Particle Acceleration by a Moving Laser Focus, Focusing Front or Ultrashort Laser Pulse Front

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The acceleration of electrons by a moving gradient of an intense light beam is considered. It is shown that ultrarelativistic particle energies can be obtained from the motion of the focal spot, focusing front, or laser pulse front. Acceleration of plasma due to the movement of the focal spot is considered. It is noted that the motion or wandering of the focal spot or hot spots in the focus may be responsible for the appearance of a group of fast particles, hard radiation, and neutrons during laser heating of matter.

RECENT research achieved high intensities of focused laser fields reaching intraatomic values of $E_a \approx 10^7$ cgs esu and large gradients of light fields (the achievable fronts in space are represented by fields rising over the length $l \approx 10\,\lambda$ across or along the focal spot, or in the front of an ultrashort pulse compressed in time). Gradient acceleration of electrons is made possible by the large gradients of the light field.

As we know[1], the average force acting on a particle in a rapidly alternating electromagnetic field with amplitude $\mathbf{E}_0(\mathbf{r})$ and frequency ω is

$$\mathbf{f} = -\frac{e^2}{2m\omega^2}\nabla(E^2)_{\text{av}}.$$

(In the quantum case this force is associated with virtual transitions.) For example, for a focused field intensity $E_f \approx (4W/c)^{1/2}a^{-1} \sim 10^7$ cgs esu and nanosecond power of $W \approx 30$ GW, and for $E_f \approx 10^8$ cgs esu and picosecond power of $W \approx 3\times10^3$ GW, given $a \approx 10^{-3}$ cm, we obtain the following equivalent electric field for the frequency $\omega \approx 2\times10^{15}\ \text{sec}^{-1}$ (neodymium laser frequency):

$$E_{\text{eff}} \approx \frac{1}{e}f_0 \approx \frac{e}{2m\omega^2}E_f^2\frac{1}{l} \approx 1 \div 100\ \text{MV/cm}.$$

Such fields can be used effectively provided the field gradient moves together with the accelerated particle. This gradient motion accompanying the motion of the focal spot can be achieved in the transverse direction by scanning the beam and in the longitudinal direction by time variation of the beam divergence or focal length of the lens.

Motion of the focusing front is also obtained when different parts of the beam are focused on different points of the axis at different times, so that the focusing front moves according to a definite law. (For example, a special lens can focus different annular strips of the ultrashort pulse beam onto different points on the axis at the times $t(r) = c^{-1}[r^2 + L_f^2(r)]^{1/2}$, where $L_f(r)$ is determined by the angle of refraction θ of the lens:

$$L_f(r) \approx r/\operatorname{tg}\theta(r).)$$

In the general case, if a time-variable force is given, the equation of motion, taking the relativistic force correction $f = f_0/\gamma$ (where $\gamma = (1-\beta^2)^{-1/2}$) into account, is

$$\frac{\dot\beta}{(1-\beta^2)^2} = \frac{e^2}{2m_0^2\omega^2c}\nabla E_f^2(t) = \frac{\alpha}{c}E_f^2(t);$$

Here $\alpha \approx e^2/2m_0^2\omega^2 l$, neglecting changes in the front width l. We readily see that

$$\gamma^2 - \gamma_0^2 \approx \frac{2\alpha}{c^2}\int_0^z E_f^2(z)\,dz \approx 2\alpha E_0^2\frac{L}{c^2} \approx 2f_0\frac{L}{m_0c^2}$$

or in greater detail

$$\left\{\frac{\beta}{2(1-\beta^2)} + \frac{1}{4}\ln\frac{1+\beta}{1-\beta}\right\}\Big|_0^t \approx \frac{\alpha}{c}\int_0^t E_f^2(t)\,dt,$$

if $E_f^2(t)$ at the location of the particle is given.

We first consider the conditions of starting acceleration from low initial velocities. For $\beta \ll 1$ we have

$$\beta \approx \frac{\alpha}{c}\int_0^t E_f^2(t)\,dt + \beta_0,$$

and the location of the particle is

$$z(t) \approx \alpha\int_0^t dt'\int_0^{t'} E_f^2(t'')\,dt'' + \beta_0ct + z_0.$$

Knowing $E_f^2(t)$ we can obtain $z(t)$, i.e., the necessary focal shift $z_f(t) = z(t)$. For example, for a linear power rise in time $(E_f^2)_{\text{av}} \approx At$, we obtain

$$z(t) = {}^1/_6\alpha At^3 + \beta_0ct + z_0,$$

For $(E_f^2)_{\text{av}} \approx$ const we have

$$z(t) = {}^1/_2\alpha E_{f0}^2t^2 + \beta_0ct + z_0.$$

Such $z_f(t)$ can be produced by scanning or focusing devices.

If the focal shift is accomplished by a nonlinear effect (for moving foci see[2] for example) we can estimate the form of the $E_f(t)$ function necessary for starting acceleration. Assuming that $z_f \approx B_K E_f^2$ (in the case of a Kerr of electronic nonlinearity), we obtain

$$\frac{d^2}{dt^2}(E_f^2) = \frac{\alpha}{B_K}E_f^2(t)$$

or

$$E_f^2(t) = \frac{z_f}{B_K} = C_1\exp\left[\left(\frac{\alpha}{B_K}\right)^{1/2}t\right] + C_2\exp\left[-\left(\frac{\alpha}{B_K}\right)^{1/2}t\right],$$

i.e., either $z(0)$ or $\dot z(0) = v_0$ must be different from zero to achieve acceleration.

For $z(0) = 0$ ($E_f(0) = 0$) we have

$$E_f^2(t) = 2C\,\operatorname{sh}\left(\left(\frac{\alpha}{B_K}\right)^{1/2}t\right) = \frac{z_f(t)}{B_K},\quad C = \frac{v_0}{L}\left(\frac{B_K}{\alpha}\right)^{1/2},$$

i.e., the power at first rises almost linearly and then exponentially. For $z(0) = z_m$, $E_f(0) = E_{f\,\max}$, and $v_0 = 0$ we obtain

$$E_f^2(t) = 2C\,\mathrm{ch}\left(\left(\frac{a}{B_\kappa}\right)^{1/2} t\right) = \frac{z_f(t)}{B_\kappa},$$

$$C = \frac{E_{f\,max}^2}{2} = \frac{z_m}{2B_\kappa}.$$

In the ultrarelativistic region the necessary law can be obtained by assuming that $\dot{z}_f = c$, i.e.,

$$E_f^2 = ct/B_\kappa + E_f^2(0).$$

In the case of thermal nonlinearity

$$z = B_T\int_0^t E_f^2\,dt + z(0),$$

i.e., in the non-relativistic region

$$\ddot{z} = B_T\frac{d}{dt}(E_f^2)$$

or

$$\frac{d}{dt}E_f^2 = \frac{a}{B_T}E_f^2,$$

$$E_f^2 = E_f^2(0)\exp\left(\frac{a}{B_T}t\right) = \frac{\dot{z}}{B_T}.$$

Thus in this case an exponential power rise is necessary for the capture of nonrelativistic particles. In the ultrarelativistic region $E^2 \approx c/B_T \approx$ const.

The focal shift is

$$z_f = F^2\theta_{\mathrm{nonl.}}/d,$$

where

$$\theta_{\mathrm{nonl.}} \approx n_2E_d^2h/d$$

in the case of Kerr nonlinearity and

$$\theta_{\mathrm{nonl.}} = \varkappa h n_T' c\int_0^t E_d^2\,dt\frac{1}{4\pi C\rho}$$

in the case of "thermal" nonlinearity (here E_d and d are field intensity and beam radius in a nonlinear cell of length h, n_2 is a nonlinear increment to the refractive index of the medium, κ is the coefficient of linear and nonlinear absorption, n_T' is the partial derivative of the refractive index in rapid heating or excitation of the medium, and $C\rho$ is heat capacity of a unit volume of the medium). Since $E_d \approx E_f\rho_f/d$, where ρ_f is the radius of the focal spot of the lens with a focal length of F($\rho_f = F\varphi$ and φ is the divergence), we have

$$B_\kappa = \frac{F^2n_2h\rho_f^2}{d^4}, \quad B_T = \frac{F\varkappa h n_T' c\rho_f^2}{4\pi d^4 C\rho},$$

i.e., $B \sim (F/d)^4$ if the beam divergence is greater than the diffraction limit. Such a strong dependence allows us to select the conditions so as to obtain a sufficiently large focal shift, $z_{fm} \sim 0.1F$ and larger. Similar means can be employed to obtain the transverse focal shift by changing the beam direction[3].

The final energies of accelerated particles are determined for $\gamma \gg 1$ from the relation

$$\gamma^2 \approx 2\frac{\varepsilon_{\mathrm{thr}}}{mc^2}\frac{L}{l}.$$

For example, for the achievable $\epsilon_{osc} \approx 10^5$ eV and $l \approx 10^{-3}$ cm we obtain $\gamma^2 \approx 0.3L/l$, i.e., for $L \approx 3$ cm, $\gamma \approx 30$. The corresponding energy exceeds the oscillation energy by several orders, thus facilitating the separation of accelerated particles even when the match between the particle motion and the focal spot motion is not very good. We note that particles can be injected in the accelerating regime from an electron cloud or a plasma created, for example by the very action of the laser beam on the target.

A focal spot moving in the medium can also be utilized. According to Kelley and others[4-6], the instantaneous position of the beam focus having an initial diffraction divergence is given by the expression

$$L = \frac{a}{2n_2^{1/2}}\frac{1}{E - E_{\mathrm{thr}}}$$

or

$$L \approx \frac{a}{2n_2^{1/2}}\frac{1}{(E^2 - E_{\mathrm{osc}}^2)^{1/2}},$$

which yields for $E \gg E_{thr}$

$$L \approx a/2n_2^{1/2}E(t),$$

i.e.,

$$z_f = L - L_m = \frac{a}{2n_2^{1/2}}\left\{\frac{1}{E(t)} - \frac{1}{E_m}\right\},$$

and taking the initial divergence θ_0 into account, the focal shift is

$$z_f \approx \frac{a}{(-\theta_0 + n_2^{1/2}E)} - \frac{a}{(-\theta_0 + n_2^{1/2}E_m)}.$$

Here E_m is the amplitude of the light pulse. A channel with a small radius can be made along the path of the focal spot in the medium to pass the particles and to reduce nonlinear absorption of light.

In the above processes, the acceleration stability, which determines the effective number of accelerated particles, is significant. Longitudinal stability is achieved near that region of the beam gradient for which $\partial f/\partial z < 0$, i.e., in the region in which the acceleration of lagging particles is increased and that of leading particles is decreased. Transverse stability can be achieved not only by external fields but also by a potential well created by a slight weakening of the light field on the acceleration axis (shadow or diffraction). The number of captured particles depends on the compensation of their Coulomb charge by the immobile ion background. In the absence of ions the number of electrons captured in the acceleration regime is determined from the condition that the Coulomb repulsion force $f_{coul} \approx 2N_1e^2/r$ should equal the transverse striction retention force:

$$f_{\nabla\perp} \sim \frac{e^2}{2m\omega^2}\nabla_\perp E^2 \approx \frac{e^2}{m_e\omega^2}\frac{E^2}{r},$$

i.e., $N_1 \sim E^2/m\omega^2$, where N_1 is the number of captured particles per unit of length, or the total number of electrons $N \approx N_1l = E^2l/m_0\omega^2 \approx 3\times10^8$. This quantity of electrons can be captured in a volume $V \approx \pi lr^2 \approx 10^{-8}$ cm^3 without compression from thermoelectronic or plasma clouds with plasma concentration of $\sim10^{17}$ cm^3.

The Coulomb force between the electrons and ions in a plasma cloud is insignificant when $E_{acc} > E_{coul}^{max} \approx n_2el$. In this case the electrons are accelerated as free electrons. A much larger number of electrons can be accelerated in the presence of an ion background.

The above estimates show that it is possible to achieve and to observe gradient acceleration of particles by moving light foci or focusing fronts of intense light beams.

If the Coulomb force binding the electrons and ions is so strong that the force acting on electrons is also transferred to the ions, then the ion acceleration energy can be estimated from the condition

$$\mathscr{E}_i \approx fL \approx eE_{\mathrm{eff}} L \approx {}^1\!/_2 Mu^2,$$

if the acting time $\tau \approx L/u < T_{\mathrm{pulse}}$, or from the condition $Mu \sim fT$, if $T < L/u$, i.e., the focal spot moving with the velocity $u(t)$ can bring about the appearance of a group of fast ions.

We note that spontaneous wandering, as well as deliberate motion, of the focal spot or hot spots in the focus can cause the appearance of accelerated particles in laser plasma responsible for hard radiation or nuclear reactions.

The focal motion can also be used to increase the effectiveness of light-reactive acceleration[7] of macroscopic particles.

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[1]A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. **34**, 242 (1958) [Sov. Phys.-JETP **7**, 168 (1958)].
[2]P. D. McWane, Nature (Lond.) **211**, 1081 (1966).
[3]A. E. Kaplan, Zh. Eksp. Teor. Fiz. Pis'ma Red. **9**, 58 (1969) [JETP Lett. **9**, 33 (1969)].
[4]P. L. Kelley, Phys. Rev. Lett. **15**, 1005 (1965).
[5]V. I. Talanov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **2**, 218 (1965) [JETP Lett. **2**, 138 (1965)].
[6]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk **93**, 19 (1967) [Sov. Phys.-Usp. **10**, 609 (1968)].
[7]G. A. Askar'yan, M. S. Rabinovich, M. M. Savchenko, V. K. Stepanov, and V. B. Studenov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **5**, 258 (1967) [JETP Lett. **5**, 208 (1967)].

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