COLLISION INTEGRAL FOR A LOW-DENSITY PLASMA (QUANTUM THEORY)

V. N. ALYAMOVSKIĬ

P. N. Lebedev Physics Institute,

Submitted April 6, 1972

Zh. Eksp. Teor. Fiz. 63, 918–929 (September, 1972)

An approximate method is proposed for solving the Bogolyubov set of coupled quantum equations in the case of a long range interaction potential between the particles which is strong at short distances. The pair correlation functions for a low density spatially homogeneous gas are found as functionals of single-particle distribution functions. On this basis, kinetic equations for a low density electron plasma are obtained which do not contain any divergences and are applicable throughout the whole nonrelativistic range of particle velocities. For $e^2/\hbar v_{av} \to \infty$ they go over into the classical-mechanics result obtained earlier by the author, and for $e^2/\hbar v_{av} \to 0$ they are identical to the asymptotic of the Balescu-Silin equations for extremely low densities.

THIS author has proposed earlier[1] a method for the approximate solution of the set of coupled equations for the reduced distribution functions of a classical gas with a long-range interaction potential that is strong at short distances (in particular, a Coulomb potential). In this paper (Secs. 1 and 2) this approach is extended to the quantum chain of Bogolyubov equations[2,3].

A weakly-nonideal electron plasma is necessarily of the "Boltzmann" ($\hbar n^{1/3}/p_{av} \ll 1$) and (or) "Born" ($e^2/\hbar v_{av} \ll 1$) type, since the condition for weak nonideality ($n^{1/3}e^2/E_{av} \ll 1$) can be replaced by the equivalent condition $(\hbar n^{1/3}/p_{av})(e^2/\hbar v_{av}) \ll 1$. In the case of a Born plasma the Balescu-Silin kinetic equation is valid[4,5], with its assumption of weak interaction between the particles (the special case $e^2/\hbar v_{av} \ll 1$, $\hbar n^{1/3}/p_{av} \ll 1$ for small deviations from equilibrium was discussed by Konstantinov and Perel'[6]). A description of the weakly nonideal Boltzmann plasma requires, in the general case, the use of exact solutions of the two-body problem; for small deviations from equilibrium this problem was treated by Gould and Devitt[7] with the aid of a diagram technique, but in summing the diagrams they introduced for the collision integral an approximation that contained the exact (unknown) quantum mechanical scattering cross section in the Debye-Hückel field.

In this paper (Sec. 3) the collision integral for the arbitrary (within the framework of stability of spatial homogeneity) non-equilibrium states of a weakly nonideal Boltzmann plasma is obtained in explicit form, and is applicable over the entire nonrelativistic interval of particle velocities. As $e^2/\hbar v_{av} \to \infty$ it reduces, as indeed it must, to the result of classical mechanics[1], while as $e^2/\hbar v_{av} \to 0$ it agrees with the asymptotic form of the Balescu-Silin integral at $n \to 0$.

1. For our purposes the coordinate representation ($\mathbf{r}$) is most convenient (initially). Taking into account the requirements of permutation (anti-)symmetry, we write the elements of the l-particle density matrix ρ_l that are diagonal in the spin states in the form

$$\rho_l(\sigma_1 \ldots \sigma_l;\, \mathbf{r}_1 \ldots \mathbf{r}_l | \sigma_1 \ldots \sigma_l;\, \mathbf{r}_1' \ldots \mathbf{r}_l') = V^{-l}\gamma^{(l)}F_l(x_1 \ldots x_l).$$

Here $x_i \equiv \{\sigma_i, \mathbf{r}_i, \mathbf{r}_i'\}$, σ_i is the spin projection, $\gamma^{(l)}$ is the (anti-)symmetrization operator defined by the relation

$$\gamma^{(l)} = \gamma^{(l-1)}\left[1 + (-1)^{2s}\sum_{i=1}^{l-1}\delta_{\sigma_i\sigma_l}P_{\mathbf{r}_i\mathbf{r}_l}\right]$$

(P is the permutation operator, S is the particle spin), and the function F_l is symmetrical with respect to the permutations $x_i \rightleftarrows x_j$; for simplicity we are considering a system of particles of one kind. Then, as $l(nV)^{-1} \to 0$, the chain of Bogolyubov equations takes the following form:

$$\left\{i\hbar\frac{\partial}{\partial t} - \mathscr{H}_l\right\}F_l(x_1 \ldots x_l, t) =$$

$$= n\sum_{i=1}^{l}\operatorname*{Sp}_{l+1}\Phi_{i,l+1}\left[1 + (-1)^{2s}\sum_{j=1}^{l}\delta_{\sigma_j\sigma_{l+1}}P_{\mathbf{r}_j,\mathbf{r}_{l+1}}\right]F_{l+1}(x_1 \ldots x_{l+1}, t),$$

where

$$\operatorname*{Sp}_i(\ldots) \equiv \sum_{\sigma_i}\iint_V d\mathbf{r}_i\, d\mathbf{r}_i'\, \delta(\mathbf{r}_i' - \mathbf{r}_i)(\ldots),$$

$$\mathscr{H}_l(\mathbf{r}_1 \ldots \mathbf{r}_l : \mathbf{r}_1' \ldots \mathbf{r}_l') \equiv H_l(\mathbf{r}_1 \ldots \mathbf{r}_l) - H_l(\mathbf{r}_1' \ldots \mathbf{r}_l'),$$

$$\Phi_{ij} \equiv \varphi(|\mathbf{r}_i - \mathbf{r}_j|) - \varphi(|\mathbf{r}_i' - \mathbf{r}_j'|),$$

H_l is the Hamiltonian of l particles, φ is the interaction potential; the functions F_l are subject to the additional conditions

$$V^{-1}\operatorname*{Sp}_l\left[1 + (-1)^{2s}\sum_{j=1}^{l-1}\delta_{\sigma_j\sigma_l}P_{\mathbf{r}_j,\mathbf{r}_l}\right]F_l(x_1 \ldots x_l) = F_{l-1}(x_1 \ldots x_{l-1}), \qquad l \geqslant 2;$$

$$V^{-1}\operatorname{Sp}F_1 = 1.$$

We introduce the correlation functions g_l:

$$F_2(12) \equiv F(1)F(2) + g(12),$$
$$F_3(123) \equiv F(1)F(2)F(3) + F(1)g(23) + F(2)g(13) + F(3)g(12) + g_3(123)$$

etc, where $F(1) \equiv F_1(x_1)$, $g(12) \equiv g_2(x_1, x_2)\ldots$ From the conditions on F_l follow the requirements on g_l:

$$V^{-1}\operatorname*{Sp}_i g_l(\ldots x_i \ldots) \to 0, \qquad V \to \infty$$

(with the exception of the case of Bose condensation). In the case of spatial homogeneity ($F(x_i) \equiv F(\sigma_i, \mathbf{r}_i - \mathbf{r}_i')$) we have the following set of equations for $F, g, \ldots$:

$$i\hbar\frac{\partial}{\partial t}F(1) = n\operatorname*{Sp}_2\Phi_{12}[1 + (-1)^{2s}\delta_{\sigma_1\sigma_2}P_{\mathbf{r}_1,\mathbf{r}_2}]g(12), \tag{1}$$

$$\left\{i\hbar\frac{\partial}{\partial t} - \mathscr{H}_2\right\}g(12) = \Phi_{12}F(1)F(2) + n\operatorname*{Sp}_3\{\Phi_{13}F(1)g(23) + \Phi_{23}F(2)g(13) + O(123)\} \tag{2}$$

etc; O designates the terms containing the permutation operator and the triple correlation. As to the function g, we make an assumption that is natural for bound states—that $g(t+\tau)$ becomes independent of the form of g(t) when $\tau\to\infty$, in any case in the zeroth approximation in the density. As is easily seen from (2), this leads to the condition

$$\exp(\tau\mathscr{H}_2/i\hbar)g\to 0,\quad \tau\to\infty, \tag{3}$$

which is analogous to the Bogolyubov condition in the classical case[2].

We transform (2) using (3), as in[1]: we apply to both sides of (2) the operator $\exp(\tau\mathscr{H}_2/i\hbar)$ and integrate with respect to τ from 0 to ∞. By virtue of (3) we obtain

$$g(12)-i\hbar(\mathscr{H}_2-i\delta)^{-1}\frac{\partial}{\partial t}g(12)=g^0(12)-(\mathscr{H}_2-i\delta)^{-1}n\,\underset{3}{\mathrm{Sp}}\{\ldots\},\quad \delta\to+0, \tag{4}$$

where

$$g^0(12t)\equiv\int_0^\infty\frac{d\tau}{i\hbar}\exp\left\{\frac{\tau}{i\hbar}\mathscr{H}_2\right\}\Phi_{12}F(1t)F(2t) \tag{5}$$
$$=\lim\left[\exp\left(\frac{\tau}{i\hbar}\mathscr{H}_2\right)-1\right]F(1t)F(2t),\qquad \tau\to\infty$$

(in view of $F(x_i)\equiv F(\sigma_i,\mathbf{r}_i-\mathbf{r}_i')$) and n Sp {...} is the second term in the right-hand side of (2).

In the low-density limit, the integral term in the right-hand side of (4) can be considerably simplified: one need keep in {...} only the first two terms, while the interaction in $\mathscr{H}_2$ can be neglected (since the integral term is important only at large distances between particles). As a result we get the equation

$$g(12)-i\hbar(\mathscr{H}_2-i\delta)^{-1}\frac{\partial}{\partial t}g(12)$$
$$+\frac{n}{\mathscr{H}_2^0-i\delta}\underset{3}{\mathrm{Sp}}\{\Phi_{13}F(1)g(23)+\Phi_{23}F(2)g(13)\}=g^0(12), \tag{6}$$

where $\mathscr{H}_2^0(12)\equiv\mathscr{H}_2(12)-\Phi_{12}$. We are assuming here that in the integral term the contribution of the strong-interaction region is small for every term in {...} (the case of an "integrally weak" potential[1], particularly a pure Coulomb potential); in the general case of a central nucleus of arbitrary intensity, only the weak long-range part of the potential need be kept in Φ.

2. We are interested in the quasistationary solution of Eq. (6), which describes the "steady-state" correlations. Inasmuch as $F(x_i)\equiv F(\sigma_i,\mathbf{r}_i-\mathbf{r}_i')$, g can be written in the form

$$g(x_1x_2)=(2\pi)^{-3}\int d\mathbf{k}\,d\mathbf{p}_1\,d\mathbf{p}_2\,\tilde g(y_1y_2\mathbf{k})\exp\left\{\frac{i}{\hbar}\mathbf{p}_1(\mathbf{r}_1-\mathbf{r}_1')\right. \tag{7}$$
$$\left.+\frac{i}{\hbar}\mathbf{p}_2(\mathbf{r}_2-\mathbf{r}_2')-i\mathbf{k}\left(\frac{\mathbf{r}_1+\mathbf{r}_1'}{2}-\frac{\mathbf{r}_2+\mathbf{r}_2'}{2}\right)\right\},$$
$$y_i\equiv\{\sigma_i;\ \mathbf{p}_i\}.$$

Discarding the term with the time derivative in (6), we get the following equation for $\tilde g$:

$$\tilde g(y_1y_2\mathbf{k})+n\tilde\varphi_k\frac{h_\mathbf{k}(y_1)S_\mathbf{k}f_{\sigma_2}(\mathbf{p}_2+\hbar\mathbf{k}/2)-h_{-\mathbf{k}}(y_2)S_\mathbf{k}f_{\sigma_1}(\mathbf{p}_1+\hbar\mathbf{k}/2)}{S_\mathbf{k}(E_{\mathbf{p}_1+\hbar\mathbf{k}/2}-E_{\mathbf{p}_2+\hbar\mathbf{k}/2})+i\delta}$$
$$=\tilde g^0(y_1y_2\mathbf{k}), \tag{8}$$

where $\tilde g^0$ is defined in terms of g^0, just as in (7), $E_\mathbf{p}\equiv E(\mathbf{p})$ is the energy of a particle with momentum $\mathbf{p}$,

$$f_{\sigma_i}(\mathbf{p}_i)\equiv\int\frac{d\mathbf{r}_iF(x_i)}{(2\pi\hbar)^3}\exp\left\{-\frac{i}{\hbar}\mathbf{p}_i(\mathbf{r}_i-\mathbf{r}_i')\right\},\qquad h_\mathbf{k}(y_i)\equiv\int dy_j\,\tilde g(y_iy_j\mathbf{k}),$$
$$\tilde\varphi_k\equiv\int d\mathbf{r}\,\varphi(r)e^{-i\mathbf{k}\mathbf{r}},\qquad S_\mathbf{k}\psi(\mathbf{k})\equiv\hbar^{-1}[\psi(\mathbf{k})-\psi(-\mathbf{k})].$$

Equation (8) differs from the one discussed in[5,8] in that the interaction is taken into account exactly in $\tilde g^0$. We solve it for the arbitrary (even) function E(p) (which is important for a solid-state plasma), reducing it to equations that coincide with the ones discussed in[1]. Integrating both sides of (8) we get an equation for h:

$$\varepsilon_\mathbf{k}(S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2})h_\mathbf{k}(y)+$$
$$+n\tilde\varphi_k\left[S_\mathbf{k}f_\sigma\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)\right]\int_{-\infty}^{\infty}\frac{d\omega H_{-\mathbf{k}}(-\omega)}{\omega-S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2}-i\delta}=h_\mathbf{k}^0(y), \tag{9}$$

where

$$H_\mathbf{k}(\omega)\equiv\int dy\,\delta(S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2}-\omega)h_\mathbf{k}(y),$$
$$\varepsilon_\mathbf{k}(\omega)\equiv1+n\tilde\varphi_k\sum_\sigma\int\frac{d\mathbf{p}S_\mathbf{k}f_\sigma(\mathbf{p}+\hbar\mathbf{k}/2)}{\omega-S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2}+i\delta}.$$

Multiplying both sides of (9) by $\delta(S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2}-\omega)$ and integrating in respect to y, we get the equation for $H_\mathbf{k}(\omega)$:

$$\varepsilon_\mathbf{k}(\omega)H_\mathbf{k}(\omega)-\frac{1}{\pi}\mathrm{Im}\,\varepsilon_\mathbf{k}(\omega)\int_{-\infty}^{\infty}\frac{d\omega'H_{-\mathbf{k}}(-\omega')}{\omega'-\omega-i\delta}=H_\mathbf{k}^0(\omega). \tag{10}$$

We introduce the functions $H_\mathbf{k}'(\omega)$ and $H_\mathbf{k}''(\omega)$:

$$2H_\mathbf{k}'(\omega)\equiv H_\mathbf{k}(\omega)+H_{-\mathbf{k}}(-\omega),\qquad 2iH_\mathbf{k}''(\omega)\equiv H_\mathbf{k}(\omega)-H_{-\mathbf{k}}(-\omega).$$

In (10) we reverse the signs of ω and k, and then subtract the resulting equation from (10) and add the result to (10); since $\epsilon_{-\mathbf{k}}(-\omega)=\epsilon_\mathbf{k}^*(\omega)$, we get the equations

$$K^+H_\mathbf{k}''(\omega)=H_\mathbf{k}^{0\prime\prime}(\omega),$$
$$K^-H_\mathbf{k}'(\omega)=|\varepsilon_\mathbf{k}(\omega)|^{-2}[H_\mathbf{k}^{0\prime}(\omega)+2\mathrm{Im}\,\varepsilon_\mathbf{k}(\omega)H_\mathbf{k}''(\omega)], \tag{11}$$

where the operators K^+ and K^- are defined by

$$K^\pm\psi_\mathbf{k}(\omega)=\mathrm{Re}\,\varepsilon_\mathbf{k}^{\pm1}(\omega)\psi_\mathbf{k}(\omega)+\frac{1}{\pi}\mathrm{Im}\,\varepsilon_\mathbf{k}^{\pm1}(\omega)\int_{-\infty}\frac{d\omega'\,\psi_\mathbf{k}(\omega')}{\omega'-\omega}.$$

By virtue of the connection between Im ϵ and Re ϵ (the dispersion relation) the operators K^+ and K^- have the same properties as those defined in[1], in particular

$$K^-K^+=1. \tag{12}$$

In the absence of zeroes of $\epsilon_\mathbf{k}(z)$ in the upper half-plane of complex z, in addition[1],

$$K^+K^-=1. \tag{13}$$

As is easily seen, Eq. (13) is necessary and sufficient for the existence of a solution to the set (11); the limitation on ϵ, however, which is required to satisfy (13), is necessary and sufficient for the stability of the spatially-homogeneous distribution (see, e.g.,[9]). Thus, a solution of Eq. (8) exists only for a "stable" gas and is determined by Eqs. (9) and (11) to (13).

The obtained solution gives, together with (1), a Markov-type kinetic equation. It follows from (1) that

$$\frac{\partial f_{\sigma_1}(\mathbf{p}_1)}{\partial t}=\frac{n}{i}\int\frac{d\mathbf{k}}{(2\pi)^3}\exp\left\{-\frac{\hbar\mathbf{k}}{2}\frac{\partial}{\partial\mathbf{p}_1}\right\}S_\mathbf{k}\{\tilde\varphi_kh_\mathbf{k}(y_1) \tag{14}$$
$$+(-1)^{2s}\int dy_2\delta_{\sigma_1\sigma_2}\tilde g(y_1y_2\mathbf{k})\,\tilde\varphi_{|\mathbf{p}_1-\mathbf{p}_2|/\hbar}\}.$$

After several transformations we obtain from (9) to (12) for $S_\mathbf{k}h_\mathbf{k}(y)$:

$$S_\mathbf{k}h_\mathbf{k}(y)=|\varepsilon_\mathbf{k}(S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2})|^{-2}S_\mathbf{k}\{\varepsilon_\mathbf{k}^*(S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2})h_\mathbf{k}^0(y) \tag{15}$$
$$-n\tilde\varphi_k\left[S_\mathbf{k}f_\sigma\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)\right]\int_{-\infty}^{\infty}\frac{d\omega H_{-\mathbf{k}}^0(-\omega)}{\omega-S_\mathbf{k}E_{\mathbf{p}+\hbar\mathbf{k}/2}-i\delta}\}.$$

In the lowest-order approximation in n, one must put $\tilde{g} = \tilde{g}^0$ in the exchange term.

In the lowest order in φ, in accord with (5) and (7), we have $\tilde{g}^0 = \tilde{g}^0_B$, where

$$g_B{}^0(y_1 y_2 \mathbf{k}) \equiv \tilde{\varphi}_k \frac{S_\mathbf{k}\{f_{\sigma_1}(\mathbf{p}_1 - \hbar\mathbf{k}/2) f_{\sigma_2}(\mathbf{p}_2 + \hbar\mathbf{k}/2)\}}{S_\mathbf{k}\{E(\mathbf{p}_1 - \hbar\mathbf{k}/2) + E(\mathbf{p}_2 + \hbar\mathbf{k}/2)\} - i\delta}. \quad (16)$$

Substituting in (14) and (15), we get

$$\frac{\partial f_{\sigma_1}(\mathbf{p}_1)}{\partial t} = \frac{n}{(2\pi)^2 \hbar^4} \int dy_2\, d\Delta\, \delta(E_{\mathbf{p}_1-\Delta} + E_{\mathbf{p}_2+\Delta} - E_{\mathbf{p}_1} - E_{\mathbf{p}_2}) \tilde{\varphi}_{\Delta/\hbar}$$
$$\times \left\{\tilde{\varphi}_{\Delta/\hbar} \left| \varepsilon_{\Delta/\hbar}\left(\frac{E_{\mathbf{p}_1} - E_{\mathbf{p}_1-\Delta}}{\hbar}\right)\right|^{-2} + (-1)^{2s} \delta_{\sigma_1\sigma_2} \tilde{\varphi}_{|\mathbf{p}_1 - \mathbf{p}_2 - \Delta|/\hbar}\right\}$$
$$\times \{f_{\sigma_1}(\mathbf{p}_1 - \Delta) f_{\sigma_2}(\mathbf{p}_2 + \Delta) - f_{\sigma_1}(\mathbf{p}_1) f_{\sigma_2}(\mathbf{p}_2)\}, \quad (17)$$

i.e., Balescu-Silin type equations[4,5] that include the exchange interaction in the absence of degeneracy. Appendix 1 contains a simplification of (17) for the Coulomb case.

3. In case of $\varphi(r) = e^2/r$ and $E(p) = p^2/2m$, Eq. (14), accurate to terms linear in the density inclusive, reduces to the following form (see Appendices 2 and 3):

$$\frac{\partial f_{\sigma_1}(\mathbf{p}_1)}{\partial t} = ne^4 \sum_{\sigma_2} \int d\mathbf{p}_2 \left\{ \frac{\partial}{\partial \mathbf{p}_1} \int d\Omega_\mathbf{q} L_{12} \mathbf{Q}_{12} - 4 \int_0^{mw_{12}} d\Delta \ln\frac{\Delta}{mw_{12}} \right. \quad (18)$$
$$\left. \times \frac{d}{d\Delta} \int \frac{d\Omega_\Delta}{\Delta} D_{12} + 4(-1)^{2s} \delta_{\sigma_1\sigma_2} \int d\Delta \frac{C(\Delta)}{\Delta^2 (m^2 w_{12}^2 - \Delta^2)} D_{12} \right\},$$

where

$$C(\Delta) \equiv \cos\left[\frac{e^2}{\hbar w_{12}} \ln\left(\frac{m^2 w_{12}^2}{\Delta^2} - 1\right)\right],$$

$$\mathbf{Q}_{12}(\mathbf{q}) \equiv \delta(\mathbf{q}\mathbf{w}_{12}) \left[\mathbf{q}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right) f_{\sigma_1}(\mathbf{p}_1) f_{\sigma_2}(\mathbf{p}_2)\right] \mathbf{q}, \quad |\mathbf{q}| = 1;$$

$$D_{12}(\Delta) \equiv \delta(\Delta \mathbf{w}_{12} - \Delta^2/m)[f_{\sigma_1}(\mathbf{p}_1 - \Delta) f_{\sigma_2}(\mathbf{p}_2 + \Delta) - f_{\sigma_1}(\mathbf{p}_1) f_{\sigma_2}(\mathbf{p}_2)];$$

$$\mathbf{w}_{12} \equiv \mathbf{v}_1 - \mathbf{v}_2, \quad \mathbf{v}_i \equiv \mathbf{p}_i/m.$$

The right-hand side of (18) differs from the Boltzmann collision integral only in the presence in the first term ("Fokker-Planck" term) L_{12} of finite value, instead of the divergent expression $2\int_0^{mw_{12}} d\Delta/\Delta$; L_{12} is equal to (see Appendix 3)

$$L_{12}(\mathbf{q}) = \ln\left|\frac{mw_{12}^2}{\zeta(e^2/\hbar w_{12}) \varkappa_\mathbf{q}(\mathbf{q}\mathbf{v}_1) e^2}\right|^2 - \gamma_\mathbf{q}(\mathbf{q}\mathbf{v}_1), \quad (19)$$

$$\varkappa_\mathbf{q}^2(u) \equiv 4\pi n e^2 \sum_\sigma \int \frac{d\mathbf{p}\, \mathbf{q}\partial f_\sigma(\mathbf{p})/\partial \mathbf{p}}{u - \mathbf{q}\mathbf{v} + i\delta},$$

$$\gamma \equiv \frac{\operatorname{Re}\varkappa^2}{|\operatorname{Im}\varkappa^2|} \operatorname{arcctg} \frac{\operatorname{Re}\varkappa^2}{|\operatorname{Im}\varkappa^2|}, \quad 0 \leqslant \operatorname{arcctg} < \pi,$$

$$\zeta(\eta) \equiv \eta^{-1} \exp\left\{-\frac{2}{\pi}\int_0^{\pi/2} d\varphi \int_0^1 \frac{dx}{x}(x^2 - 1)^2 \frac{\cos[\eta R(x,\varphi)] - 1}{(x^2+1)^2 - 4x^2\cos^2\varphi}\right\},$$

$$R(x,\varphi) \equiv \ln\frac{x^2 + 2x\cos\varphi + 1}{x^2 - 2x\cos\varphi + 1}.$$

The limiting expression for $\zeta(\eta)$ are

$$\zeta(\eta) \cong \begin{cases} \eta^{-1} & \eta \to 0; \\ \exp C, \ C = 0.577\ldots, & \eta \to \infty \end{cases}$$

(of which the first is obvious; concerning the second, see Appendix 4). The fraction under the logarithm sign in L_{12} has the meaning of the ratio of the maximum possible momentum transfer (mw_{12}) to the minimum effective value ($\sim \zeta |\kappa| e^2/w_{12}$) for scattering in a Coulomb field, with the classical screening length $|\kappa|^{-1}$. The second term in L_{12}, which has a purely classical form, describes plasma-wave exchange[1].

In the classical limit $e^2/\hbar v_{av} \gg 1$, there follows from (18) for $f(p) \equiv \sum_\sigma f_\sigma(p)$ an equation that agrees with the result of classical mechanics (the exchange term vanishes in view of the fast oscillations of the cosine). In the Born limit ($e^2/\hbar v_{av} \ll 1$) the right-hand side of (18) coincides with the Balescu-Silin integral, accurate to terms linear in n inclusive (see Appendix 1). We note that corrections to the latter for the finiteness of the parameter $e^2/\hbar v_{av}$ cannot be obtained simply from perturbation theory in e^2, since L_{12} is expanded in terms of $e^4/\hbar^2 w_{12}^2$ and divergencies arise upon integrating over the momentum.

APPENDIX 1

We calculate the right-hand side of (17) accurate to terms linear in n for $\tilde{\varphi}_k = 4\pi e^2/k^2$ and $E(p) = p^2/2m$. We write (17) in the form

$$\frac{\partial f_{\sigma_1}(\mathbf{p}_1)}{\partial t} = 4ne^4 \int dy_2 \left\{ \int \frac{d\mathbf{k}\, D_{12}(\hbar\mathbf{k})}{\hbar k^4 |\varepsilon_\mathbf{k}(\mathbf{k}\mathbf{v}_1 - \hbar k^2/2m)|^2} \right. \quad (1.1)$$
$$\left. + (-1)^{2s} \delta_{\sigma_1\sigma_2} \int \frac{}{\Delta^2 (m^2 w_{12}^2 - \Delta^2)} \right\}.$$

Taking into account the presence of the δ-function in D_{12}, we can set the upper limit of integration with respect to k in the first term of (1.1) equal to $mw_{12}/\hbar$; then, separating in $D_{12}(\hbar\mathbf{k})$ the first two terms of its expansion in $\hbar k$ (1.2)

$$D_{12}(\hbar\mathbf{k}) = -\mathbf{Q}_{12}\left(\frac{\mathbf{k}}{k}\right)\frac{\mathbf{k}}{k} + \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)\mathbf{Q}_{12}\left(\frac{\mathbf{k}}{k}\right)\hbar k + D_{12}'(\hbar\mathbf{k})$$

and recognizing that $\mathbf{Q}(\mathbf{q})$ is even in $\mathbf{q}$, we rewrite the first term in the curly brackets in (1.1) in the form (1.3)

$$\int_0^{mw_{12}\hbar^{-1}} \frac{dk}{k} \int d\Omega_\mathbf{k} \left\{ \left[-\frac{\mathbf{k}}{2k^2} S_\mathbf{k} \left|\varepsilon_\mathbf{k}\left(\mathbf{k}\mathbf{v}_1 - \frac{\hbar k^2}{2m}\right)\right|^{-2} - \frac{1}{2}\frac{\partial}{\partial \mathbf{p}_1} |\varepsilon_\mathbf{k}^{(c)}(\mathbf{k}\mathbf{v}_1)|^{-2} \right.\right.$$
$$\left. + \frac{1}{2}\left(\left|\varepsilon_\mathbf{k}\left(\mathbf{k}\mathbf{v}_1 - \frac{\hbar k^2}{2m}\right)\right|^{-2} - |\varepsilon_\mathbf{k}^{(c)}(\mathbf{k}\mathbf{v}_1)|^{-2}\right)\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)\right] \mathbf{Q}_{12}\left(\frac{\mathbf{k}}{k}\right)$$
$$\left. + \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)\frac{\mathbf{Q}_{12}(\mathbf{k}/k)}{|\varepsilon_\mathbf{k}^{(c)}(\mathbf{k}\mathbf{v}_1)|^2} + \frac{D_{12}'(\hbar\mathbf{k})}{\hbar k |\varepsilon_\mathbf{k}(\mathbf{k}\mathbf{v}_1 - \hbar k^2/2m)|^2} \right\},$$

where

$$\varepsilon_\mathbf{k}^{(c)}(\omega) \equiv 1 + k^{-2}\varkappa_{\mathbf{k}/k}^2(\omega/k).$$

Since $\epsilon_{-\mathbf{k}}(-\omega) = \epsilon_\mathbf{k}^*(\omega)$, we have

$$S_\mathbf{k} |\varepsilon_\mathbf{k}(\mathbf{k}\mathbf{v}_1 - \hbar k^2/2m)|^{-2} = \hbar^{-1}\{|\varepsilon_\mathbf{k}(\mathbf{k}\mathbf{v}_1 - \hbar k^2/2m)|^{-2}$$
$$- |\varepsilon_\mathbf{k}(\mathbf{k}\mathbf{v}_1 + \hbar k^2/2m)|^{-2}\}.$$

Therefore, if we change to the variable $n^{-1/2}k$, we see easily that the first two terms in the square brackets in (1.3) make finite and mutually cancelling contributions when n = 0. It is easy to verify similarly that at n = 0 the contribution from the third term also vanishes, by representing $|\epsilon|^{-2} - |\epsilon^{(c)}|^{-2}$ in the form

$$\frac{1 - \operatorname{Re}\varepsilon}{|\varepsilon|^2} - \frac{1 - \operatorname{Re}\varepsilon^{(c)}}{|\varepsilon^{(c)}|^2} + \frac{\varepsilon^{(c)*} - \varepsilon}{2\varepsilon^{(c)*}\varepsilon} - \frac{\varepsilon^* - \varepsilon^{(c)}}{2\varepsilon^*\varepsilon^{(c)}}.$$

In the next-to-last term in the curly brackets in (1.3) the operator $(\partial/\partial p_1) - (\partial/\partial p_2)$ can be taken outside of the integral sign (since $\mathbf{Q}_{12}(\mathbf{q})$ contains $\delta(\mathbf{q}\cdot\mathbf{w}_{12})$, and integration is carried out with respect to k; discarding quantities that vanish at n = 0, we have

$$\int_0^{mw_{12}\hbar^{-1}} \frac{dk}{k} |\varepsilon_\mathbf{k}^{(c)}(\mathbf{k}\mathbf{v}_1)|^{-2} \underset{n\to 0}{\to} \ln\left|\frac{mw_{12}}{\hbar\varkappa_\mathbf{q}(\mathbf{q}\mathbf{v}_1)}\right| - \frac{1}{2}\gamma_\mathbf{q}(\mathbf{q}\mathbf{v}_1); \quad \mathbf{q} = \mathbf{k}/k.$$

The integral of the last term in (1.3) is finite at n = 0; integrating by parts with respect to k we get

$$-\int_0^{mw_{12}\hbar^{-1}} dk \ln\frac{\hbar k}{mw_{12}}\frac{d}{dk}\int\frac{d\Omega_{\mathbf{k}}}{\hbar k}D_{12}'(\hbar\mathbf{k}).$$

It is seen from (1.2) that D can be replaced here by D.

As a result, we get for (1.1), accurate to terms linear in n,

$$\frac{\partial f_{\sigma_1}(\mathbf{p}_1)}{\partial t} = ne^4\int dy_2\left\{\frac{\partial}{\partial\mathbf{p}_1}\int d\Omega_{\mathbf{q}}\left[\ln\left|\frac{mw_{12}}{\hbar\varkappa_q(\mathbf{qv}_1)}\right|^2 - \gamma_{\mathbf{q}}(\mathbf{qv}_1)\right]\mathbf{Q}_{12}(\mathbf{q})\right.$$
$$\left.-4\int_0^{mw_{12}} d\Delta\ln\frac{\Delta}{mw_{12}}\frac{d}{d\Delta}\int\frac{d\Omega_\Delta}{\Delta}D_{12}(\Delta) + 4(-1)^{2s}\delta_{\sigma_1\sigma_2}\int\frac{d\Delta D_{12}(\Delta)}{\Delta^2(m^2w_{12}^2-\Delta^2)}\right\}$$

APPENDIX 2

We reduce the exchange term in (14) to explicit form in an approximation linear in n, i.e., we assume $\tilde{g} = \tilde{g}^0$. Expanding $F(x_1)$ and $F(x_2)$ in (5) in Fourier integrals, we have

$$g^0(x_1x_2) = (2\pi\hbar)^6\int d\mathbf{p}_1 d\mathbf{p}_2 f_{\sigma_1}(\mathbf{p}_1)f_{\sigma_2}(\mathbf{p}_2)\ \{\psi_{\mathbf{p}_1\mathbf{p}_2}(\mathbf{r}_1,\mathbf{r}_2)\psi^*_{\mathbf{p}_1\mathbf{p}_2}(\mathbf{r}_1',\mathbf{r}_2')$$
$$-\psi_{\mathbf{p}_1}^0(\mathbf{r}_1)\psi_{\mathbf{p}_2}^0(\mathbf{r}_2)\psi_{\mathbf{p}_1}^{0*}(\mathbf{r}_1')\psi_{\mathbf{p}_2}^{0*}(\mathbf{r}_2')\},$$
$$\psi_{\mathbf{p}}^0(\mathbf{r}) \equiv (2\pi\hbar)^{-3/2}\exp(i\mathbf{pr}/\hbar), \tag{2.1}$$
$$\psi_{\mathbf{p}_1\mathbf{p}_2}(\mathbf{r}_1,\mathbf{r}_2) \equiv \lim\exp\left\{\frac{\tau}{i\hbar}(H_2 - E_{p_1} - E_{p_2})\right\}\psi_{\mathbf{p}_1}^0(\mathbf{r}_1)\psi_{\mathbf{p}_2}^0(\mathbf{r}_2),\quad \tau\to\infty.$$

For convenience we consider the "truncated" potential $\varphi(r) \equiv \Theta_+(R-r)e^2/r$ (Θ_+ is the step function) with the consequent transition to $R\to\infty$. Representing the exchange term in (14) as the Fourier transform of the exchange integral in (1), expanding the produces of the form $\varphi(|\mathbf{r}_1-\mathbf{r}_2|)\psi(\mathbf{r}_1,\mathbf{r}_2)$ in Fourier integrals, and separating in H_2 the motion of the mass center, we have

$$(-1)^{2s}\frac{n}{i\hbar}(2\pi\hbar)^3\int d\mathbf{p}_2 d\Delta f_{\sigma_1}(\mathbf{p}_1-\Delta)f_{\sigma_1}(\mathbf{p}_2+\Delta)\ \{\langle\psi^0_{-\mathbf{p}_{12}}|\varphi|\psi_{\mathbf{p}_{12}-\Delta}\rangle$$
$$\times\langle\psi_{\mathbf{p}_{12}-\Delta}|\psi^0_{\mathbf{p}_{12}}\rangle - \langle\psi^0_{-\mathbf{p}_{12}}|\psi_{\mathbf{p}_{12}-\Delta}\rangle\langle\psi_{\mathbf{p}_{12}-\Delta}|\varphi|\psi^0_{\mathbf{p}_{12}}\rangle\}, \tag{2.2}$$

where

$$\psi_{\mathbf{p}} \equiv \lim_{\tau\to\infty}\exp\left\{\frac{\tau}{i\hbar}(H-E_p')\right\}\psi_{\mathbf{p}}^0 = \psi_{\mathbf{p}}^0 + \frac{\varphi\psi_{\mathbf{p}}^0}{E_p'+i\delta-H}, \tag{2.3}$$
$$\mathbf{p}_{12} = \mu\mathbf{w}_{12},\quad \mu = \frac{m}{2},\quad E_p' = \frac{p^2}{2\mu},\quad H = -\frac{\hbar^2\nabla^2}{2\mu}+\varphi.$$

According to (2.3), ψ_p is the solution of the stationary Schrödinger equation with a diverging scattered wave. The meaning of "truncating" the potential is that the matrix elements in (2.2) are not defined in the Coulomb case when the momenta are equal in absolute value.

We make use of the following two relations. It is directly evident from (2.3) that

$$\langle\psi_{\mathbf{p}'}^0|\psi_{\mathbf{p}}\rangle = \delta(\mathbf{p}'-\mathbf{p}) + \frac{\langle\psi_{\mathbf{p}'}^0|\varphi|\psi_{\mathbf{p}}\rangle}{E_p'-E_{p'}'+i\delta}. \tag{2.4}$$

From the equality

$$\int d\mathbf{p}''\langle\psi_{\mathbf{p}'}|\overset{0}{\psi}_{\mathbf{p}''}\rangle\langle\overset{0}{\psi}_{\mathbf{p}''}|\psi_{\mathbf{p}}\rangle = \langle\psi_{\mathbf{p}'}|\psi_{\mathbf{p}}\rangle = \delta(\mathbf{p}'-\mathbf{p})$$

we have by virtue of (2.4)

$$\langle\psi^0_{\mathbf{p}'}|\varphi|\psi_{\mathbf{p}}\rangle - \langle\psi_{\mathbf{p}'}|\varphi|\psi_{\mathbf{p}}^0\rangle = \int d\mathbf{p}''\langle\psi_{\mathbf{p}'}|\varphi|\psi_{\mathbf{p}''}^0\rangle\langle\psi_{\mathbf{p}''}^0|\varphi|\psi_{\mathbf{p}}\rangle$$
$$\times\left(\frac{1}{E_{p''}'-E_{p'}'+i\delta} - \frac{1}{E_{p''}'-E_p'-i\delta}\right). \tag{2.5}$$

Substituting (2.4) in (2.2) and taking into account (2.5) and the spherical symmetry of φ we obtain:

$$(-1)^{2s}(2\pi)^4 n\hbar^2\int d\mathbf{p}_2 d\Delta\,\mathrm{Re}\,[\langle\psi_{-\mathbf{p}_{12}}|\varphi|\psi^0_{\mathbf{p}_{12}-\Delta}\rangle\langle\psi^0_{\mathbf{p}_{12}-\Delta}|\varphi|\psi_{\mathbf{p}_{12}}\rangle]$$
$$\times\delta(E'_{p_{12}} - E'_{p_{12}-\Delta})\{f_{\sigma_1}(\mathbf{p}_1-\Delta)f_{\sigma_1}(\mathbf{p}_2+\Delta) - f_{\sigma_1}(\mathbf{p}_1)f_{\sigma_1}(\mathbf{p}_2)\}. \tag{2.6}$$

To take the limit as $R\to\infty$, we use the fact that

$$\langle\psi_{\mathbf{p}'}^0|\varphi|\psi_{\mathbf{p}}\rangle|_{p'=p} = -\frac{1}{(2\pi)^2\hbar\mu}A_p(\widehat{\mathbf{p}'\mathbf{p}})$$

(see, for example[10]), where $A_p(\widehat{\mathbf{p}'\mathbf{p}})$ is the amplitude of scattering through the angle $\widehat{\mathbf{p}'\mathbf{p}}$. If A is written in the form of a partial expansion, it can be replaced by A′ when substituted in (2.6):

$$A_p'(\theta) = \frac{\hbar}{2ip}\sum_{l=0}^{\infty}(2l+1)e^{2i\delta_l}P_l(\cos\theta),$$

since the function

$$\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)$$

is equal to zero everywhere except at $\theta = 0$[11] (and is integrable). It follows from the asymptotic form of the radial wave function in a Coulomb field, at $r \gg \hbar l/p$[11], that

$$\delta_l \cong -\frac{\mu e^2}{\hbar p}\ln 2\frac{pR}{\hbar} + \arg\Gamma\left(1+l+\frac{i\mu e^2}{\hbar p}\right),\quad l \ll \frac{pR}{\hbar}.$$

Therefore as $R\to\infty$ we have

$$A_p'(\theta)A_p'^*(\theta') \to A_p^{(c)}(\theta)A_p^{(c)*}(\theta'),$$

where

$$A_p^{(c)}(\theta) = \frac{\hbar}{2ip}\sum_{l=0}^{\infty}(2l+1)\frac{\Gamma(1+l+i\mu e^2/\hbar p)}{\Gamma(1+l-i\mu e^2/\hbar p)}P_l(\cos\theta)$$
$$= -\frac{\mu e^2}{2p^2\sin^2(\theta/2)}\exp\left\{-i\frac{\mu e^2}{\hbar p}\ln\sin^2\frac{\theta}{2}\right\}\frac{\Gamma(1+i\mu e^2/\hbar p)}{\Gamma(1-i\mu e^2/\hbar p)}$$

(see, for example,[11]). As a result we obtain from (2.6) the exchange term in (18).

APPENDIX 3

We reduce the direct interaction term in (14) to an explicit form, neglecting all terms of order higher than first in the density. Since $\tilde{g} = \pi(\tilde{g}^0)$, where Π is a linear functional, it follows that if we take into account the contribution of the function $\Pi(\tilde{g}^0_B)$ ($\tilde{g}^0_B$ defined by equality (16)), which was calculated in Appendix 1, it remains to calculate the contribution of the function $\Pi(\tilde{g}^0 - \tilde{g}^0_B)$. We can assume, as the following calculations confirm, that the latter contribution is finite if polarization effects are neglected, i.e., in the lowest order in n the direct interaction term in (14) is equal to

$$I_B(y_1) + \frac{n}{i}\int\frac{dy_2\,d\mathbf{k}}{(2\pi)^3}\tilde{\varphi}_k\exp\left\{-\frac{\hbar\mathbf{k}}{2}\frac{\partial}{\partial\mathbf{p}_1}\right\}S_{\mathbf{k}}[g^0(y_1y_2\mathbf{k}) - g_B^0(y_1y_2\mathbf{k})], \tag{3.1}$$

where $I_B(y_1)$ is the right-hand side of (1.4) without the exchange term.

If we represent the second term in (3.1) in the form of the Fourier transform of an integral of type (1), use the fact that g^0 is the solution of Eq. (2) without the time derivative and the integral term, and then transform to $\tilde{g}^0$ again, we can write it in the form

$$\frac{n}{i}\int\frac{dy_2\,d\mathbf{k}}{(2\pi)^3}d\mathbf{r}\,e^{i\mathbf{kr}}\left[\mathbf{kw}_{12}g^0(y_1y_2\mathbf{k})\right. \tag{3.2}$$
$$\left.-\int\frac{d\mathbf{k}'}{(2\pi)^3}\tilde{\varphi}_{k'}\exp\left\{-\frac{\hbar\mathbf{k}'}{2}\left(\frac{\partial}{\partial\mathbf{p}_1}-\frac{\partial}{\partial\mathbf{p}_2}\right)\right\}S_{\mathbf{k}'}g_B^0(y_1y_2,\mathbf{k}+\mathbf{k}')\right].$$

Just as in Appendix 2, we consider the potential

$\varphi(\mathbf{r}) \equiv \Theta_+(R - r)e^2/r$. After separating the motion of the center of mass, we have from (2.1)

$$\tilde{g}^0(y_1y_2\mathbf{k}) = (2\pi\hbar)^3 \int d\Delta f_{\sigma_1}(\mathbf{p}_1 - \Delta) f_{\sigma_2}(\mathbf{p}_2 + \Delta)\{\langle\psi^0_{\mathbf{p}_{12}-\hbar\mathbf{k}/2}|\psi_{\mathbf{p}_{12}-\Delta}\rangle \times \langle\psi_{\mathbf{p}_{12}-\Delta}|\psi^0_{\mathbf{p}_{12}+\hbar\mathbf{k}/2}\rangle - \delta(\Delta - \hbar\mathbf{k}/2)\delta(\Delta + \hbar\mathbf{k}/2)\}.$$

The integration with respect to $\mathbf{r}$ is carried out in cylindrical coordinates with the axis along $\mathbf{w}_{12}$:

$$\int d\mathbf{r}\, e^{i\mathbf{k}\mathbf{r}} = (2\pi)^2 \frac{\alpha w_{12}\,\delta(\mathbf{k}\mathbf{w}_{12})}{[k^2 - (\mathbf{k}\mathbf{w}_{12}/w_{12})^2 + \alpha^2]^{3/2}}, \quad \alpha \to +0.$$

By virtue of (2.4) and (2.5)

$$\int d\Omega_{\mathbf{k}}\delta(\mathbf{k}\mathbf{w}_{12})\mathbf{k}\mathbf{w}_{12}\tilde{g}^0(y_1y_2\mathbf{k}) = (2\pi)^4 i\hbar^2 \int d\Omega_{\mathbf{k}}\delta(\mathbf{k}\mathbf{w}_{12})\int d\Delta\delta(E'_{\mathbf{p}_{12}-\Delta} - E'_{\mathbf{p}_{12}-\hbar\mathbf{k}/2})\,\mathrm{Re}\,[\langle\psi_{\mathbf{p}_{12}-\hbar\mathbf{k}/2}|\varphi|\psi^0_{\mathbf{p}_{12}-\Delta}\rangle\langle\psi^0_{\mathbf{p}_{12}-\Delta}|\varphi|\psi_{\mathbf{p}_{12}+\hbar\mathbf{k}/2}\rangle] \times\{f_{\sigma_1}(\mathbf{p}_1 - \Delta)f_{\sigma_2}(\mathbf{p}_2 + \Delta) - f_{\sigma_1}(\mathbf{p}_1 - \hbar\mathbf{k}/2)f_{\sigma_2}(\mathbf{p}_2 + \hbar\mathbf{k}/2)\}.$$

The integral with respect to $\Omega_{\mathbf{k}}$ of the second term in the curly brackets in (3.2) can be reduced to an analogous form by replacing ψ with ψ^0. As a result, taking the limit as $R \to \infty$ by the same method as in Appendix 2, we obtain

$$\frac{2}{\pi} ne^4 \int dy_2\, d\Delta_1\, d\Delta \frac{\alpha w_{12}\delta(\Delta_1\mathbf{w}_{12})}{(\Delta_1^2 + \alpha^2)^{3/2}}\,\delta\left(\Delta\mathbf{w}_{12} - \frac{\Delta^2 - \Delta_1^2}{m}\right) \times\left\{|\Delta - \Delta_1|^{-2}|\Delta + \Delta_1|^{-2}\left[\cos\left(\frac{2e^2}{\hbar\sqrt{w_{12}^2 + \Delta_1^2/\mu^2}}\ln\frac{|\Delta - \Delta_1|}{|\Delta + \Delta_1|}\right) - 1\right]\right\} \times[f_{\sigma_1}(\mathbf{p}_1 - \Delta)f_{\sigma_2}(\mathbf{p}_2 + \Delta) - f_{\sigma_1}(\mathbf{p}_1 - \Delta_1)f_{\sigma_2}(\mathbf{p}_2 + \Delta_1)], \quad \alpha \to +0. \tag{3.3}$$

We make the substitution $\Delta = x\Delta_1$ in (3.3); by virtue of the equality

$$\alpha\int_0^\infty \frac{\Delta_1\, d\Delta_1}{(\Delta_1^2 + \alpha^2)^{3/2}}\psi(\Delta_1) = \psi(0), \quad \alpha \to +0,$$

and recognizing that the expression in the curly brackets in (3.3) is even in Δ/Δ and that Δ_1/Δ_1 is not changed by the substitution $\Delta_1/\Delta_1 \rightleftarrows \Delta/\Delta$, we get

$$\frac{ne^4}{\pi}\frac{\partial}{\partial\mathbf{p}_1}\int dy_2\, d\Omega_{\mathbf{q}}\int_0^\infty dx\,(x^2 - 1)x\int_0^{2\pi} d\varphi[(x^2 + 1)^2 - 4x^2\cos^2\varphi]^{-1} \times\left[\cos\left(\frac{e^2}{\hbar w_{12}}\ln\frac{x^2 - 2x\cos\varphi + 1}{x^2 + 2x\cos\varphi + 1}\right) - 1\right]\mathrm{Q}_{12}(\mathbf{q}). \tag{3.4}$$

Combining this expression with I_B in (3.1), we get the first terms of the right-hand side of (18) (we have changed over in the integral over x in (3.4), in the interval 1 to ∞, to the variable $1/x$).

APPENDIX 4

Let us find the limit of $\zeta(\eta)$ in (19) as $\eta \to \infty$. We write the integral with respect to x in the form

$$\int_0^1 dx\left(\frac{1}{x} - \frac{4x\sin^2\varphi}{(x^2 - 1)^2 + 4x^2\sin^2\varphi}\right)\{\cos[\eta R(x, \varphi)] - 1\}$$

and consider separately the integrals from the terms in the first bracket. In the first we integrate by parts and then take $R(x, \varphi)$ as the variable of integration. We get

$$\eta\,\mathrm{Im}\int_0^{R(1,\varphi)} dz\ln\left(\frac{\cos\varphi}{\mathrm{th}(z/2)} - \sqrt{\frac{\cos^2\varphi}{\mathrm{th}^2(z/2)} - 1}\right)e^{i\eta z} \underset{\eta\to\infty}{\to} -C - \ln(4\eta\cos\varphi),$$

(on deformation of the integration contour to the upper half-plane, a nonvanishing contribution is made as $\eta \to \infty$ only by a segment of the straight line Re z = 0). When integrating the second term in the parentheses, the contribution of the cosine vanishes as $\eta \to \infty$ because of the fast oscillations; the remaining integral equals $(\pi/2 - \varphi)\tan\varphi$. As a result we obtain:

$$\ln\zeta(\eta) \underset{\eta\to\infty}{\to} C + 2\ln 2 - \frac{2}{\pi}\int_0^{\pi/2} d\varphi\left[-\ln\cos\varphi + \left(\frac{\pi}{2} - \varphi\right)\mathrm{tg}\,\varphi\right] = C.$$

[1] V. N. Alyamovskiĭ, Zh. Eksp. Teor. Fiz. 60, 1672 (1971) [Sov. Phys.-JETP 33, 906 (1971)].

[2] N. N. Bogolyubov, Problemy dinamicheskoĭ teorii v statisticheskoĭ fizike (Problems of Dynamic Theory in Statistical Physics), Gostekhizdat, 1946.

[3] N. N. Bogolyubov and G. P. Gurov, Zh. Eksp. Teor. Fiz. 17, 614 (1947).

[4] R. Balescu, Phys. Fluids, 4, 94 (1961).

[5] V. P. Silin, Zh. Eksp. Teor. Fiz. 40, 1768 (1961) [Sov. Phys.-JETP 13, 1244 (1961)]; Fiz. Met. Metallov, 11, 805 (1961).

[6] O. V. Konstantinov and V. I. Perel', Zh. Eksp. Teor. Fiz. 39, 861 (1960) [Sov. Phys.-JETP 12, 597 (1961)].

[7] H. A. Gould and H. E. DeWitt, Phys. Rev., 155, 68 (1967).

[8] Yu. L. Klimontovich and S. V. Temko, Zh. Eksp. Teor. Fiz. 33, 132 (1957) [Sov. Phys.-JETP 6, 102 (1958)].

[9] R. Balescu, Statistical Mechanics of Charged Particles, Int. Pub., 1963.

[10] A. S. Davydov, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963.

[11] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963 [Addison-Wesley, 1965].

Translated by J. G. Adashko
97