathbf{k}'$ and $\mathbf{k}''$ into a transverse electromagnetic wave t with frequency ω and wave vector $\mathbf{k}$ is forbidden [10], since it is impossible to satisfy the synchronization conditions (the decay conditions).

$$\omega' + \omega'' = \omega, \quad \mathbf{k}' + \mathbf{k}'' = \mathbf{k}. \tag{1}$$

The generation of a transverse electromagnetic wave with the doubled frequency $\omega = 2\omega_0$ ($\omega' = \omega'' \equiv \omega_0$) in such a plasma is possibly only if at least one of the combining waves is a longitudinal (potential) oscillation l with a frequency ω_0 equal to the frequency of the transverse wave. The processes which are then possible, of coalescence of a transverse wave t with an electron Langmuir oscillation l to form a transverse wave t, or coalescence of two Langmuir oscillations l into a transverse wave t, can be written in the form

$$t + l \to t, \quad l + l \to t. \tag{2}$$

In a weakly inhomogeneous plasma the wave vector still depends on the coordinates, so that the synchronization condition $\mathbf{k} = \mathbf{k}' + \mathbf{k}''$ is satisfied only for certain points in space. If there are no such points on the electron density profile, then the probability of the effect is exponentially small. The second-harmonic energy flux is proportional to the energy fluxes of the coalescing waves and is significant only when the intensity levels of the electron Langmuir oscillations and the transverse waves are high enough. Thus, in order to generate the second harmonic effectively it is necessary, first, that a powerful longitudinal wave be formed and, second, that there exist a point on the electron density profile where the synchronization condition (1) is fulfilled for one of the processes (2).

It must be emphasized that the combination (2) has been discussed frequently and in great detail in the theory of homogeneous, weakly turbulent plasmas (see, for example,[10-12]). As a rule, however, in doing so the question of the mechanism producing the powerful Langmuir oscillation was completely ignored. We shall discuss jointly both of these effects, viz., the generation of an electron Langmuir oscillation of great amplitude and the subsequent formation of the second harmonic of the laser radiation incident on the plasma. This is the approach that seems to us most adequate for the experimental situation that has presently developed in studies aimed at heating laser plasma to thermonuclear temperatures. Powerful light fluxes in the plasma are provided by the laser source. Appreciable longitudinal fields arise in the plasma through the linear transformation of light into electron Langmuir oscillations[13]. The effectiveness of such a transformation in an inhomogeneous plasma when the incident wave frequency ω_0 approaches the electron Langmuir frequency ω_{Le} has been demonstrated both theoretically[14] and experimentally[15] in the case of the interaction with a plasma of electromagnetic waves in the microwave hand.

Another cause of generation of intense electron Langmuir oscillation is the parametric buildup of

[1)]Observation of anomalous absorption of electromagnetic waves in the microwave band was first described by Gekker and Sizukhin [3].

plasma oscillations in the presence of a powerful electromagnetic wave [2,16]. The mechanism of parametric buildup consists, for example, in decay of an incident electromagnetic wave into electron Langmuir oscillations and ion-sound oscillations (i.e., a nonlinear transformation). There the second harmonic may arise as a result of the second coalescence process in (2). According to the results of [2] the longitudinal wave amplitude is at a maximum along the electric field intensity vector of the radiation incident on the plasma. For this reason the second harmonic radiation of a laser plasma due to parametric resonance should be directed (for $l + l \to t$ coalescence) perpendicular to the incident light beam.

In this paper we confine ourselves to the case of a linear transformation, assuming that the laser radiation intensity is lower than the threshold value necessary for the development of parametric instability. As we shall see below, in linear transformation the second harmonic is generated along the light beam, and can thus be distinguished from the second harmonic due to parametric resonance.

2. INITIAL RELATIONS

Let p be a polarized plane electromagnetic wave of frequency ω_0, obliquely incident from vacuum or an inhomogeneous plasma layer. We choose the z axis in the direction of the electron-density inhomogeneity of the plasma $N_e(z)$, while the xz plane coincides with the plane of polarization of the wave. We assume that at the origin of this coordinate system ($z = 0$) the density equals the critical density: $N_0 = m\omega_0^2/4\pi e^2$ (e is the electron charge and m is the electron mass), and positive values of z correspond to densities higher than N_0. As we know from linear theory, in the absence of Coulomb collisions and of thermal motion of the plasma electrons, the component $E_z(\omega_0, z)$ of the electric field of the wave along the inhomogeneity is infinitely large at the critical point $z = 0$ and has a singularity of the pole type[17]. If one of these factors is taken into account, the electric field at the critical point proves to be finite, but the way in which it varies differs greatly, depending on which of the factors mentioned plays the principal role. Physically this means that a portion of the transverse wave energy near $z = 0$ either becomes dissipated or is transformed into electron Langmuir oscillations. The total energy lost by the wave does not depend on the exact mechanism that limits the field at the critical point.

The coefficient of reflection of the electromagnetic wave from the plasma layer is determined only by the geometry of the problem and can be found in the cold-plasma approximation[19]. On the other hand, the nonlinear sources that govern the generation of the second harmonic are obviously strongly dependent on the structure of the electric field at the first harmonic. It is necessary then to describe correctly that vicinity of the critical point at which the field amplitude has a maximum. If the electron temperature T_e is small, so that $(\nu/\omega_0) \gg (\beta c/a\omega_0)^{2/3}$ (c is the speed of light, ν is the electron-ion collision frequency $\beta \equiv 3^{1/2} v_{Te}/c$, $v_{Te} \equiv (\kappa T_e/m)^{1/2}$ is the thermal velocity of the electrons, κ is Boltzmann's constant, and a is the characteristic dimension of the electron density inhomogeneity), then the field first-harmonic structure is determined by the Coulomb collisions. Generation of the second harmonic in this case was considered by Erokhin et al.[18]

In this paper we consider the opposite limiting case $(\nu/\omega_0) \ll (\beta c/a\omega_0)^{2/3}$ which is encountered in a high-temperature plasma formed by focusing powerful laser pulses on solid targets[13]. It is then possible to neglect Coulomb collisions and confine oneself exclusively to the thermal motion of the particles. The equations for the fundamental frequency of the field take the form[14]:

$$\beta^2 \frac{\omega_{Le}^2}{\omega_0^2} \frac{d^2 E_z(\omega_0, z)}{dz^2} + \left(\frac{\omega_0}{c}\right)^2 \varepsilon(\omega_0) E_z(\omega_0, z) = -\left(\frac{\omega_0}{c}\right)^2 B_y(\omega_0, z) \sin\theta, \tag{3}$$

$$\frac{d^2 B_y(\omega_0, z)}{dz^2} - \frac{d \ln \varepsilon(\omega_0)}{dz} \frac{dB_y(\omega_0, z)}{dz} + \left(\frac{\omega_0}{c}\right)^2 [\varepsilon(\omega_0) - \sin^2\theta] B_y(\omega_0, z)$$
$$= \beta^2 \left[\frac{\omega_{Le}^2}{\omega_0^2} \frac{d \ln \varepsilon(\omega_0)}{dz} - \frac{1}{\omega_0^2} \frac{d\omega_{Le}^2}{dz}\right] \frac{dE_z(\omega_0, z)}{dz}, \tag{3a}$$

$$E_x(\omega_0, z) = -i \frac{c}{\omega_0} \frac{1}{\varepsilon(\omega_0)} \frac{dB_y(\omega_0, z)}{dz}. \tag{3b}$$

Here $B_y(\omega_0, z)$ is the only nonvanishing component of the magnetic field at the given polarization of the electromagnetic wave; $\epsilon(\omega_0) \equiv (1 - \omega_{Le}^2/\omega_0^2)$ is the dielectric constant at the frequency ω_0 of the cold isotropic plasma with Langmuir frequency ω_{Le}; θ is the angle of incidence, i.e., the angle between the wave vector of the incident radiation and the z axis.

For the second harmonic of the field, the thermal motion does not play a leading role. Therefore the field equations can be obtained in the cold plasma approximation ($v_{Te} = 0$), for example by starting with the hydrodynamic equations for the electrons and assuming the ions to be stationary (cf.[18]):

$$\Delta \mathbf{B}(2\omega_0) - [\operatorname{rot} \mathbf{B}(2\omega_0),\ \operatorname{grad} \ln \varepsilon(2\omega_0)] + \left(\frac{2\omega_0}{c}\right)^2 \varepsilon(2\omega_0) \mathbf{B}(2\omega_0) = \mathbf{A}, \tag{4}$$

$$\mathbf{A} \equiv -i \frac{e}{m\omega_0 c} \Big\{ i \frac{\omega_0}{c} \mathbf{B}(\omega_0) \operatorname{div} \mathbf{E}(\omega_0) - [\mathbf{E}(\omega_0), \operatorname{grad} \operatorname{div} \mathbf{E}(\omega_0)]$$
$$+ [\mathbf{E}(\omega_0), \operatorname{grad} \ln \varepsilon(2\omega_0)] \operatorname{div} \mathbf{E}(\omega_0) + [\operatorname{grad} \mathbf{E}^2(\omega_0), \operatorname{grad} \ln \varepsilon(2\omega_0)] \Big\} \tag{5}$$

Here $\epsilon(2\omega_0) = 1 - \omega_{Le}^2/4\omega_0^2$ is the dielectric constant of the plasma at the second harmonic; $\mathbf{B}(2\omega_0)$ is the magnetic field of the wave at frequency $2\omega_0$. We note that the right-hand side (5) of Eq. (4) is due not only to nonlinear effects but also to the potential character of the electric field $\mathbf{E}(\omega_0)$ at the fundamental frequency ω_0. In a homogeneous plasma the field at the fundamental frequency is completely transverse, for which reason $\mathbf{A} = 0$ and no generation of the second harmonic takes place.

The set of equations (3–5) does not take into account the effect of the second harmonic on the first. This effect must be taken into consideration at large intensities of the incident radiation. In particular, second harmonic generation itself (and not thermal motion or Coulomb collisions) may limit the field at the critical point (cf. [18]). However, in what follows we assume that the coefficient of transformation into the second harmonic is small and such effects do not play any role.

It follows from symmetry considerations that the non-zero field components at the second-harmonic frequency are the same as at the fundamental frequency, $B_x(2\omega_0) = B_z(2\omega_0) = E_y(2\omega_0) = 0$, while the magnetic field $\mathbf{B}(2\omega_0)$ depends on the x coordinate only through the multiplier $\exp(i2\omega_0 x \sin\theta/c)$, i.e., the second harmonic is "reflected" by the plasma at the mirror angle (relative to the incident radiation at the fundamental frequency ω_0). Taking this circumstance into account, we rewrite Eq. (4) in the form

$$\frac{d^2B_y(2\omega_0,z)}{dz^2}-\frac{d\ln\varepsilon(2\omega_0)}{dz}\frac{dB_y(2\omega_0,z)}{dz}+\left(\frac{2\omega_0}{c}\right)^2[\varepsilon(2\omega_0)-\sin^2\theta]B_y(2\omega_0,z)=A_y(z), \quad (6)$$

$$A_y(z)\equiv A_{t+l}+A_{l+l}+A_{t+t}; \quad (7)$$

$$A_{t+l}=-\frac{ie}{m\omega_0 c}\left\{i\frac{\omega_0}{c}B_y(\omega_0,z)\frac{dE_z(\omega_0,z)}{dz}+E_x(\omega_0,z)\frac{d^2E_z(\omega_0,z)}{dz^2}-\frac{d\ln\varepsilon(2\omega_0)}{dz}E_x(\omega_0,z)\frac{dE_z(\omega_0,z)}{dz}\right\},$$

$$A_{l+l}=-\frac{e}{mc^2}\left\{E_z(\omega_0,z)\frac{dE_z(\omega_0,z)}{dz}\sin\theta+2\frac{d\ln\varepsilon(2\omega_0)}{dz}E_z^2(\omega_0,z)\sin\theta\right\},$$

$$A_{t+t}=\frac{ie}{m\omega_0 c}\left\{2\left(\frac{\omega_0}{c}\right)^2B_y(\omega_0,z)E_x(\omega_0,z)\sin\theta+3i\frac{\omega_0}{c}\frac{d\ln\varepsilon(2\omega_0)}{dz}E_x^2(\omega_0,z)\sin\theta\right\}.$$

The partition (7) of the nonlinear source $A_y(z)$ is in strict correspondence to the three coalescence processes discussed in the Introduction. The first two terms describe the coalescence (2) of a transverse wave with an electron Langmuir oscillation, or of two Langmuir oscillations, into an electromagnetic wave of frequency $2\omega_0$. The third term determines the continuation from the coalescence of two transverse waves into one transverse wave.

We can write the definition of the coefficient of transformation of an electromagnetic wave into the second harmonic. We designate by $B_y^+(2\omega_0, z)$ and $B_y^-(2\omega_0, z)$ the two solutions of the homogeneous Eq. (6) (at $A_y = 0$), describing the free propagation of transverse waves of frequency $2\omega_0$ in directions parallel and antiparallel to the incident radiation. If the plasma density inhomogeneity is sufficiently weak, the effect of the critical point on the second harmonic ($2\omega_0 = \omega_{Le}$) is slight, and we can write for the magnetic field $B_y(2\omega_0, z)$ in Eq. (6) as the boundary conditions

$$B_y(2\omega_0,z)\sim\alpha^-B_y^-(2\omega_0,z),\quad z\to-\infty,$$
$$B_y(2\omega_0,z)\sim\alpha^+B_y^+(2\omega_0,z),\quad z\to+\infty,$$

where α^- and α^+ are the amplitudes of the reflected and the transmitted waves. Then, solving (6), we can represent the reflected and transmitted energy fluxes $S^-(2\omega_0)$ and $S^+(2\omega_0)$ at the second harmonic in the form

$$\frac{S^-(2\omega_0)}{S(\omega_0)}=\left|\frac{c}{4\omega_0}\frac{1}{B_y(\omega_0,-\infty)\cos\theta}\int_{-\infty}^{+\infty}dz\frac{B_y^+(2\omega_0,z)A_y(z)}{\varepsilon(2\omega_0)}\right|^2, \quad (8)$$

$$\frac{S^+(2\omega_0)}{S(\omega_0)}=\left|\frac{c}{4\omega_0}\frac{1}{B_y(\omega_0,-\infty)\cos\theta}\int_{-\infty}^{+\infty}dz\frac{B_y^-(2\omega_0,z)A_y(z)}{\varepsilon(2\omega_0)}\right|^2. \quad (9)$$

Here $S(\omega_0)$ is the energy flux of the incident radiation. Equations (8) and (9) indeed determine the sought coefficient for transformation (in energy terms) into the second harmonic. To calculate this coefficient it is necessary to determine the form of the field in the vicinity of the critical point.

3. STRUCTURES OF ELECTROMAGNETIC FIELDS AT THE FIRST AND SECOND HARMONICS

The electric and magnetic fields at the fundamental frequency are determined by Eqs. (3) and (3a). However, since the parameter $\beta = 3^{1/2} v_{Te}/c$, which characterizes the role of the thermal motion, is small, the latter need be taken into account only to find the field component $E_z(\omega_0, z)$ which has a pole at the point $z = 0$ when $v_{Te} = 0$. The quantities $B_y(\omega_0, z)$ and $E_x(\omega_0, z)$ can be gotten from the field equations in the cold plasma approximation, assuming $\beta = 0$ in Eq. (3a). To this end, we use the geometrical-optics approximation. As the criterion of applicability we have the condition that the wavelength of the incident radiation be small compared with the characteristic dimension a of the plasma inhomogeneity $(a\omega_0/c) \gg 1$, as in the case with the laser plasma we are considering ($a \approx 10^{-2}$ cm). The magnetic field $B_y(\omega_0, z)$ that satisfies Eq. (3a) at $\beta = 0$ is given in this approximation by[19]

$$B_y(\omega_0,z)=B_y(\omega_0,-\infty)[\varepsilon(\omega_0)\cos\theta]^{1/2}[\varepsilon(\omega_0)-\sin^2\theta]^{-1/4}\times\left\{\exp\left[i\frac{\omega_0}{c}\int^z dz'(\varepsilon(\omega_0,z')-\sin^2\theta)^{1/2}\right]+R(q)\exp\left[-i\frac{\omega_0}{c}\int dz'(\varepsilon(\omega_0,z')-\sin^2\theta)^{1/2}\right]\right\}, \quad (10)$$

where $R(q)$ is the reflection coefficient at frequency ω_0, and $q \equiv (a\omega_0/c)^{1/3}\sin\theta$.

In accord with the foregoing considerations, the second-harmonic generation is more effective the greater the fraction of the incident-radiation energy that is transformed into electron Langmuir oscillations. On the other hand, according to the results of Piliya[14] and Omel'chenko et al.[20] the reflection coefficient has a sharp minimum at $q^2 = 0.4$. We therefore confine ourselves henceforth to small angles of incidence ($\cos\theta \sim 1$, $q \sim 1$), where the linear transformation coefficient is maximal.

In the region where $N_e(z)$ is a linear function of the coordinate

$$N_e(z)=N_0(1+z/a),\quad \varepsilon(\omega_0)\equiv\varepsilon(\omega_0,z)=-z/a,$$
$$\varepsilon(2\omega_0)={}^3/_4-z/4a\approx{}^3/_4, \quad (11)$$

formula (10) can be represented in a form more convenient for subsequent analysis ($\rho \equiv (z/a)(a\omega_0/c)^{2/3}$):

$$B_y(\omega_0,z)=B_y(\omega_0,-\infty)(a\omega_0/c)^{-1/6}f(q,\rho)\cos\theta,$$

$$f(q,\rho)\equiv(-\rho)^{1/2}(-\rho-q^2)^{-1/4}\left\{\exp\left[i\int^{\rho}d\rho'(-\rho'-q^2)^{1/2}\right]+R(q)\exp\left[-i\int^{\rho}d\rho'(-\rho'-q^2)^{1/2}\right]\right\}. \quad (12)$$

The electric field component perpendicular to the density gradient, $E_z(\omega_0, z)$, is given, according to (3b) and (12) by the relation

$$E_z(\omega_0,z)=i\left(\frac{a\omega_0}{c}\right)^{1/6}\frac{\cos\theta}{\rho}\frac{df(q,\rho)}{d\rho}B_y(\omega_0,-\infty). \quad (13)$$

Although in the derivation of formulas (12) and (13) we made extensive use of the geometrical-optics approximation, these equations remain valid both near the turning point $\epsilon(\omega_0) = \sin^2\theta$ and near the critical point $\epsilon(\omega_0) = 0$. In the vicinity of these points only the explicit form of the function $f(q, \rho)$ changes, but not its order of magnitude.

Since the effect of the critical point $z = 0$ on the wave with doubled frequency $2\omega_0$ is small, the geometrical optics approximation is valid for the field $B_y^{\pm}(2\omega_0, z)$ in the whole region under consideration

$$B_y^{\pm}(2\omega_0, z) = [\varepsilon(2\omega_0)\cos\theta]^{1/2}[\varepsilon(2\omega_0) - \sin^2\theta]^{-1/4} \times \exp\left[\pm i\frac{2\omega_0}{c}\int^{z} dz'(\varepsilon(2\omega_0) - \sin^2\theta)^{1/2}\right]. \tag{14}$$

The longitudinal component of the electric field $E_z(\omega_0, z)$ in a plasma with thermal motion can, according to the results of Piliya[14], be written in the form

$$E_z(\omega_0, z) = q(a\omega_0/c)^{1/6}\beta^{-2/3}\varphi(\zeta),$$

$$\varphi(\zeta) = -i\left(\frac{a\omega_0}{c}\right)^{1/6} B_y(\omega_0, 0)\int_0^{\infty} dt \exp\left(-it\zeta - i\frac{t^3}{3}\right), \quad \zeta = \frac{z}{a}\left(\frac{a\omega_0}{c\beta}\right)^{2/3} \tag{15}$$

Here $B_y(\omega_0, 0)$ is the magnetic field of the first harmonic at the critical point. It follows at once from (15) that in the cold plasma $v_{Te} = 0$ the longitudinal component of the electric field at the point $z = 0$ becomes arbitrarily large. In the plasma region $(-\zeta) \gg 1$, where the electron Langmuir oscillation that propagates counter to the incident radiation is already formed, the geometrical optics approximation is also valid for $E_z(\omega_0, z)$:

$$E_z(\omega_0, z) = -i\frac{\sin\theta}{\beta^{2/3}}\left(\frac{a\omega_0}{c}\right)^{2/3} B_y(\omega_0, 0)(-i\pi)^{1/2}(-\zeta)^{-1/4} \times \exp\left[\frac{2}{3}i(-\zeta)^{3/2}\right]. \tag{16}$$

4. SYNCHRONIZATION CONDITIONS

Once the structure of the electromagnetic fields has been found, we proceed directly to the determination of the coefficient of conversion into the second harmonic. We consider first backward generation of the second harmonic. The integrand in (8) contains the product of three rapidly oscillating functions $B_y^{+}(2\omega_0, z)$, $B_y(\omega_0, z)$, and $E_z(\omega_0, z)$ (or $E_x(\omega_0, z)$). The chief contribution to the integral is made by the stationary-phase points, at which the integrand varies most slowly. It is at these points that the synchronization conditions (1) are fulfilled. The law of wave-number conservation in the direction transverse to the density gradient of the plasma, $k_x' + k_x'' = k_x$, is observed in all space, just as in a homogeneous plasma (cf. the remarks made above about the specular reflection of the second harmonic). The synchronization conditions (1) reduce therefore to the condition of equality of the projections of the wave vectors of the interacting waves in the direction of the density gradient: $k_z' + k_z'' = k_z$. The very existence of such z-dependent wave numbers presupposes the use of the geometrical optics approximation. We shall verify later on that the points at which the synchronization conditions are fulfilled are really located in the region where geometrical optics is applicable, i.e., far enough from the critical point.

We consider now in detail the contribution from each of the components of (7) to the coefficient of transformation into the second harmonic (8).

1) $t + l \to t$. The coalescence of a transverse wave with an electron Langmuir oscillation leads to the following synchronization condition:

$$\pm(\varepsilon(\omega_0) - \sin^2\theta)^{1/2} - \frac{1}{\beta}(\varepsilon(\omega_0))^{1/2} = -2(\varepsilon(2\omega_0) - \sin^2\theta)^{1/2}. \tag{17}$$

Here, as everywhere above, the signs + and − refer to waves propagating into and out of the plasma. Since the thermal motion of the electrons is relatively weak, $\beta \ll 1$, and the dielectric constant at the doubled frequency $\epsilon(2\omega_0)$ is, according to (11), of the order of unity, it is necessary to satisfy equality (17) in order to have $\epsilon(\omega_0) \ll 1$.

For small incidence angles ($\sin^2\theta \ll 1$), where the transformation is most effective, synchronization condition (17) reduces in fact to the condition that the more vectors of the transverse electromagnetic wave at the doubled frequency and the electron Langmuir oscillation at the fundamental frequency coincide:

$$(\varepsilon(\omega_0))^{1/2} = 2\beta(\varepsilon(2\omega_0) - \sin^2\theta)^{1/2},$$

which is realized at the point $z = z_1 \equiv -3a\beta^2$.

2. $l + l \to t$. The coalescence of two electron Langmuir oscillations corresponds to the synchronization condition

$$-2(\varepsilon(\omega_0))^{1/2} = -2\beta(\varepsilon(2\omega_0) - \sin^2\theta)^{1/2}, \tag{18}$$

fulfilled at the point $z = z_2 \equiv -3(a\beta^2)/4$.

3. $t + t \to t$. The last term in (7) corresponds to a coalescence process that is forbidden in a homogeneous plasma. It should be discarded in the geometrical optics approximation, since its contribution to (8) is exponentially small compared to the two preceding terms.

The synchronization conditions similar to (17) and (18) for forward generation of the second harmonic (into the plasma) cannot be realized, since an electron Langmuir oscillation of frequency ω_0 does not propagate in that direction. Thus no forward generation of the second harmonic takes place in the approximation we are discussing.

5. COEFFICIENTS OF CONVERSION INTO THE SECOND HARMONIC

Using formulas (12) to (15) and the conditions of the applicability of geometrical optics $a\omega_0/c \ll 1$, of weak thermal motion $\beta \ll 1$, and of maximum transformation $q \sim 1$, it is easy to verify that of all the terms in the nonlinear source (7) the largest contribution to the total coefficient of conversion into the second harmonic is made by the terms:

$$E_x(\omega_0, z)d^2E_z(\omega_0, z)/dz^2, \quad (t + l \to t),$$

$$-i\frac{\omega_0}{c}E_x(\omega_0, z)\frac{dE_z(\omega_0, z)}{dz}\sin\theta, \quad (l + l \to t).$$

We calculate the contribution of each of them, neglecting the effect of the others. As will become clear from what follows, these are really quantities of different order of magnitude and it is the process $t + l \to t$ which is the more effective.

1) $t + l \to t$. The coefficient of transformation into the second harmonic due to the coalescence of a transverse wave with an electron Langmuir oscillation takes, according to formula (8), the following form:

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \left| \frac{e}{4m\omega_0^2} \frac{1}{B_y(\omega_0, -\infty)\cos\theta} \right.$$
$$\times \left. \int_{-\infty}^{+\infty} dz \frac{B_y^+(2\omega_0, z)}{\varepsilon(2\omega_0)} E_x(\omega_0, z) \frac{d^2 E_z(\omega_0, z)}{dz^2} \right|^2. \quad (19)$$

The transverse component of the electric field of the wave $E_x(\omega_0, z)$ has a smaller wave vector and varies considerably more slowly than $B_y^+(2\omega_0, z)$ or $E_z(\omega_0, z)$. Therefore $E_x(\omega_0, z)$ can be taken outside the integral sign at the stationary phase point $z = z_1$. It is necessary here to take into account the change in the behavior of the transverse component of the electric field $E_x(\omega_0, z)$ as it passes through the turning point $z = z_0 \equiv -a \sin^2\theta$.

If the temperature of the plasma electrons is not too high ($3^{1/2}\beta < \sin\theta$), then the stationary phase point z_1 lies to the right of the turning point ($|z_0| \gg |z_1|$), and one may expand $E_x(\omega_0, z)$ near the critical point[17]

$$E_x(\omega_0, z) \approx -i\frac{a\omega_0}{c} B_y(\omega_0, 0) \ln\left(\frac{\omega_0 |z|}{c}\sin\theta\right)\sin^2\theta.$$

Taking this expansion into account and calculating the integral that appears then in (19) by the stationary-phase method, we get

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \frac{2\pi^2}{3^{1/3}} \left| \frac{e}{ma\omega_0^2} \left(\frac{a\omega_0}{c}\right)^{4/3} \beta^{-2/3} \right.$$
$$\times \left. \sin^3\theta \frac{B_y^2(\omega_0, 0)}{B_y(\omega_0, -\infty)} \ln\left(3\frac{a\omega_0}{c}\beta^2 \sin\theta\right) \right|^2.$$

The magnetic field $B_y(\omega_0, 0)$ at the critical point is conveniently expressed in terms of $A(q) = 1 - |R(q)|^2$, the linear coefficient of conversion of an electromagnetic wave at the fundamental frequency ω_0 into the electron Langmuir oscillation[13]:

$$A(q) = \frac{\pi}{\cos\theta}\left(\frac{a\omega_0}{c}\right)^{1/3} q^2 \left| \frac{B_y(\omega_0, 0)}{B_y(\omega_0, -\infty)} \right|^2.$$

As a result we obtain the following final formula for the coefficient of conversion into the second harmonic:

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \frac{4}{3^{1/3}} q^2 A^2(q) \frac{\mathscr{E}}{mc^2}\left(\frac{a\omega_0}{c}\right)^{2/3} \beta^{-4/3}$$
$$\times \ln^2\left(3\frac{a\omega_0}{c}\beta^2 \sin\theta\right), \quad \sin\theta > 3^{1/2}\beta, \quad (20)$$

where $\mathscr{E} \equiv [e^2 B_y^2(\omega_0, -\infty)/2m\omega_0^2]$ is the energy of the electron oscillations in the incident-radiation field. We note that the linear transformation coefficient $A(q)$ was tabulated in[20] by solving Eq. (3a) numerically for $B_y(\omega_0, z)$ in the cold-plasma approximation, $v_{Te} = 0$.

If, however, the temperature of the plasma electrons is sufficiently high ($\sin\theta < 3^{1/2}\beta$), then the stationary phase point z_1 lies to the left of the turning point z_0 ($|z_1| \gg |z_0|$). In this region $B_y(\omega_0, z)$, and consequently also the electric field $E_x(\omega_0, z)$, is a superposition of incident and reflected waves. Once again carrying out the integration in (19) and taking (12) and (13) into account, we find

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \frac{4\pi}{3} qA(q) \frac{\mathscr{E}}{mc^2}\left(\frac{a\omega_0}{c}\right)^{1/3} \beta^{-7/3}, \quad \sin\theta < 3^{1/2}\beta. \quad (21)$$

It is easy to verify that at incidence angles $\sin\theta \sim 3^{1/2}\beta$ and at $q \sim 1$ formulas (20) and (21) give results of identical order of magnitude, as indeed they should.

2. $l + l \to t$. The coalescence of two electron Langmuir oscillations into a transverse electromagnetic wave gives the following relation for the coefficient of transformation into the second harmonic:

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \left| \frac{e}{4m\omega_0 c} \frac{\operatorname{tg}\theta}{B_y(\omega_0, -\infty)} \int_{-\infty}^{+\infty} dz \frac{B_y^+(2\omega_0, z)}{\varepsilon(2\omega_0)} E_z(\omega_0, z) \frac{dE_z(\omega_0, z)}{dz} \right|^2.$$

The final expression, which is obtained, as above, after integrating by the stationary phase method, is of the form

$$\frac{S^-(2\omega_0)}{S(\omega_0)} = \frac{32\pi}{27} q^2 A(q) \frac{\mathscr{E}}{mc^2}\left(\frac{a\omega_0}{c}\right)^{-1} \beta^{-14/3}. \quad (22)$$

6. DISCUSSION OF RESULTS

Formulas (20) to (22) give the dependence of the coefficient of transformation of laser radiation into the second harmonic on the plasma temperature and on the electron concentration gradient. It was assumed everywhere in the derivation of $S^-(2\omega_0/S(\omega_0)$ that the synchronization conditions are satisfied at points $z_{1,2}$ located sufficiently far from the critical point, or more accurately, in the region where the potential electric field has already been formed into a traveling longitudinal wave. This assumption imposes a limitation on the electron temperature of the plasma:

$$\varkappa T_e > \frac{8}{9\sqrt{3}}\left(\frac{a\omega_0}{c}\right)^{-1} mc^2.$$

It can be shown that when this condition is satisfied the coalescence process $t + l \to t$ (see (20) and (21) is more effective than $l + l \to t$ (see (20)).

Allowance for electron-ion collisions leads to the appearance of the following factors in the right-hand sides of (20) to (22):

$$\exp\left[-2\frac{a\nu}{c\beta}\left(-\frac{z_{1,2}}{a}\right)^{1/2}\right],$$

describing the absorption of the longitudinal move on its way from the critical point to the points $z_{1,2}$. According to Sec. 4, we have $z_{1,2} \sim a\beta^2$, and thus the argument of the exponential is $\sim a\nu/c$. Under conditions where there is considerable transformation of light into Langmuir oscillations, this quantity is always small, since it coincides with the optical density of the plasma. Thus, Coulomb collisions do not play any role in our problem.

With increasing temperature ($\beta > a\nu/c$), the absorption of the longitudinal waves becomes due to Landau damping. Comparing the damping length $l \approx a \ln^{-1}[(2\pi)^{1/2} a\omega_0/v_{Te}]$ in[13] with the distance to the next stationary phase point $z_2 = -3(a\beta^2)/4$, we find that transformation into the second harmonic occurs prior to absorption of the longitudinal move by Landau damping if

$$\varkappa T_e < {}^4/_9 mc^2 \ln^{-1}[(2\pi)^{1/2} a\omega_0/v_{Te}].$$

This inequality is satisfied as a rule and therefore neither Coulomb collisions nor Landau damping hinder the emission at the doubled frequency.

The intensity of the second-harmonic radiation, as well as the linear transformation coefficient $A(q)$, depends strongly on the angle of incidence θ (cf.[18,21]).

For typical laser plasma parameters $\omega_0 \approx 1.8 \times 10^{15}$ sec^{-1}, $a \approx 10^{-2}$ cm and $\kappa T_e \approx 1$ keV, we have $\beta \approx 0.073$ and $a\omega_0/c = 6 \times 10^2$. The linear transformation coefficient reaches a maximum $A(q) \approx 0.5$ at $q^2 \approx 0.4$[20], i.e., at the incidence angle $\sin\theta \approx 8.5 \times 10^{-2}$. Since $\sin\theta < 3^{1/2}\beta$, one should employ formula (21). At this incidence angle and at the laser radiation intensity 2×10^{13} W/cm^2 ($\mathscr{E} \approx 4$ eV), the coefficient of transformation into the second harmonic is equal to 1%.

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