DAMPING OF LONG-WAVE PLASMA OSCILLATIONS

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Formulas which are exact in the long-wave limit are derived for plasma oscillations damping due to collisions between charged particles. In a one-component plasma with a homogeneous compensating background the damping is $\sim k^2$ for $k \to 0$, whereas in a two-component plasma with particles of unequal masses it is constant for $k \to 0$. The integrals defining the damping are calculated explicitly for the one and two-component models in the limiting cases of a hot and cold Boltzmann plasma.

1. The purpose of the paper is an exact calculation, in the limit of long waves (small wave vectors **k**), of the damping of plasma oscillations by particle collisions. We consider a model of a system of electrons against the background of a homogeneous compensating charge, and also a more realistic model of a two-component plasma. In the first model, and also in a two-component plasma of particles of equal mass, the calculated part of the damping decreases in proportion to k^2 as $k \to 0$. On the other hand, if the masses of the positive and negative charges are different, then the damping tends to a constant γ_0 as $k \to 0$. In any case, at $k \to 0$ the collision part of the damping is larger than the collisionless Landau damping[1].

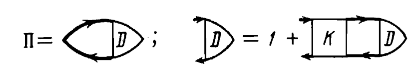
In diagram perturbation theory (cf.[2]), the problem reduces to finding the imaginary part of the polarization operator $\Pi(\mathbf{k}, E)$. In the one-component model, we express this function in terms of a vertex part that satisfies an equation of the kinetic type. In a multi-component system it is necessary to consider several vertex parts and a system of kinetic equations. The sought damping is given by a second-approximation correction in the solution of the kinetic equation (system). The cross sections for the scattering of the particles in the medium, which enter in the collision integrals, are taken in an approximation that takes into account the dynamic polarization of the medium[3].

The kinetic equations and a general formula for the damping are obtained in Secs. 2 and 3 for a single-component system, and are generalized then in Sec. 4 to a two-component system. In Sec. 5 we calculate the integrals that determine the damping for one-component and two-component systems in the limiting cases of a "hot" and "cold" Boltzmann plasma. In the intermediate calculations we used a system of units $\hbar = k_B = 1$, where $\hbar$ and k_B are the Planck and Boltzmann constants, and we return to the ordinary units in the final answers.

2. As is well known, the plasma-oscillation spectrum is determined by the poles of the density-oscillation Green's function $G_\rho(\mathbf{k}, E)$, which is expressed in terms of the polarization operator $\Pi(\mathbf{k}, E)$ by the formula

$$G_\rho(\mathbf{k}, E) = \Pi(\mathbf{k}, E)\left(1 - \frac{4\pi e^2}{k^2}\Pi(\mathbf{k}, E)\right)^{-1}. \tag{2.1}$$

We consider first a system of electrons against the background of a uniform compensating charge. The following exact diagram equations hold:

 (2.2)

where the thick lines correspond to the exact electronic Green's function. The first of these equations makes it possible to express the function $\Pi(\mathbf{k}, E)$ in terms of the electronic Green's function and the vertex part D. The second is an equation for the vertex part, in which K denotes an irreducible four-pole diagram (the sum of the contribution of diagrams that cannot be cut vertically by breaking two lines).

Equations of the type (2.2) were considered in[4] for an electron-phonon system. Following[4], we can reduce the second equation of (2.2) at small k and $E \sim \omega_0$, where ω_0 is the plasma frequency, to an equation of the form

$$(E - \mathbf{k}\mathbf{k}_1/m)h + iI(h) = 1 \tag{2.3}$$

for a new unknown function $h(\mathbf{k}_1, \mathbf{k}, E)$. By $I(h)$ we denote the expression

$$I(h) = \frac{(1 + e^{-\beta\varepsilon_1})}{(2\pi)^5}\int d^3k_2\, d^3k_3\, d^3k_4\, n_2 n_3 n_4 e^{\beta(\varepsilon_1+\varepsilon_2)} \times |\Gamma|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)\,\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times (h_1 + h_2 - h_3 - h_4). \tag{2.4}$$

Here $\epsilon_i \equiv \epsilon(\mathbf{k}_i) = k_i^2/2m - \mu$ is the energy spectrum of the electrons (μ is the chemical potential), $h_i = h(\mathbf{k}_i, \mathbf{k}, E)$, $n_i = (e^{\beta\epsilon_i} + 1)^{-1}$ is the Fermi distribution function. The factor $|\Gamma|^2$ in (2.4) is the square of the modulus of the four-point diagram and is proportional to the differential cross section for the scattering of two electrons in the medium.

Equation (2.3) has the structure of an inhomogeneous linearized kinetic equation in which $I(h)$ plays the role of the collision integral. The function $\Pi(k, E)$ of interest to us is expressed in terms of the solution h of Eq. (2.3) with the aid of the formula

$$\Pi(\mathbf{k}, E) = \frac{\beta}{(2\pi)^3}\int d^3k_1 \frac{e^{\beta\varepsilon_1}}{(e^{\beta\varepsilon_1} + 1)^2}(Eh_1 - 1) \equiv \beta\langle Eh - 1\rangle, \tag{2.5}$$

to which the first of the diagram equations in (2.2) reduces (see[4]).

3. Let us examine Eq. (2.3). At small k and $E \sim \omega_0$, the collision integral $I(h)$ is small in com-

parison with the other terms of the equation, so that in the first approximation we have

$$h \approx h^{(1)}(\mathbf{k}_1, \mathbf{k}, E) = (E - \mathbf{k}\mathbf{k}_1/m)^{-1}. \tag{3.1}$$

Substituting $h^{(1)}$ in (2.5), we obtain an expression corresponding to the simplest second-order diagram, namely an electron loop

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which will henceforth be denoted Π_0. The imaginary part of Π_0 determines the Landau damping (cf, e.g.,[5]).

The second approximation $h^{(2)}$ for h is obtained by replacing h in the collision integral by its first approximation $h^{(1)}$

$$h^{(2)} = (E - \mathbf{k}\mathbf{k}_1/m)^{-1}(1 - iI(h^{(1)})) = h^{(1)} - ih^{(1)}I(h^{(1)}). \tag{3.2}$$

Substitution of $h^{(2)}$ in (2.5) yields the second approximation for Π in the form

$$\Pi_0 - iE\beta\langle h^{(1)}I(h^{(1)})\rangle = \Pi_0 + \Delta\Pi. \tag{3.3}$$

For the imaginary part of the correction $\Delta\Pi$, which is of interest to us, we obtain the expression

$$\begin{aligned} |\mathrm{Im}\Delta\Pi| = {} & \frac{\beta E}{4(2\pi)^8}\int d^3k_1\, d^3k_2\, d^3k_3\, d^3k_4 \\ & \times e^{\beta(\varepsilon_1+\varepsilon_2)} n_1 n_2 n_3 n_4 |\Gamma|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \\ & \times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4)(h_1^{(1)} + h_2^{(1)} - h_3^{(1)} - h_4^{(1)})^2. \end{aligned} \tag{3.4}$$

where $h_i^{(1)} = h^{(1)}(\mathbf{k}_i, \mathbf{k}, E)$. At small values of k we use the expansion

$$h_i^{(1)} = \left(E - \frac{\mathbf{k}\mathbf{k}_i}{m}\right)^{-1} = E^{-1}\left(1 + \frac{\mathbf{k}\mathbf{k}_i}{mE} + \frac{(\mathbf{k}\mathbf{k}_i)^2}{(mE)^2} + \ldots\right), \tag{3.5}$$

the third term of which makes the main contribution to $(h_1^{(1)} + h_2^{(1)} - h_3^{(1)} - h_4^{(1)})$, since the contribution of the first two terms vanishes identically.

The damping γ determined from (3.4) is given by the formula

$$\begin{aligned} \gamma = \frac{4\pi e^2}{k^2} E\,|\mathrm{Im}\,\Delta\Pi|_{E=\omega_0} = {} & k^2 \frac{\beta e^2}{2(2\pi)^7(m\omega_0)^4}\int d^3k_1\, d^3k_2\, d^3k_3\, d^3k_4 e^{\beta(\varepsilon_1+\varepsilon_2)} \\ & \times n_1 n_2 n_3 n_4 |\Gamma|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \cdot \\ & \times ((\mathbf{n}\mathbf{k}_1)^2 + (\mathbf{n}\mathbf{k}_2)^2 - (\mathbf{n}\mathbf{k}_3)^2 - (\mathbf{n}\mathbf{k}_4)^2)^2 \end{aligned} \tag{3.6}$$

where $\mathbf{n}$ is a unit vector parallel to $\mathbf{k}$. The damping (3.6) is proportional to k^2 and exceeds the Landau damping when $k \to 0$. We shall see later on that formula (3.6) holds true also for a two-component system of particles of equal mass.

4. The generalization of the results to include a two-component system entails no difficulty. We have here two vertex parts in place of one, and accordingly two functions h and g in place of the function h; these functions satisfy a system of kinetic equations in the form

$$\begin{aligned} (E - \mathbf{k}\mathbf{k}_1/m_1)h + i(I_{11}(h) + I_{12}(h, g)) &= 1, \\ (E - \mathbf{k}\mathbf{k}_1/m_2)g + i(I_{21}(h, g) + I_{22}(g)) &= 1. \end{aligned} \tag{4.1}$$

Here I_{ik} are the collision integrals describing the scattering of particles of one sort at $i = k$ and of different sorts at $i \neq k$. The formula for $\Pi(\mathbf{k}, E)$ (the analog of (2.5) is

$$\begin{aligned} \Pi(\mathbf{k}, E) = {} & \frac{\beta}{(2\pi)^3}\int d^3k_1 \frac{e^{\beta\varepsilon_1}}{(e^{\beta\varepsilon_1} + 1)^2}(Eh_1 - 1) \\ & + \frac{\beta}{(2\pi)^3}\int d^3k_1 \frac{e^{\beta\tilde{\varepsilon}_1}}{(e^{\beta\tilde{\varepsilon}_1} + 1)^2}(Eg_1 - 1) \equiv \beta(\langle Eh - 1\rangle_1 + \langle Eg - 1\rangle_2), \end{aligned} \tag{4.2}$$

where

$$\varepsilon_1 = k_1^2/2m_1 - \mu_1, \quad \tilde{\varepsilon}_1 = k_1^2/2m_2 - \mu_2,$$
$$h_1 = h(\mathbf{k}_1, \mathbf{k}, E), \quad g_1 = g(\mathbf{k}_1, \mathbf{k}, E).$$

A solution of the system (4.1) in accordance with the scheme indicated in Sec. 3, gives an expression for $|\mathrm{Im}\,\Delta\Pi|$ in the form of a sum of integrals of the type (3.4). The integrands contain as factors the squares

$$(h_1^{(1)} + h_2^{(1)} - h_3^{(1)} - h_4^{(1)})^2, \quad (g_1^{(1)} + g_2^{(1)} - g_3^{(1)} - g_4^{(1)})^2 \quad \text{or}$$
$$(h_1^{(1)} + g_2^{(1)} - h_3^{(1)} - g_4^{(1)})^2.$$

The functions

$$\begin{aligned} h_i^{(1)} &= \left(E - \frac{\mathbf{k}\mathbf{k}_i}{m_1}\right)^{-1} = E^{-1}\left(1 + \frac{\mathbf{k}\mathbf{k}_i}{m_1 E} + \frac{(\mathbf{k}\mathbf{k}_i)^2}{(m_1 E)^2} + \ldots\right), \\ g_i^{(1)} &= \left(E - \frac{\mathbf{k}\mathbf{k}_i}{m_2}\right)^{-1} = E^{-1}\left(1 + \frac{\mathbf{k}\mathbf{k}_i}{m_2 E} + \frac{(\mathbf{k}\mathbf{k}_i)^2}{(m_2 E)^2} + \ldots\right), \end{aligned} \tag{4.3}$$

are the first approximations in the solution of the system (4.1). It follows from (4.3) that the expressions $h_1^{(1)} + h_2^{(1)} - h_3^{(1)} - h_4^{(1)}$, $g_1^{(1)} + g_2^{(1)} - g_3^{(1)} - g_4^{(1)}$ are quadratic in k at small k, and the expression $h_1^{(1)} + g_2^{(1)} - h_3^{(1)} - g_4^{(1)}$ is linear in k when $m_1 \neq m_2$. Therefore at $m_1 \neq m_2$ the contribution of the integral with $(h_1^{(1)} + g_2^{(1)} - h_3^{(1)} - g_4^{(1)})^2$ is predominant. Physically this means that the main contribution to the damping is made by collisions of particles of different masses. As $k \to 0$, the damping tends to the limit

$$\begin{aligned} \gamma_0 = {} & \frac{\beta e^2}{(2\pi)^7 \omega_0^2}\int d^3k_1 d^3k_2 d^3k_3 d^3k_4 e^{\beta(\varepsilon_1+\tilde{\varepsilon}_2)} n_1\tilde{n}_2 n_3\tilde{n}_4 |\Gamma_{12}|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \\ & \times \delta(\varepsilon_1 + \tilde{\varepsilon}_2 - \varepsilon_3 - \tilde{\varepsilon}_4)\left(\frac{\mathbf{n}\mathbf{k}_1}{m_1} + \frac{\mathbf{n}\mathbf{k}_2}{m_2} - \frac{\mathbf{n}\mathbf{k}_3}{m_1} - \frac{\mathbf{n}\mathbf{k}_4}{m_2}\right)^2 \\ = {} & \left(\frac{1}{m_1} - \frac{1}{m_2}\right)^2 \frac{\beta e^2}{(2\pi)^7 \omega_0^2}\int d^3k_1 d^3k_2 d^3k_3 d^3k_4 e^{\beta(\varepsilon_1+\tilde{\varepsilon}_2)} n_1\tilde{n}_2 n_3\tilde{n}_4 |\Gamma_{12}|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 \\ & - \mathbf{k}_3 - \mathbf{k}_4)\delta(\varepsilon_1 + \tilde{\varepsilon}_2 - \varepsilon_3 - \tilde{\varepsilon}_4)(\mathbf{n}, \mathbf{k}_1 - \mathbf{k}_3)^2. \end{aligned} \tag{4.4}$$

On the other hand, if the masses of the positive and negative charges are equal, then $\gamma \sim k^2$. Formula (3.6) remains in force in this case if the cross sections for the stamping of particles having charges of one sign and of opposite signs can be regarded as equal.

5. We consider in greater detail the case of a Boltzmann plasma. In this case $n_i \approx e^{-\beta\epsilon_i}$, $\tilde{n}_i \approx e^{-\beta\tilde{\epsilon}_i}$.

For the one-component system, expression (3.6) in terms of the variables

$$\mathbf{P} = \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2), \quad \mathbf{Q} = \tfrac{1}{2}(\mathbf{k}_3 + \mathbf{k}_4), \quad \mathbf{p} = \mathbf{k}_1 - \mathbf{k}_3, \quad \mathbf{q} = \mathbf{k}_1 - \mathbf{k}_4 \tag{5.1}$$

takes the form

$$\begin{aligned} \gamma = {} & k^2 \frac{2me^2\beta e^{2\beta\mu}}{(2\pi)^7(m\omega_0)^4}\int d^3P d^3Q d^3p d^3q |\Gamma|^2 \cdot \\ & \times \exp\left\{-\frac{\beta P^2}{m} - \frac{\beta(p^2 + q^2)}{4m}\right\}(\mathbf{n}\mathbf{p})^2(\mathbf{n}\mathbf{q})^2\delta(\mathbf{p}\mathbf{q})\delta(\mathbf{P} - \mathbf{Q}). \end{aligned} \tag{5.2}$$

For a "hot" plasma we use the approximation

$$\Gamma \approx \frac{4\pi e^2}{p^2}\left(1 - \frac{4\pi e^2}{p^2}\Pi_0\right)^{-1}, \tag{5.3}$$

which takes into account the dynamic polarization of the medium[3] and corresponds to the Born approximation for the amplitude for the scattering of two electrons in a medium. The function (5.3) depends on the variables p and $P|\cos(\mathbf{p}, \mathbf{P})|$.

The presence of the δ-function $\delta(\mathbf{P} - \mathbf{Q})$ eliminates the integration with respect to $\mathbf{Q}$, and the integral with respect to $\mathbf{q}$ is equal to

$$\int d^3q(\mathbf{nq})^2\delta(\mathbf{pq})\exp\left(-\frac{\beta q^2}{4m}\right)=\frac{\pi}{2}\left(\frac{4m}{\beta}\right)^2p^{-3}(p^2-(\mathbf{np})^2). \quad (5.4)$$

The integral with respect to P reduces to a one-dimensional one

$$\int d^2P|\Gamma(p,P|\cos(\widehat{\mathbf{p},\mathbf{P}})|)|^2\exp\left(-\frac{\beta P^2}{m}\right)$$
$$=\frac{2\pi m}{\beta}\int_0^\infty dP|\Gamma(p,P)|^2\exp\left(-\frac{\beta P^2}{m}\right), \quad (5.5)$$

and the integral with respect to the angular part of the momentum $\mathbf{p}$ is

$$\int d\Omega_{\mathbf{p}}(\mathbf{np})^2(p^2-(\mathbf{np})^2)=8\pi p^4/15. \quad (5.6)$$

Expression (5.2) takes the form

$$\gamma=k^2\frac{2e^2e^{2\beta\mu}}{15(\pi\omega_0)^4\beta^2}\int_0^\infty dP\exp\left(-\frac{\beta P^2}{m}\right)\int_0^\infty p^3dp|\Gamma(p,P)|^2\exp\left(-\frac{\beta p^2}{4m}\right). \quad (5.7)$$

In terms of the approximation (5.3) we have

$$|\Gamma(p,P)|^2=\frac{(4\pi e^2)^2}{(p^2+a(P))^2+b^2(P)}, \quad (5.8)$$

where the functions a and b are defined as the real and imaginary parts of the expression

$$c=a+ib=a(0)\left\{\left(1-2e^{-t^2}\int_0^t e^{u^2}du\right)+i\pi^{1/2}te^{-t^2}\right\}\equiv a(0)(\varphi(t)+i\psi(t)), \quad (5.9)$$

in which

$$a(0)=me^2e^{\beta\mu}\left(\frac{2m}{\pi}\right)^{1/2},\quad t=P\left(\frac{\beta}{2m}\right)^{1/2}. \quad (5.10)$$

In the integral with respect to p we can replace $\exp(-\beta p^2/4m)$ by unity for small p ($p < k_0$), and neglect a and b in (5.8) for large p ($p > k_0$):

$$\int_0^\infty p^3dp|\Gamma(p,P)|^2\exp\left(-\frac{\beta p^2}{4m}\right)$$
$$=(4\pi e^2)^2\left(\int_0^{k_0}\frac{p^3dp}{(p^2+a)^2+b^2}+\int_{k_0}^\infty\frac{dp}{p}\exp\left(-\frac{\beta p^2}{4m}\right)\right)$$
$$=\frac{(4\pi e^2)^2}{2}\left(\frac{1}{2}\ln\frac{16m^2}{\beta^2(a^2+b^2)}-\frac{a}{b}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{a}{b}\right)-C\right), \quad (5.11)$$

where C = 0.577 ... is Euler's constant.

In the remaining one-dimensional integral with respect to P, we change over to the dimensionless $t = P(\beta/2m)^{1/2}$, and eliminate the factors $e^{\beta\mu}$ and ω_0 by using the formulas

$$\rho=(2\pi)^{-3}\int e^{-\beta\varepsilon}d^3k=e^{\beta\mu}\left(\frac{m}{2\pi\beta}\right)^{3/2},\quad \omega_0^2=\frac{4\pi e^2\rho}{m}. \quad (5.12)$$

As the result we obtain an expression for the damping in the form

$$\gamma=k^2\frac{4e^2}{15(\pi mk_BT)^{1/2}}\left[\ln\frac{mk_B^2T^2}{\pi e^2\rho\hbar^2}-C-2J\right], \quad (5.13)$$

where

$$J=\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-2t^2}\left[\frac{2\varphi(t)}{\psi(t)}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{\varphi(t)}{\psi(t)}\right)+\ln(\varphi^2(t)+\psi^2(t))\right]dt. \quad (5.14)$$

We consider now the case of a classical (cold) Boltzmann plasma. The right-hand side of (5.3), which has for a hot plasma the meaning of a scattering amplitude, now plays the role of a retarded interaction potential of two electrons in a medium in the Fourier representation. Without dwelling in detail on the calculation of the integral (5.2) in this case, we present the final result

$$\gamma=k^2\frac{8e^2}{15(\pi mk_BT)^{1/2}}\left[\ln\left(\frac{4r_Dk_BT}{e^2}\right)+\frac{1}{2}-2C-J\right], \quad (5.15)$$

where $r_D = (4\pi e^2\rho/k_BT)^{-1/2}$ is the Debye radius.

Formulas (5.13) and (5.15) are given in ordinary units. We note that the damping (5.15) for a classical (cold) plasma, unlike (5.13), does not contain the quantum constant $\hbar$, as indeed it should not.

Formulas (3.6) and the ensuing formulas (5.13) and (5.15) hold true also for a two-component plasma of particles with equal mass, if ρ is taken to mean the total particle density.

We now obtain the damping (4.4) for a two-component system under the condition

$$m_1\equiv m\ll m_2\equiv M, \quad (5.16)$$

which is characteristic of a real electron-ion plasma.

In the approximation (5.3) for Γ it is necessary to replace Π_0 (the electron loop) by the sum of electrons ($m_1 = m$) and ion ($m_2 = M$) loops. In all other respects, the calculations for the Boltzmann plasma are analogous to those for the one-component system and leads to the results

$$\gamma_0=\frac{2\omega_0}{3}\left(\frac{e^2\rho^{1/3}}{k_BT}\right)^{3/2}\left[\ln\frac{2\sqrt{2}k_BT}{\hbar\omega_0}-\frac{C}{2}-J_1\right] \quad (5.17)$$

for a hot plasma and

$$\gamma_0=\frac{2\omega_0}{3}\left(\frac{e^2\rho^{1/3}}{k_BT}\right)^{3/2}\left[\ln\left(\frac{4r_Dk_BT}{e^2}\right)-2C-J_1\right] \quad (5.18)$$

for a cold plasma. Here $\rho = \rho_1 + \rho_2$ is the total particle density, $r_D = (4\pi e^2\rho_1/k_BT)^{-1/2}$ is the "electronic" Debye radius, and

$$J_1=\frac{1}{2\sqrt{\pi}}\int_0^\infty e^{-t^2}\left[\frac{2(1+\varphi(t))}{\psi(t)}\operatorname{arctg}\frac{\psi(t)}{1+\varphi(t)}+\ln((1+\varphi(t))^2+\psi^2(t))\right]dt. \quad (5.19)$$

A numerical calculation yields $J \approx 0.30$ and $J_1 \approx 0.69$.

The factor in the square brackets in (5.17) (the Coulomb logarithm for a hot plasma) coincides with that obtained by Perel' and Éliashberg[6] in the calculation of the absorption of electromagnetic waves in a plasma.

[1] L. D. Landau, Zh. Eksp. Teor. Fiz. 16, 574 (1946); Sobranie Trudov (Collected Works) 2, Nauka, 1969, p. 7.
[2] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v staticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[3] R. Balescu, Statistical Mechanics of Charged Particles, Interscience, 1963.
[4] V. N. Popov, Zh. Eksp. Teor. Fiz. 58, 257 (1970) [Sov. Phys.-JETP 31, 140 (1970)].
[5] A. I. Larkin, ibid. 37, 264 (1959) [10, 186 (1960)].
[6] V. I. Perel' and G. M. Éliashberg, ibid. 41, 885 (1961) [13, 620 (1962)].

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