RESONANCE ABSORPTION OF ELECTROMAGNETIC ENERGY BY IONS ON A VORTEX IN SUPERFLUID HELIUM

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The possibility of observing resonance absorption of the energy of an electromagnetic field by vortex-ion complexes in a rotating He II is considered theoretically. It is shown that at temperatures $T < 0.5°K$ such resonance may appear at frequencies $\sim 10^{12}$ sec^{-1}. The dependence of the resonance frequency on the ion and vortex-core radii and on the density of the superfluid component is derived. The deviation of the vortex ion spectrum from an equidistant one is estimated. Possible causes of the resonance line broadening—viscosity due to the interaction of the ion with the vortex excitations and Coulomb interaction between the ions—are considered, and the relevant estimates are presented. The absorption coefficient is computed.

1. INTRODUCTION

HALLEY and Cheung[1] have investigated theoretically the problem of the resonance perturbation by an alternating electric field of the motion of ions in rotating superfluid helium. It was discovered that the given resonance can appear at a definite relationship between the eigenfrequencies of the vortex oscillations and the frequency of the perturbing field. They showed also that this resonance can be detected by studying the mobility of the ions along the vortex filaments. It turns out that the resonance will manifest itself in a sharp decrease of this mobility.

In the present paper we undertake to study this phenomenon from a different point of view, namely, to investigate the problem as to how the absorption (or emission) of electromagnetic energy occurs in such a system, and to compute the intensity of these processes in the first nonvanishing order. In contrast to the resonance at vortex oscillation frequencies $\omega_{lkm} \sim 10^7$–10^9 sec^{-1} studied in[1], we investigate the possibility of observing a resonance at frequencies $\omega \sim 10^{12}$ sec^{-1} which characterize, at definite temperatures, the oscillations of an ion on a vortex line. We postulate that an experimental investigation of this resonance will enable us to obtain additional information (and to determine more accurately the existing data) about the properties of the vortex motion of He II, in particular, about vortex structure and intervortex and ion-vortex interactions.

The fruitfulness of the various types of resonance methods in the physics of solids and ordinary liquids is well known, and we assume that the application of these methods to the investigation of liquid helium will be promising. In the present paper we concentrate mainly on the computation of the steady-state effects of the interaction between the electromagnetic field and the vortex ions. In this case, as can easily be shown, we can neglect the coherent properties of the system under consideration and consider the contributions from the ions to the intensity to be independent of each other. It is clear that such a simplification is justified only as a first step in the estimate of the magnitude of the supposed effects. Indeed, the correlations in the motion of the ions that determine the coherent properties largely depend on the interactions obtaining in helium, and carry a great deal of information about these interactions. Thus, practically the most interesting of the experiments of this sort would be those experiments in which the role of these correlations were more fully exhibited. As is well known[2], such experiments involve the observation of nonstationary superradiant responses of the system: induction and echo.

2. COMPUTATION OF THE SPECTRUM AND LINE WIDTHS

In the presence of an electromagnetic field the ion-vortex system is described by the Hamiltonian[1]

$$H = H_1 + H_2 + H'_{12} + H''_{12} + H_3 + H_{23}, \qquad (1)$$

where H_1 is the Hamiltonian of a free vortex, H_2 that of a free ion, and H_3 that of the free electromagnetic field; H'_{12} is the ion-vortex interaction potential energy which commutes with H_1; H''_{12} is the ion-vortex interaction potential energy which does not commute with H_1 and is responsible for the energy exchange between the ions and vortices (for more details in connection with the forms of the interactions H'_{12} and H''_{12}, see[1,3]); H_{23} takes account of the ion-field (electromagnetic) interaction.

We now note that in the absence of energy exchange between the three subsystems which make up the whole ion-vortex-field system (i.e., for $H''_{12} = H_{23} = 0$), the energy of a vortex ion will be determined by the time-independent Hamiltonian $\tilde{H}_2 = H_2 + V(r_j)$ ($[H, \tilde{H}_2]_- = 0$ for $H_{12} = H_{23} = 0$), where r_j is the distance of the center of the ion from the vortex line. The spectrum of this Hamiltonian is determined by the form of $V(r_j) = H'_{12}$[1]:

$$V(r_j) = \frac{\hbar^2}{2\mu} \int_j d\mathbf{r}\, \psi^* \nabla^2 \psi; \qquad (2)$$

here μ is the mass of the helium atom, ψ the wave function of the unexcited condensate with a vortex, and the symbol j indicates that the integral is evaluated over the volume of the ion.

Generally speaking, the determination of the wave function ψ involves the solution of the nonlinear differential equation[4,5]

$$\frac{\hbar^2}{2\mu}\nabla^2\psi = V_0(n - n_0)\psi, \tag{3}$$

where V_0 is a constant describing the interaction between helium atoms, $n = n(r) = |\psi|^2$ is the density of the superfluid component as a function of the distance from the vortex line, and n_0 is the density of the superfluid component far from the vortex (it is a constant and is assumed to be equal to the density in the absence of the vortex). This solution was found by numerical methods[5]; its radial part vanished on the vortex line and tended to some constant limit far from it. Subsequently, Fetter[4] proposed for the radial part of the wave function ψ a simple analytical expression which behaved as the exact solution in the indicated limits

$$f(r) = n_0^{1/2} r (r^2 + a^2)^{-1/2}, \tag{4}$$

where $f(r)$ is the radial part of the wave function ($\psi = e^{i\vartheta} f(r)$) and $a = \hbar/(2mn_0V_0)^{1/2}$ is the radius of the vortex core. Fetter also showed that the use of this approximation did not introduce significant changes into the results of the computation of the quantities characterizing the vortex (thus, the exact value of the energy per unit vortex length differs by only a small logarithmic constant from the value computed by the Fetter method).

Let us, using (3) and (4), now transform (2). After a number of steps in the calculation we obtain

$$V(r_j) = -\frac{\hbar^2 n_0}{2\mu}\int_0^R dr\, r[R^2 - r^2]^{1/2}\int_0^{2\pi}\frac{(r^2 + r_j^2 + 2rr_j\cos\vartheta)\,d\vartheta}{(a^2 + r^2 + r_j^2 + 2rr_j\cos\vartheta)^2}, \tag{5}$$

where R is the effective radius of the ion[3]. This potential has a sharp maximum at $r_j = 0$, and in the vicinity of this point it may be written in the form

$$V(r_j) = V(0) + \frac{1}{2}\frac{d^2V(0)}{dr_j^2}r_j^2. \tag{6}$$

Thus, for small oscillations near the vortex line the ion behaves like a harmonic oscillator.

The results of the computation of the constants entering into (6) are somewhat different for positive and negative ions, and this is connected with the difference in the effective dimensions ($R_+/a \approx 4.6$; $r_-/a \approx 10$ [3]). In the case of negative ions

$$V_-(0) = -\frac{\hbar^2 n_0 \pi R_-}{2\mu}\left(2\ln\frac{R_-}{a} - 2{,}3\right) \approx -8\cdot 10^{-11} n_0\ [\sec^{-1}],$$

$$\omega_- = \left(\frac{1}{M}\frac{d^2V(0)}{dr_j^2}\right)^{1/2} = \frac{\hbar R_-^{3/2}}{a^2\mu}\left(\frac{\pi n_0}{2\alpha}\right)^{1/2} \approx 3\sqrt{n_0}\ [\sec^{-1}]; \tag{7}$$

$\alpha \equiv M/\mu$. For positive ions

$$V_+(0) = -\frac{\hbar^2 n_0 \pi R_+}{2\mu}\left[2\ln\frac{R_+}{a} - 2.3 + \left(0.93 + 2\ln\frac{R_+}{a}\right)\left(\frac{a}{R_+}\right)^2\right] \approx$$
$$\approx -1{,}3\cdot 10^{-11} n_0\ [\text{сек}^{-1}],$$

$$\omega_+ = \frac{\hbar R_+^{3/2}}{a^2\mu}\left(\frac{\pi n_0}{2\alpha}\right)^{1/2}\left(1 - \frac{3}{2}\frac{a^2}{R_+^2}\right) \approx 1.5\sqrt{n_0}\ [\sec^{-1}]. \tag{8}$$

Thus, we see that at sufficiently low temperatures $T < 0.5°K$, when the density of the superfluid component can be assumed to be approximately equal to the total particle density in He II[6] ($n_0 \sim 10^{24}$), an ion on a vortex behaves like an oscillator of frequency $\omega \sim 10^{12}$ sec^{-1}. More precisely, we may assert that at least the lower levels of an ion on a vortex will have an equidistant vibrational character (this follows from the large ratio of the depth of the potential well to $\hbar\omega$). When the ion is in equilibrium in the indicated temperature range only the lowest level is preponderantly populated and, therefore, we can, in computing the absorption rate, neglect the deviation from equal spacing at the higher parts of the spectrum. Such situations can, however, arise when many levels are comparably populated. In that case the uneven spacing of the spectrum will, if it is small (the condition for smallness is the comparability of the difference between two frequencies of the spectrum with the homogeneous width of a line), manifest itself in a nonuniform broadening of the resonance line. It turns out that consideration of the next term ($\sim r_j^4$) results in a shift of the levels by the amount

$$\hbar^{-1}\Delta E \approx 10^7[n^2 + (n+1)^2 + n(n+1)]. \tag{9}$$

The homogeneous width of a line may be estimated in the following way. We shall consider an ion on a vortex as an oscillator immersed in a liquid of viscosity η. In that case the width of its line

$$\gamma = \eta/M.$$

The viscosity of helium with respect to an ion is easily estimated from mobility measurements[1]. In the temperature range of interest to us, $\gamma \sim 10^7$ sec^{-1}, i.e., comparable with the frequency shifts (9).

We have not taken into account in the theory developed here the Coulomb repulsion between ions. In other words, we have neglected in the ion gas the relaxation processes caused by an interaction of the form q^2/r_{jk}, where r_{jk} is the distance between a pair of ions. It turns out that such neglect is justified at ion densities $\sim 10^{15}$ cm^{-3}. Indeed, let us consider two charged linear oscillators at a distance r_0 from each other. It turns out that if in the beginning one of the oscillators was displaced from the equilibrium position while the other remained unexcited, then in the presence of the Coulomb interaction the difference between the energies of the oscillators will decrease with the time, and in the limit of long intervals of time this difference executes small oscillations about zero. The time τ required for the establishment of such an equilibrium is given by the formula $\tau = q^{-2}\pi\omega M r_0^3$ (ω is the frequency of the oscillator, M is its mass, and q its charge). If we take this time as the characteristic time for the establishment of equilibrium in a system of oscillators with a mean distance r_0 between them, then in the case of interest to us $\tau \sim 10^{-5}$ sec for the indicated ion density. We see that the contribution to the line width due to this mechansim is much smaller than the previously computed width resulting from viscous friction.

The Coulomb repulsion will manifest itself again in the motion of the ions along the vortex tubes, and this will result in a continuous decrease of the number of absorbing ions. It can, however, be shown that for a linear ion density $\lambda \sim 10^3$ cm^{-1} on a vortex the distribution of the ions is practically constant. Indeed, it is sufficient if this distribution changed little during times comparable with the relaxation time, which we estimated above to be $\sim 10^{-7}$ sec. During this time the distance $r_0 \sim 10^{-3}$ cm between the ions increases, owing to the Coulomb repulsion, by an amount $\sim 10^{-6}$ cm (for a mobility $\mu = 10^2\ cm^2 \times V^{-1} \times sec^{-1}$ [1]), i.e. by an extremely small amount. Thus, for vortex densities of $\sim 10^{10}\ cm^{-2}$ and a linear ion density on a vortex of $\sim 10^3\ cm^{-1}$, the Coulomb repulsion will not have any effect on the resonance pattern.

The data obtained are quite sufficient for an estimate of the absorption coefficient. Since the above-considered temperatures $T \ll k^{-1}\hbar\omega$ (where k is the Boltzmann constant), the overwhelming majority of the ions are in the lowest energy state. In this case the absorption will be determined, in the main, by transitions to the first excited state. The rate of absorption by a layer dl will therefore be determined by the formula $dI = -(2cM)^{-1} Iq^2 Nf(\omega) dl$, where I is the intensity of the incident wave, N the concentration of ions entrapped by the vortices, c the velocity of light, $f(\omega)$ a function of the form of a line, and the absorption coefficient in the resonance

$$\sigma = q^2 N (2cM\pi\gamma)^{-1}. \quad (10)$$

Generally speaking, the absorption coefficient depends on the angular velocity of the container with the helium. Indeed, the concentration of the entrapped ions can be expressed in terms of the vortex density ν and the linear ion density λ along a vortex: $N = \nu\lambda$. Owing to the Coulomb repulsion (see above), λ has a tendency to attain the saturation value, but[7] $\nu = 2\Omega/\kappa$, where Ω is the angular velocity of the helium container, and $\kappa = 10^{-3}\ cm^2/sec$ is the circulation quantum, Then we obtain finally

$$\sigma = q^2 \Omega\lambda / cM\pi\gamma\kappa$$

and in the case of saturation (λ = const), the absorption coefficient depends only on the angular velocity Ω. If we assume $\lambda = 10^3\ cm^{-1}$ and $\gamma = 10^7\ sec^{-1}$, then

$$\sigma = 10^{-9}\ \Omega\ [cm^{-1}].$$

At low angular velocities ~1 rpm the absorption coefficient becomes a quite measurable quantity.

Thus, we have established that at temperatures $T < 0.5°K$ there exists in the absorption spectrum of a rotating liquid helium with ions a reasonably narrow line of frequency $\sim 10^{12}\ sec^{-1}$, due to oscillations of ions on the vortex lines in He II.

The frequency at which absorption occurs depends on the structure constant of the vortex-ion complex, as well as on the density of the superfluid component.

At angular velocities of the He-II container up to $10^7\ sec^{-1}$ and at linear ion densities along a vortex line up to $10^3\ cm^{-1}$, the various types of interaction between an ion and the vortex excitations which lead to the appearance in He II of viscous friction with respect to the ion constitute the predominant mechanism of the broad-

The absorption coefficient depends decisively on the angular velocity of the He-II container if the linear ion density along the vortex tubes has reached the saturation point as a result of the Coulomb repulsion between the ions.

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