Theory of the Intermediate State of Antiferromagnets

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A theory is constructed of the coherent intermediate state (IS) produced in an antiferromagnet (AFM) of finite size when the magnetic sublattices are inverted in a magnetic field. It is shown that, in the IS, an antiferromagnetic plate breaks up into periodically repeating domains of phases $\Phi_{\parallel}$ and $\Phi_{\perp}$, the dimensions of which depend on the external magnetic field. The critical domain dimensions at which the IS ceases to be stable and goes over into a state with a uniform magnetization distribution are calculated. It is shown that the magnetic susceptibility in the IS is determined by the magnetic dipole interaction and appreciably exceeds the magnetic susceptibility of uniform states of the AFM. The role of the magnetic dipole interaction in the formation of the AFM domain structure is analyzed.

1. INTRODUCTION

It is well known that antiferromagnets in an external magnetic field can be found in different ground states or phases, depending on the magnitude and direction of the magnetic field. The states of antiferromagnets (AFM's) in the case when the external magnetic field is directed along the anisotropy axis have been investigated particularly thoroughly. In this case, the following ground states are possible: the state with the antiferromagnetism vector **l** parallel to the anisotropy axis and with the magnetic moment **m** equal to zero (the phase $\Phi_{\parallel}$), the state with the vector **l** perpendicular to the anisotropy axis and with the magnetic moment $\mathbf{m} \neq 0$ (the phase $\Phi_{\perp}$), the state with the vector $\mathbf{l} = 0$ and with the magnetic moment parallel to the anisotropy axis (the phase Φ_f) and, finally, a state in which the vectors **l** and **m** are oriented at a certain angle to the anisotropy axis. The possibility of the existence of such phases follows simply from symmetry considerations and from the fact that a uniform state of an AFM can be described by the vectors **l** and **m**.

The question which naturally arises concerns the nature of the transitions from one phase to another and the phase-transition curves. This question has been discussed repeatedly in the literature.[1] In a paper by the authors[2], it was shown that the transition $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$ in solids of finite size occurs via an intermediate state (IS), which can be represented intuitively as a state in which domains of the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ coexist. The idea of the existence of an IS in AFM's received confirmation in the experiments of Galkin and Kovner[3] on the antiferromagnetic resonance in copper chloride dihydrate at low frequencies. This idea made it possible to develop a theory[3] which explains the features of the AFM resonance in copper chloride dihydrate during inversion of the magnetic sublattices at both high[4] and low frequencies. The principal results, which refer to the static magnetic properties of an AFM in the IS, received full confirmation in the experiments of Dudko, Eremenko and Fridman on manganese chloride[5].

The present paper is devoted to an account of a consistent theory of the intermediate state.

2. FORMULATION OF THE PROBLEM AND CHOICE OF A MODEL

The whole set of phases mentioned in the Introduction can be obtained if we confine ourselves, in the expression for the thermodynamic potential, to terms of second order in the vectors **m** and **l**, i.e., represent Φ in the form

$$\Phi\{\mathbf{m},\mathbf{l}\} = \int dV\,\{(\alpha+\alpha')(\partial m_i/\partial x_k)^2 + (\alpha-\alpha')(dl_i/\partial x_k)^2 + 2\delta m^2 + (\beta+\beta')m_x^2 + (\rho+\rho')m_y^2 + (\beta-\beta')l_x^2 + (\rho-\rho')l_y^2 - 2m_k h_k - m_k h_{mk}\}, \tag{1}$$

where α, α' and δ are exchange constants ($\delta \sim T_N/\mu M_0$, $\alpha \sim \alpha' \sim a^2\delta$, T_N is the Néel temperature, μ is the Bohr magneton, M_0 is the magnetic moment of a sublattice, and a is the lattice constant); β, β', ρ and ρ' are the magnetic-anisotropy constants ($\beta \sim \rho \sim 1$); $\mathbf{h} = \mathbf{H}M_0^{-1}$, $\mathbf{h}_m = \mathbf{H}_m M_0^{-1}$; the antiferromagnetism vector **l** and the magnetization vector **m** are connected with the magnetic moments of the sublattices by the relations l = ½ $\mathbf{l} = \frac{1}{2}(\mathbf{M}_1 - \mathbf{M}_2)M_0^{-1}$ and $\mathbf{m} = \frac{1}{2}(\mathbf{M}_1 + \mathbf{M}_2)M_0^{-1}$. As usual, we shall assume that

$$\mathbf{l}^2 + \mathbf{m}^2 = 1, \quad \mathbf{ml} = 0. \tag{2}$$

The external magnetic field **H** is determined by given currents, and the fields $\mathbf{H}_m$ by the distribution of magnetization in the solid, i.e., by the magnetostatic equations and the continuity conditions for the tangential components of $\mathbf{H}_m$ and the normal components of $\mathbf{B}_m = M_0(\mathbf{h}_m + 8\pi\mathbf{m})$ at the interfaces.

Varying the thermodynamic potential Φ and using the magnetostatic equations

$$\operatorname{div}(\mathbf{h} + 8\pi\mathbf{m}) = 0, \quad \operatorname{rot}\mathbf{h} = 0,$$

we can determine the possible phases of the AFM in an external field. This problem is far from simple for nonuniform distributions, since in this case the relation between $\mathbf{h}_m$ and **m** turns out, as is well known, to be nonlocal. But to neglect the field $\mathbf{h}_m$ in investigating the intermediate state (IS) is, in principle, impossible, as the domain sizes of the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ in the IS are determined by the competition between the energy of the field $\mathbf{h}_m$ and the energy associated with the formation of the domain walls.

The problem is made easier by the fact that the magnetic susceptibility of an AFM is small and serves as a small parameter in which the quantities **m** and $\mathbf{h}_m$ can be expanded. It is not difficult to develop a perturbation theory which makes it possible to calculate the distribution $\mathbf{m}(\mathbf{r})$ in the IS with arbitrary exactness. If we assume that the distributions of **l** and **m** are uniform, then it is easy to find the ranges of magnetic fields in which one or another phase of the AFM is stable. Below, we give the results for the case when the AFM is ellipsoidal in shape, the anisotropy axis coincides with one of the axes of the ellipsoid and the external magnetic field is applied along this axis (the 3-axis or z-axis):

For $0 < h < h_1$, the phase $\Phi_{\|}$ is stable and

$$\Phi_{\|} = 0, \quad m = 0, \quad l = l_z = 1, \quad h_m = 0; \tag{3a}$$

for $h_2 < h < h_f$, $\Phi_{\perp}$ is stable and

$$\Phi_{\perp} = {}^1/_2 V\chi_{\perp}(\tilde{h}_{tr}^2 - h^2), \quad m = m_z = {}^1/_2\chi_{\perp}h; \quad l = l_y = (1 - m^2)^{1/2}, \quad h_m = h_{mz} = -8\pi N_3 m_z \tag{3b}$$

and, finally, for $h > h_f$, the ferromagnetic state with

$$\Phi_f = V(h_f + h_a - 2h), \quad m = m_z = 1, \quad l = 0, \quad h_m = h_{mz} = -8\pi N_3. \tag{3c}$$

is stable. Here we have used the notation

$$h_f = 2\delta - \rho + \rho' + 8\pi N_3, \quad \tilde{h}_e = 2\delta + \rho + \rho' + 8\pi N_2,$$
$$h_a = \rho - \rho', \quad \chi_{\perp} = 2/h_f, \quad h_1^2 = \tilde{h}_e h_a, \quad h_2 = h_1 h_f / h_e,$$
$$\tilde{h}_{tr}^2 = h_f h_a, \quad h_e = 2\delta - \rho + \rho';$$

and N_1, N_2 and N_3 are the demagnetization factors. It was assumed in the investigation of the stability of the phases that $\rho + 4\pi(N_2 - N_3) > 0$ and $\beta - \beta' > \rho - \rho' > 0$[6].

From the definition of the critical fields it can be seen that $h_2 < h_1$, i.e., the regions of stability of the phases $\Phi_{\|}$ and $\Phi_{\perp}$ overlap. Further, it can be seen that: 1) $\Phi_{\|} < \Phi_{\perp}$ for $h_2 < h < \tilde{h}_{tr}$, i.e., the stable phase is $\Phi_{\|}$ and the phase $\Phi_{\perp}$ is metastable; 2) $\Phi_{\perp} < \Phi_{\|}$ for $\tilde{h}_{tr} < h < h_1$, i.e., the phase $\Phi_{\perp}$ is stable and $\Phi_{\|}$ is metastable. It would appear that, in a field $h = \tilde{h}_{tr}$, a phase transition $\Phi_{\|} \rightleftarrows \Phi_{\perp}$ should occur, in accordance with the expressions for $\Phi_{\|}$ and $\Phi_{\perp}$ given above. However, as will be shown below, the transition from the phase $\Phi_{\|}$ to the phase $\Phi_{\perp}$ and vice versa is complicated and occurs via the IS. The energy of the IS in a certain range of fields about $\tilde{h}_{tr}$ turns out to be less than the energy of either of the phases $\Phi_{\|}$ and $\Phi_{\perp}$. If the dimensions of the sample in the direction of the anisotropy axis are appreciably greater than the other dimensions, i.e., $N_3 \ll N_1, N_2$, then the IS is not formed in practice and the phase transition $\Phi_{\|} \rightleftarrows \Phi_{\perp}$ occurs in the field $\tilde{h}_{tr}$.

First we shall consider the IS in a plane-parallel plate whose surface is perpendicular to the anisotropy axis.

3. PERIODIC DOMAIN STRUCTURE AND DOMAIN WALLS

We proceed now to the determination of the nonuniform distribution of **m** and **l** in the plate. For this, we must vary the free energy (1) of the AFM with respect to the variables **m** and **l**, or, which is the same thing, with respect to the variables $\mathbf{M}_1$ and $\mathbf{M}_2$. We then obtain the following equations:

$$\alpha\Delta\mathbf{M}_1 + \alpha'\Delta\mathbf{M}_2 - \delta\mathbf{M}_2 - \beta\mathbf{e}_x M_{1x} - \rho\mathbf{e}_y M_{1y} - \beta'\mathbf{e}_x M_{2x} - \rho'\mathbf{e}_y M_{2y} + \mathbf{h} + \mathbf{h}_m - \lambda_1\mathbf{M}_1 = 0,$$
$$\alpha\Delta\mathbf{M}_2 + \alpha'\Delta\mathbf{M}_1 - \delta\mathbf{M}_1 - \beta\mathbf{e}_x M_{2x} - \rho\mathbf{e}_y M_{2y} - \beta'\mathbf{e}_x M_{1x} - \rho'\mathbf{e}_y M_{1y} + \mathbf{h} + \mathbf{h}_m - \lambda_2\mathbf{M}_2 = 0, \tag{4}$$

where λ_1 and λ_2 are Lagrange multipliers introduced to take into account the conditions $M_1^2 = M_2^2 = M_0^2$, and $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along the x- and y-axes.

As we shall see below, in the interior of the plate the magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$ depend, to a good approximation, only on the x coordinate, and the field $\mathbf{h}_m$ is uniform. We shall also examine that solution of the system of equations (4) which describes the rotation of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ in the zy-plane on passage from a domain of the phase $\Phi_{\|}$ to a domain of the phase $\Phi_{\perp}$. This choice for the plane of rotations of $\mathbf{M}_1$ and $\mathbf{M}_2$ corresponds to the condition $\beta - \beta' > \rho - \rho' > 0$, according to which, less energy is required for the formation of a domain wall parallel to the zy-plane than for any other domain wall. So that we may disregard the conditions $M_1^2 = M_2^2 = M_0^2$, it is convenient to change to the variables m and θ, connected with the vectors **m** and **l** by the relations

$$m_x = 0, \quad m_y = m\cos\theta, \quad m_z = m\sin\theta,$$
$$l_x = 0, \quad l_y = (1 - m^2)^{1/2}\sin\theta, \quad l_z = (1 - m^2)^{1/2}\cos\theta, \tag{5}$$

for which we obtain the equations

$$\frac{d}{dx}\left\{\left[(\alpha + \alpha') + (\alpha - \alpha')\frac{m^2}{1 - m^2}\right]\frac{dm}{dx}\right\} - (\alpha - \alpha')\frac{m}{(1 - m^2)^2}\left(\frac{dm}{dx}\right)^2$$
$$- 2\alpha' m\left(\frac{d\theta}{dx}\right)^2 - (2\delta + \rho\cos 2\theta + \rho')m + (h + h_m^{(0)})\sin\theta = 0,$$
$$\frac{d}{dx}\left\{[2\alpha' m^2 + (\alpha - \alpha')]\frac{d\theta}{dx}\right\} - (\rho - \rho')\sin\theta\cos\theta + \rho m^2 \sin 2\theta$$
$$+ (h + h_m^{(0)})m\cos\theta = 0. \tag{6}$$

Since we are interested in the phase-transition region ($h \approx h_{tr}$), the magnetic field h satisfies the condition $h \ll h_e$. Noting also that the scale of the gradients in the coupled equations for m and θ is of order

$$\left|\frac{\alpha}{m}\frac{d^2 m}{dx^2}\right| \sim \left|\frac{\alpha}{\theta}\frac{d^2\theta}{dx^2}\right| \sim \rho,$$

we obtain the following expression for m:

$$m = \frac{h_i}{h_e}\sin\theta\left[1 - \frac{2\rho\cos^2\theta}{h_e}\right], \tag{7}$$

where $h_e = 2\delta - \rho + \rho'$ and $h_i = h + h_m^{(0)}$ is the uniform field inside the plate. Using (7), we can represent the equation for θ in the form

$$d^2\theta/dx^2 - A(h, h_m^{(0)})\sin\theta\cos\theta - 2B(h, h_m^{(0)})\sin 2\theta\cos 2\theta = 0, \tag{8}$$

where

$$A(h, h_m^{(0)}) = \frac{h_{\text{п}}^2 - h_i^2}{(\alpha - \alpha')h_e}, \quad B(h, h_m^{(0)}) = \frac{\rho h_i^2}{2(\alpha - \alpha')h_e^2}, \quad h_{tr} = \sqrt{h_e h_a}. \tag{8'}$$

In writing Eq. (8), we have confined ourselves to terms of order δ^{-2}.

The first integral of Eq. (8) is easily found and is equal to

$$(d\theta / dx)^2 = A\sin^2\theta + B\sin^2 2\theta + D, \tag{9}$$

where D is an integration constant. The distribution (9) will describe the coexisting domains of the phases $\Phi_{\|}$ and $\Phi_{\perp}$ only when the quantities A and D are simultaneously small. Putting

$$D = 0, \quad A = 0, \tag{10}$$

we can find the distribution $\theta(x)$, and, consequently, **m** and **l**, in the transitional layer between the phases $\Phi_{\|}$ and $\Phi_{\perp}$ (at the domain boundaries):

$$\operatorname{tg}\theta = \exp(-x/x_1), \quad x_1^{-1} = 2\sqrt{B}. \tag{11}$$

The formula (11) for $\theta(x)$ is given in a coordinate frame in which the origin on the x-axis coincides with the middle of the domain boundary, i.e., $\theta = \pi/4$ when $x = 0$. One of the conditions for the coexistence of the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ is the condition $A(h, h_m^{(0)}) = 0$, which means that the internal magnetic field h_i in an AFM in the IS remains constant and is approximately equal to h_{tr} in the whole range of external fields in which the IS exists.

The distribution function (11) for $\theta(x)$ has the important defect that it is not a periodic function and, consequently, cannot describe the domain structure as a whole. In order to obtain a periodic distribution for $\theta(x)$, we must study the solution of Eq. (9) when A and D are non-zero. A rigorous treatment, naturally, shows that the intermediate state corresponds to small A and D, i.e., $|A| \ll 1$ and $|D| \ll 1$. In other words, the formulas (11) describe well the rotation of the antiferromagnetism vector at the domain boundary. Integrating Eq. (9), we obtain

$$\operatorname{tg}\theta = \frac{1}{p}\frac{\operatorname{sn}(xx_1^{-1}+\psi)}{\operatorname{cn}(xx_1^{-1}+\psi)} \tag{12}$$

where ψ is a second integration constant, and $\operatorname{sn}(xx_1^{-1} + \psi)$ and $\operatorname{cn}(xx_1^{-1} + \psi)$ are the elliptic sine and cosine,

$$x_1^{-1} = \{{}^1/_2A + 2B + D + [({}^1/_2A + 2B + D)^2 - D(D+A)]^{1/2}\}^{1/2}, \quad p = x_1^{-1}D^{1/2} \tag{13}$$

and the modulus κ of the elliptic functions is given by

$$\kappa^2 = 2x_1^2[({}^1/_2A + 2B + D)^2 - D(D+A)]^{1/2}. \tag{14}$$

The formulas (12)—(14) determine $\theta(x)$ as a periodic function $\theta(x + d) = \theta(x)$, the period d of which is proportional to the period of the elliptic sine, i.e., is equal to

$$d = 4Kx_1 = 4x_1\int_0^{\pi/2} d\alpha/[1-\kappa^2\sin^2\alpha]^{1/2}. \tag{15}$$

It can be seen from the formulas (12)—(15) that the period d of the function d(x) is determined by the integration constant D.

Thus, the periodic distribution (12) is determined by the parameters h, $h_m^{(0)}$ and the integration constant D, which remains undetermined for the present. In place of the three quantities h, $h_m^{(0)}$ and D, it is convenient to choose h, $h_m^{(0)}$ and K. We put $\psi = K$, i.e., we displace the coordinate origin to the point where $\theta = \pi/2$. Then (12) takes the form

$$\operatorname{ctg}\theta = p\kappa'\left(\operatorname{sn}\frac{x}{x_1}\Big/\operatorname{cn}\frac{x}{x_1}\right), \quad \kappa' = \sqrt{1-\kappa^2}. \tag{16}$$

We shall define the points $x = \pm a$ as the points at which $\cot\theta = \pm 1$ ($\theta = {}^1/_4\pi$, ${}^3/_4\pi$). If the thickness $\Delta s \approx x_1$ of the domain boundary is much smaller than the domain dimensions, we can assume that the interval $-a \le x \le a$ is occupied by the phase $\Phi_{\perp}$. It is easy to convince oneself that there are two such intervals in the period d; therefore, the parameter $\xi = 4a/d$ determines the fraction of material in the phase $\Phi_{\perp}$. Assuming the parameter K to be sufficiently large, we can expand the elliptic functions in series in the small parameter $(\kappa)^2 \approx e^{-2K}$. We then obtain[1)]

$$\operatorname{ctg}\theta = p\kappa'\operatorname{sh}(x/x_1). \tag{17}$$

The expression (17) leads to a finite value of the derivative $d\theta/dx$ at all points in space occupied by both the phase $\Phi_{\perp}$ and the phase $\Phi_{\parallel}$. At $x = 0$, the derivative $d\theta/dx$ is, in order of magnitude, equal to $d\theta/dx \approx -2x_1^{-1}e^{-K}$. We find the connection between the concentration ξ and h, $h_m^{(0)}$ and K from (16) by putting $x = a$:

$$p\kappa' = \frac{\operatorname{cn}K\xi}{\operatorname{sn}K\xi} \approx \frac{1}{\operatorname{sh}K\xi}. \tag{18}$$

Using the expression (7) for m(x), the formula (17) determining $\theta(x)$ and also the condition $K\xi \gg 1$, we can find formulas relating the magnetic fields $\mathbf{h}_m$ and $\mathbf{h}$:

$$h_{mx} = 16m_{\perp}\pi^2\sum_{n=1}^{\infty}\left(2K\operatorname{sh}\frac{\pi^2n}{2K}\right)^{-1}e^{-4\pi nc/d}\sin\pi n\xi\sin\frac{4\pi nx}{d}\operatorname{sh}\frac{4\pi nz}{d},$$
$$h_{mz} = h_m^{(0)} - 16\pi^2m_{\perp}\sum_{n=1}^{\infty}\left(2K\operatorname{sh}\frac{\pi^2n}{2K}\right)^{-1}e^{-4\pi nc/d}\sin\pi n\,\xi\cos\frac{4\pi nx}{d}\operatorname{ch}\frac{4\pi nz}{d}, \tag{19}$$

where

$$m_{\perp} = {}^1/_2\chi_{\perp}h = (2\delta - \rho + \rho' + 8\pi)^{-1}h, \quad h_m^{(0)} = -8\pi m_{\perp}\xi \tag{20}$$

and 2c is the thickness of the plate. As can be seen from these formulas, the nonuniform part of the field h_m in the interior of the plate is considerably smaller than the uniform part. In order of magnitude, h_m is $h_m \sim \chi_{\perp}h$. The nonuniform part of the field h_m leads to a distortion of the magnetization distribution **m** described by formulas (7) and (12) at the surface of the plate.

This deviation is manifested in a deformation of the domain boundaries at the surface. The angle of deformation of a plane boundary is of order $\theta \approx m_x/m_z$ (this estimate follows from the continuity of the normal component of the induction). Further, we note that $m_x \approx \chi_{\perp}h_{mx} \approx \chi_{\perp}^2h$ and $m_z \approx \chi_{\perp}h$, so that $\theta \sim \chi_{\perp} \ll 1$, i.e., the deformation is small, and we shall not take it into account below,[2)] since it gives no appreciable contribution to the energy of the AFM.

To conclude this Section we again return to the elucidation of the conditions for which the distribution $\theta(x)$ corresponds to alternating domains of the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$, separated by narrow transitional regions. Analysis of formula (12) shows that this is possible when $K \gg 1$ and $p \gg 1$. It is not difficult to convince oneself (see (13)—(15)) that these two conditions are equivalent to the conditions, already mentioned earlier,

$$|D| \ll 1, \quad |A(h, h_m^{(0)})| \ll 1. \tag{21}$$

Assuming the quantities K and ξ to be independent and using formulas (18), (13), and (14), and also the relation between the elliptic modulus κ and K, we obtain

$$D = 8\rho\frac{h_i^2}{h_e^2(\alpha-\alpha')}e^{-2K(1-\xi)}, \quad A = \frac{8\rho}{\alpha-\alpha'}\frac{h_i^2}{h_e^2}[e^{-2K\xi} - e^{-2K(1-\xi)}] \tag{22}$$

The second of the formulas (22) shows that the distribution, determined by formulas (11), of material over the phases, is valid with exponential exactness. The condition $h_i = h_{tr}$ for the coexistence of the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ is fulfilled with the same exactness, and leads, if we take (20) into account, to a relation between ξ and the field h:

$$\xi = (h - h_{tr})(4\pi\chi_{\perp}h_a)^{-1}. \tag{23}$$

Allowance for the fact that the period d is bounded en-

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1)We note that $\operatorname{sn}u = \operatorname{th}u\left[1 + \frac{(\kappa')^2}{2}\left\{1 - \frac{2u}{\operatorname{sh}2u}\right\}\right]$ if $e^{2u} \ll e^{2K}$. Neglecting the quantities of order e^{-2K} for $u \lesssim 1$ and of order u^2e^{-2K} for $u \ll 1$, we can put $\operatorname{sn}u = \tanh u$.

2)The problem of the shape of the interface can be solved in more detail for domains in metals, by the method used in[7].

ables us to find the dependence of the parameter ξ on the internal field h_i from the second formula of (22):

$$\xi=\frac{1}{2}-\frac{1}{2K}\operatorname{Arsh}\left[\frac{h_{tr}^2-h_i^2}{16h_i^2}\frac{h_e}{\rho}e^{K}\right] \tag{24}$$

and to determine the dependence of the internal field on the external field h:

$$h_i=h_{tr}-8(h_t-h_n)e^{-K}\operatorname{sh}K\left(1-\frac{h-h_{tr}}{2\pi\chi_\perp h_{tr}}\right). \tag{25}$$

Since the quantities D and A are exponentially small, the parameter x_1 characterizing the width of the domain wall is equal to $x_1 \approx \frac{1}{2}B^{-1/2}$. Using (8′) for B and substituting the field h_{tr} for h, we obtain[2]

$$x_1=[(\alpha-\alpha')\delta/\rho(\rho-\rho')]^{1/2}. \tag{26}$$

Putting $\delta \sim \chi_\perp^{-1} \sim 10^3$, $\rho \sim \rho' \sim 1$, and $(\alpha - \alpha') \sim a^2\delta$ (a is the lattice constant), we obtain

$$x_1 \sim \chi_\perp^{-1}a \sim 10^{-5}\text{ cm}.$$

4. ENERGY OF THE INTERMEDIATE STATE AND DOMAIN DIMENSIONS

Knowing the distribution of **m** and **l** in the intermediate state, we can find the value of the thermodynamic potential Φ in this state. Assuming that the formulas (7) and (12) describe the distributions of θ and m not only in the interior of the plate but also near to its surface, using the relations (5) and performing the integration in (1), we obtain

$$\Phi_{IS}=V[\xi(h_n^2-hh_i)/h_e+2\rho h_n^2/Kh_e^2+8\pi h^2Kx_1f(\xi)/ch_e^2], \tag{27}$$

where

$$f(\xi)=\sum_{n=1}^{\infty}\frac{\pi}{4nK^2}\frac{\sin^2(\pi n\xi)}{\operatorname{sh}^2(\pi^2n/2K)}. \tag{28}$$

In formula (27), the second term is the energy associated with the domain walls, the effective surface energy of which is given by the formula[2]

$$\sigma=\rho h_{tr}^2h_e^{-2}x_1 \approx [(\alpha-\alpha')\rho(\rho-\rho')\delta^{-1}]^{1/2}. \tag{29}$$

The third term in formula (27) is the energy of the nonuniform field h_m at the surface of the plate.

Equating $d\Phi_{IS}/dK$ to zero, we find the equilibrium dimensions of the domains

$$d=4Kx_1=2[\rho cx_1/\pi f(\xi)]^{1/2}. \tag{30}$$

The dependence of the period d on the magnetic field means that, with change of the magnetic field in a plate of length l_1, the number of domain boundaries $N = l_1/d$ first increases and then begins to decrease. The quantity N attains its maximum value in a field $h = h_{tr} + 2\pi\chi_\perp h_{tr}$, i.e., approximately in the middle of the range of existence ($h_{tr} \ll h \ll h_{tr}(1 + 4\pi\chi_\perp)$) of the IS.

In order to determine more exactly the range of existence of the IS, we substitute the value of K determined by formula (30) into the expression for Φ_{IS}. We then obtain

$$\Phi_{IS}=V\left[-2\pi(\xi)^2+\frac{\rho}{K}\right]\frac{4h_n^2}{h_e^2}. \tag{31}$$

Equating this expression to zero and using formulas (30) and (28), we find that value of ξ at which the thermodynamic potentials of the phases, $\Phi_\parallel$ and Φ_{IS}, are equal:

$$\xi=\xi_1=(\rho/2\pi K_1)^{1/2}; \tag{32}$$

here, K_1 is a solution of the equation

$$K_1(1.8-\ln 2\pi\rho+\ln 2\pi K_1)=\pi cx_1^{-1}$$

and in order of magnitude is equal to $K_1 \approx c/x_1$. Corresponding to this value of ξ, we have the following external magnetic field (see (23)) and domain period:

$$h=h_\parallel=h_n[1+\chi_\perp(8\pi\rho/K_1)^{1/2}],\quad d=d_\parallel=4\pi x_1K_1\approx c. \tag{33}$$

For $h > h_\parallel$, the IS is stable, and the phase $\Phi_\parallel$ is metastable. Equating the expressions for Φ_{IS} from (31) and $\Phi_\perp$ from (3b), we find the value of ξ at which the thermodynamic potentials of the phases, $\Phi_\perp$ and Φ_{IS}, are equal:

$$\xi=\xi_2=1-(\rho/2\pi K_\perp)^{1/2}. \tag{34}$$

Corresponding to this value of ξ are the magnetic field and domain-structure period:

$$h=h_\perp=h_n(1+4\pi\chi_\perp)-\chi_\perp h_n(8\pi\rho/K_1)^{1/2},\quad d=d_\perp=4x_1K_1\approx c. \tag{35}$$

For $h < h_\perp$, the IS is stable, and the phase $\Phi_\perp$ is metastable. It can be seen from formulas (33) and (35) that the dimensions of the domains which form during the transition to the IS, both from the phase $\Phi_\parallel$ and from the phase $\Phi_\perp$, are the same and are determined by the thickness of the plate. The number of domain boundaries produced in the plate in this transition is of order l_1/c. In a field $h \approx \frac{1}{2}(h_\parallel + h_\perp)$, the number of domain walls considerably exceeds the value l_1/c.

The range of external magnetic fields in which the IS is stable is determined by the inequality

$$h_\parallel \leqslant h \leqslant h_\perp. \tag{36}$$

For $h = h_\parallel$ and for $h = h_\perp$, the first-order phase transitions $\Phi_\parallel \rightleftarrows \Phi_{IS}$ and $\Phi_\perp \rightleftarrows \Phi_{IS}$, respectively, occur. These transitions are accompanied by jumps in the magnetization, which are easily determined by making use of the formula

$$m_{av}=\chi_\perp\xi h_i. \tag{37}$$

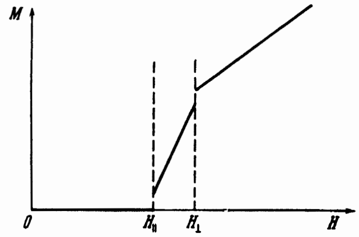
Hence, we have

$$\Delta m_1=m_1=\chi_\perp h_{tr}(\rho/2\pi K_1)^{1/2}\sim\chi_\perp h_{tr}(c/x_1)^{-1/2},$$
$$\Delta m_2=\chi_\perp h_\perp-\chi_\perp\xi_2h_\perp\approx\chi_\perp(\rho/2\pi K_1)^{1/2}\sim\chi_\perp h_{tr}(x_1/c)^{1/2}. \tag{38}$$

It can be seen from this that the jumps in the magnetization in the phase transitions $\Phi_\perp \rightleftarrows \Phi_{IS}$ and $\Phi_\parallel \rightleftarrows \Phi_{IS}$ are appreciably smaller ($(x_1/c)^{1/2} \ll 1$) than in the phase transition $\Phi_\perp \rightleftarrows \Phi_\parallel$, when the jump Δm in the magnetization is of order $\Delta m \sim \chi_\perp h_{tr}$.

In conclusion, we note that if we do not use the condition (23) determining the fraction ξ of material in the phase $\Phi_\perp$, then the expression (27) describes the nonequilibrium thermodynamic potential, as a function of ξ and K, of an antiferromagnet divided into regions occupied by the phases $\Phi_\parallel$ and $\Phi_\perp$. If we omit in (27) the second and third terms, which take into account the energy connected with surface effects, minimization of the remaining first term with respect to ξ again leads, as can easily be seen, to the condition (23) determining the equilibrium value of ξ.

The above study enables us to describe the behavior of the magnetization of an AFM in a broad interval of values of the external magnetic field h. The dependence of the magnetization on h is depicted schematically in the Figure. The average magnetization in the interval



Dependence of the magnetization M on the external magnetic field H in the phase transitions $\Phi_\parallel \rightleftharpoons \Phi_{IS}$ and $\Phi_\perp \rightleftharpoons \Phi_{IS}$. The quantity $M = \chi_\parallel H$, if $H < H_\parallel$, and $M = \chi_\perp H$ ($\chi_\parallel \ll \chi_\perp \ll 1$), if $H > H_\perp$.

$h_\parallel \le h \le h_\perp$ is determined by the formula (37) and is equal to

$$M = 2M_0 m_{av} = \frac{h - h_{tr}}{4\pi} M_0, \quad (39)$$

whence it can be seen that the external magnetic susceptibility χ_e of an AFM in the IS is equal to $1/4\pi$.

To conclude, we give an expression for the internal magnetic susceptibility χ_i. Using (39) and (37), we obtain

$$\chi_i = \frac{\partial M}{\partial H_i} = \frac{1}{\rho K}\left[\left(\frac{h_i - h_{tr}}{h_1 - h_{tr}}\right)^2 + (8e^{-K})^2\right]^{-1/2}. \quad (40)$$

If $|h_i - h_{tr}| \ll |h_1 - h_{tr}| e^{-K}$, then $\chi_i \approx e^K / 8\rho K$. The exponentially large value of χ_i in an antiferromagnet in the IS is entirely connected with the domain-structure property of being magnetized by means of displacement of the domain boundaries on exponentially small change of the external field.

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