Theory of the Pinning Effect in Type-II Superconductors

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The interaction between superconducting vortices and a set of cylindrical cavities (pinning) is analyzed electrodynamically. Critical currents in a type-II syperconductor located in an external magnetic field perpendicular to the current are found providing (1) the cavities form a wall (the cavities are parallel to each other and to the external magnetic field), (2) the cavities are at the sites of a regular square lattice, (3) pinning occurs for a set of spherical cavities forming a regular cubic lattice.

1. INTRODUCTION

THE flow of nondissipative current through a type-II superconductor in the mixed state is possible only if the superconductor contains some inhomogeneities on which the superconducting vortices can be pinned.

In spite of the many papers devoted to pinning, little is known about the mechanism whereby the vortices interact with the inhomogeneities. No attempt is made here to cover all the possible inhomogeneities with which the vortices interact, or to study the interaction of vortices with dislocations. Only inhomogeneities for which the interaction can be described purely electrodynamically are considered here.

The main idea of the paper is as follows. The interaction between the vortices and the inhomogeneities, the pinning, is a purely electrodynamic interaction between the vortex and the surface of the superconductor. The results of this paper are thus applicable to inhomogeneities whose surfaces are internal surfaces in the superconductor and separate the normal (or dielectric) volume from the superconducting one.

The present purpose develops further the ideas of earlier papers[1,2] devoted to the possible flow of nondissipative current in a perfectly homogeneous type-II superconductor in the mixed state, due to interaction between vortices and the external surface of the superconductor.

Several models of pinning will be considered. The simplest, from our point of view, and at the same time significant from the point of view of application, is the following. Empty cylindrical channels are disposed in the superconductor parallel to one another in such a way that their axes are all on one plane. An external magnetic field is applied parallel to the channels. This model will be called "a wall of channels." Pinning on the wall of channels is considered in Sec. 2. Section 3 deals with pinning on a system of parallel empty channels forming a regular two-dimensional lattice on a plane perpendicular to their axes. In conclusion we consider pinning on a system of spherical cavities.

2. PINNING ON A WALL OF CHANNELS

We consider first qualitatively the processes that lead to the pinning on a wall of channels. Let channels of radius r be located in an infinite ideally-homogeneous type-II superconductor ($\kappa \gg 1$) parallel to the 0z axis. The channel axes lie in the plane x = 0 and intersect the 0y axis at the points y = nd, n = 0, ± 1, ± 2, ..., where d is the distance between the channels. It is assumed that $r \ll d \ll \delta_0(T)$, where $\delta_0(T)$ is the depth of penetration of the weak magnetic field. We apply an external magnetic field H_0 parallel to the 0z axis. It is assumed that $H_{c1} \ll H_0 \ll H_{c2}$. In this case, a regular triangular lattice of superconducting vortices is produced in the entire volume of the superconductor. Let the side of the triangle forming the unit cell be equal to a. We assume that H_0 is strong enough, so that the inequality $r \ll a \ll d$ is satisfied.

It was shown in[3] that an empty channel always attracts to itself a vortex, and the latter, on reaching the surface of the channel, penetrates into the channel. As a result, the channel captures a magnetic-flux quantum. It was shown in the same paper that when $r \ll \delta_0$ the capture of a second quantum is hindered energywise. This statement is apparently valid also when $r \ll a$, as will henceforth be assumed.

We assume therefore that each of the channels making up the wall is "charged" by one magnetic-flux quantum. We impose now the following boundary conditions: $H(x \to \infty) = H_0 - H_I$, $H(x \to -\infty) = H_0 + H_I$, where H_I is the field produced by the transport current flowing in the 0y direction. It is assumed, of course, that $H_0 \gg H_I$. Owing to these boundary conditions, the period of the vortex lattice in the region $x < 0$ is somewhat smaller than the period in the region $x > 0$. In other words, the vortex density changes jumpwise at x = 0 (Fig. 1). In the absence of channels, such a vortex distribution ρ would be patently not in equilibrium, since a nonzero pressure on the vortex system is produced in the plane x = 0 in the positive 0x direction. This pressure is balanced by the reaction of the "charged" channels. We have assumed ρ = const at $x \gtrless 0$, by virtue of the so called rigidity of the vortex lattice. In[2] and in[4] it is shown that the vortex density remains constant even for a thin film placed in an external magnetic field. There is all the more no reason for doubting that in our wall of channels the vortex density ρ remains constant to the right and to the left of the x = 0 plane.

But such a picture of the vortex distribution leads automatically to the presence of a transport current flowing in the 0y direction and distributed in a layer of thickness $\sim 2\delta_0$ about the plane x = 0 (Fig. 1).

Our problem consists precisely of finding a transport current I_c such that the stability of the picture outlined above is violated.

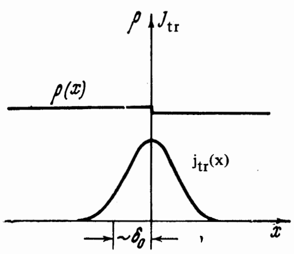


FIG. 1

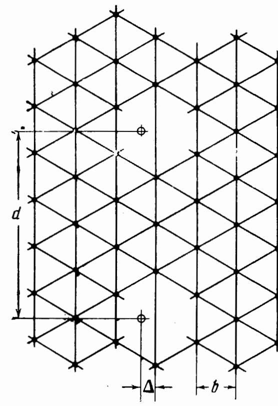


FIG. 2

FIG. 1. Density of vortices (ρ) and distribution of the transport current (j) in the case of a wall of channels.

FIG. 2. Schematic arrangement of the vortices (●) and of the channels (○) in the case of flow of a weak transport current. Turning on the transport current displaces the vortices by a distance Δ relative to the channels.

We illustrate our reasoning with the aid of Fig. 2, which shows a triangular vortex lattice (the vortices are represented by the dark circles), and where the channels are represented by light circles. The negligible difference in the vortex density Δ, due to the flow of the transport current, is not shown in the figure. We consider a field H_0 at which the ratio d/a is an integer. It is then clear that in the absence of transport current the channels will be at the points of a regular triangular lattice.

Turning on the transport current produces pressure on the vortex system in the 0x direction, and the system is shifted by a certain amount Δ. This is precisely the situation shown in Fig. 2. In the absence of the transport current the channels with their own vortices were aligned with the other vertices on a single straight line, but in the presence of the transport current they "step out" of their vortex row to the left, to a distance Δ. This deformation produces the reaction that balances the pressure due to the transport current.

What is the critical displacement? It is clear that if the lattice is displaced to the right by an amount $\Delta = b$, which is the distance along the x axis between the nearest rows, then the channels will be already located in the next row of vortices, the symmetry will be restored, the reaction will vanish, and the lattice will again move to the right, and vortex flow, i.e., a resistive state, will set in. Without claiming special accuracy, it is therefore quite natural to define the critical displacement as $\Delta_c = b/2$.

In this section we use exactly the same definition of the critical current as for a film[2].

We now proceed to the calculations. We find first the force exerted on one "charged" channel by the entire vortex system displaced by an amount Δ to the right from the equilibrium position. To this end, we must first calculate the energy of the interaction of a channel with any one vertical row of vortices.

Thus, we consider a row of vortices with coordinates (0, ma), m = 0, ±1, ±2, ... and a channel that has captured one flux quantum and has its axis at the point (x, y). It is required to find the interaction energy of this system. According to[3], the energy of interaction of a channel "charged" with one quantum and a vortex is equal to $4\pi K_0(\rho_0)/\kappa^2$, where $K_0(z)$ is a Hankel function of imaginary argument and of zero order. We shall use henceforth the relative units of the Ginzburg-Landau theory, viz., the unit of length $\delta_0(T)$ and the magnetic-field unit $\sqrt{2}H_{cm}$, where H_{cm} is the critical thermodynamic magnetic field. All the final formulas will be given in the absolute Gaussian system of units. ρ_0 is the distance from the vortex to the center of the channel.

The sought energy of interaction between a channel and a row of vortices is then

$$\varepsilon_1 = \frac{4\pi}{\kappa^2} \sum_{m=-\infty}^{\infty} K_0(\rho_m), \quad \rho_m = \sqrt{x^2 + (ma - y)^2}. \tag{1}$$

We represent $K_0(\rho)$ in the form of a Fourier integral

$$\sum_{m=-\infty}^{\infty} e^{-ik_2 ma} = 2\pi \sum_{n=-\infty}^{\infty} \delta(k_2 a - 2\pi n). \tag{2}$$

Substituting (2) in (1), we have

$$\varepsilon_1 = \frac{2}{\kappa^2} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_1\, dk_2 \frac{\exp\{i(k_1 x + k_2 y - k_2 ma)\}}{1 + k_1^2 + k_2^2}. \tag{3}$$

We sum this expression using the Poisson formula:

$$K_0(\rho) = \frac{1}{2\pi} \int d^2k \frac{e^{ik\rho}}{1 + k^2}.$$

Summing now (3) with respect to m and then integrating with respect to k_2, we get

$$\varepsilon_1 = \frac{4\pi}{\kappa^2 a} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 \frac{\exp\{i(k_1 x + 2\pi n y/a)\}}{1 + k_1^2 + (2\pi n/a)^2}.$$

Integrating this expression with respect to k_1 and recognizing that $a \ll 1$, we obtain

$$\varepsilon_1 = \frac{4\pi^2}{\kappa^2 a} e^{-|x|} + \frac{4\pi}{\kappa^2} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left\{-\frac{2\pi n}{a}|x|\right\} \cos\frac{2\pi n}{a} y.$$

Carrying out the summation in the last term, we obtain finally the following expression for the energy of interaction between a channel and a row of vortices:

$$\varepsilon_1(x, y) = \frac{4\pi^2}{\kappa^2 a} e^{-|x|} - \frac{2\pi}{\kappa^2} \ln\left|1 - 2e^{-2\pi|x|/a} \cos\frac{2\pi y}{a} + e^{-4\pi|x|/a}\right|.$$

We return now to our wall of channels and consider some particular channel.

Turning to Fig. 2, we see that the channel in question interacts with the adjacent left row of vortices with energy

$$\frac{4\pi^2}{\kappa^2 a} e^{-(b-\Delta)} - \frac{4\pi}{\kappa^2} \ln(1 + e^{-2\pi(b-\Delta)/a}).$$

The energy of interaction with the next row of vortices is

$$\frac{4\pi^2}{\kappa^2 a} e^{-(2b-\Delta)} - \frac{4\pi}{\kappa^2} \ln(1 - e^{-2\pi(2b-\Delta)/a}).$$

Thus, the energy of interaction of one channel with all the vortices to its left is

$$w_{\text{лев}} = \frac{4\pi^2}{\kappa^2 a} \sum_{n=1}^{\infty} e^{-(nb-\Delta)} - \frac{4\pi}{\kappa^2} \sum_{n=1}^{\infty} \ln\left(1 + \exp\left\{-\frac{2\pi}{a}[(2n-1)b - \Delta]\right\}\right)$$

$$- \frac{4\pi}{\kappa^2} \sum_{n=1}^{\infty} \ln\left(1 - \exp\left\{-\frac{2\pi}{a}(2nb - \Delta)\right\}\right).$$

We confine ourselves to small deformations, i.e., we assume that $\Delta \ll b$. Then all the exponentials in the sum under the logarithm sign are small. Using this, we make the substitution $\ln(1 + x) \to x$ everywhere and carry out the summation:

$$w_{\text{лев}} = \frac{4\pi^2}{\varkappa^2 a} e^{\Delta} \frac{e^{-b}}{1-e^{-b}} - \frac{4\pi}{\varkappa^2} e^{2\pi(b+\Delta)/a} \frac{e^{-4\pi b/a}}{1-e^{-4\pi b/a}} + \frac{4\pi}{\varkappa^2} e^{2\pi\Delta/a} \frac{e^{-4\pi b/a}}{1-e^{-4\pi b/a}}.$$

This expression can be greatly simplified by recognizing that $b \ll 1$ and $b/a = \sqrt{3}/2$, and retaining in the final expression only the principal term:

$$w_{\text{left}} = 4\pi^2 e^{\Delta} / \varkappa^2 ab.$$

The energy of interaction between the channel and all the right-hand vortices is calculated by the same method. The final result is

$$w_{\text{rt}} = 4\pi^2 e^{-\Delta} / \varkappa^2 ab.$$

This yields directly the energy of interaction of one channel with the entire system of vortices:

$$w = w_{\text{left}} + w_{\text{rt}} = 8\pi^2 \operatorname{ch} \Delta / \varkappa^2 ab. \tag{5}$$

The sought force exerted on one channel by the entire system is equal to

$$-dw / d\Delta = -8\pi^2 \Delta / \varkappa^2 ab.$$

This yields directly the force acting on a unit area of the wall of channels, i.e., the pressure

$$p = 8\pi^2 \Delta / \varkappa^2 abd.$$

According to our condition, the critical pressure at which the system of channels becomes unstable corresponds to a displacement $\Delta_c = b/c$, i.e., $p_c = 4\pi^2/\kappa^2 ad$. Changing over to absolute Gaussian units, we obtain

$$p_c = \frac{\pi}{\varkappa^2} H_{cm}^2 \frac{\delta_0^2(T)}{ad}. \tag{6}$$

To find the critical current, p_c must be equated to the Lorentz force exerted by the transport current on a unit area in the $x = 0$ plane:

$$f_L = I_c B / c, \tag{7}$$

where I_c is the total transport current flowing along the wall of channels in the 0y direction, and B is the induction inside the superconductor.

For a triangular lattice we have

$$a = 2^{1/2} \Phi_0^{1/2} / 3^{1/4} B^{1/2}, \tag{8}$$

where $\Phi_0 = \pi\hbar c/e$ is the quantum of magnetic flux.

Substituting (8) in (6) and equating (6) and (7), we obtain after trivial transformations

$$I_c = \frac{3^{1/4}}{4} \frac{cH_{cm}}{\varkappa d} \sqrt{\frac{\Phi_0}{B}}. \tag{9}$$

We have thus obtained the total critical current per unit height (along the 0z axis) flowing along the wall of channels in the vicinity of the $x = 0$ plane. As already indicated, this current is distributed in a layer of thickness on the order of $2\delta_0$ about the plane $x = 0$, so that the average density of the critical current in this layer is obtained by dividing (9) by $2\delta_0$:

$$j_c = \frac{3^{1/4}}{8} \frac{cH_{cm}}{\varkappa d \delta_0} \sqrt{\frac{\Phi_0}{B}}. \tag{10}$$

We present some estimates. Let the wall be made up of channels spaced $d \sim 10^{-5}$ cm apart, and let the superconducting material be characterized by the values $H_{cm} \sim 10^3$ Oe, $\kappa \sim 100$, and $\delta_0 \sim 10^{-4}$ cm. Then at $B \sim 10^4$ G, according to (9) and (10), we have $I_c \sim 10$ A/cm and $j_c \sim 10^5$ A/cm^2. Very many hard superconductors have critical-current densities of this order of magnitude.

We discuss now one more possibility of upsetting the stability of the vortex system in our model. At a sufficiently large external field H_0, it may turn out that before the vortex system shifts by an amount equal to the critical displacement Δ_c it will develop such a high pressure on the wall of channels, that the channels will be unable to retain the captured flux quanta, and the wall will break up.

It is easy to visualize when this can occur. According to[3], the force needed to extract a vortex from a channel (per unit channel length) is $H_{cm}^2 \xi/2$. It follows therefore that the pressure needed to break up the wall is $H_{cm}^2 \xi/2d$. Equating this value to the pressure due to the Lorentz force ($I_c B/c$), we obtain a new expression for the critical current and its density

$$I_c = cH_{cm}^2 \xi / 2dB, \tag{11}$$

$$j_c = cH_{cm}^2 \xi / 4d\delta_0 B. \tag{12}$$

Equating the right-hand sides of (9) and (11) and solving the resultant equation with respect to B, we obtain the field B_1 at which the "seepage" of the vortices through the channel wall gives way to the breakdown of the channels themselves:

$$B_1 = H_{c2} / \pi\sqrt{3}. \tag{13}$$

Thus, for the channel wall model under consideration, the vortex system should become unstable at $B < B_1$ as a result of "seepage" of the vortices through the wall, and the critical current is determined by (9), whereas at $B > B_1$ the stability is lost as a result of breakdown of the wall, and the critical current is given by (11).

3. PINNING ON CHANNELS FORMING A REGULAR LATTICE

We consider now a model in which the empty cylindrical channels are located at the points of a regular quadratic lattice. The length of the unit cell side is d, i.e., the distance between the nearest channels, satisfies the inequality $r \ll d \ll \delta_0(T)$, where r is the radius of the channel. We assume as before that the constant of the Ginzburg-Landau theory is $\kappa \gg 1$.

Let the external magnetic field (parallel to the channels) be $H_0 > H_{c1}$, but let H_0 always be weaker than the field at which the period a of the vortex lattice becomes of the order of r. If $a > r$, then each channel captures not more than one flux quantum. The transport current flows perpendicular to the channels along one of the sides of the quadratic unit cell.

It is assumed throughout that the external field is so strong, or that the transverse dimension of the superconductor is so small, that the field produced by the transport current on the surface of the superconductor $H_I \ll H_0$.

Let the external field H_0 be such that all the channels have captured one flux quantum each. Further increase of H_0 causes the vortices to orient themselves parallel to the channels, filling gradually the entire superconducting space between the channels. If the distance between the vortex in question and the channel is much larger than the channel radius, then the vortex becomes repelled from the channel[3]. It is clear that the unit cell made up of the four nearest channels acts like a magnetic "trap" in which the vortex in question is captured.

We consider here two cases, weak and strong fields.

We consider first the case of a weak field, which we take to mean a field such that each cell contains no more than one vortex, i.e., $B \leq 2\Phi_0/d^2$. The coefficient 2 in this inequality is the result of the fact that if one superconducting vortex is captured in each unit cell, then, taking into account the flux produced by the channels, the total magnetic flux per cell is equal to $2\Phi_0$.

It is obvious from symmetry considerations that the captured vortex will occupy the center of the cell. Let the external field H_0 be parallel to the 0z axis. We now turn on the transport current in the 0y direction. The resultant Lorentz force displaces the vortex in the 0x direction, and is large enough at a certain (critical) current to let the vortex negotiate the "pass" on the boundary of the cell. The same will obviously occur, at the same value of the current, for all the remaining vortices captured in the other unit cells. The result is a flow of the vortex structure through the system of channels via the "passes" on the boundaries of the unit cells.

Let us determine the critical current that leads to a flow of the vortex structure. We determine first the energy of interaction between the vortex with coordinates $(\Delta, 0)$ and the entire system of channels. The origin of the coordinate system is chosen at the center of the unit cell of the channels. The interaction of the vortex in question with any vertical row of channels located at a distance x from the vortex is obtained immediately by putting $y = d/2$ in (4) and making there the substitution $a \to d$:

$$w_1 = \frac{4\pi^2}{\varkappa^2 d} e^{-x} - \frac{4\pi}{\varkappa^2} \ln(1 + e^{-2\pi x/d}).$$

Summing now this expression over all the vertical rows of channels, we obtain the energy of interaction of the vortex in question with all the channels:

$$W(\Delta) = \frac{4\pi^2}{\varkappa^2 d} \frac{\operatorname{ch}\Delta}{\operatorname{sh}(d/2)} - \frac{4\pi}{\varkappa^2} \ln\left(1 + 2e^{-\pi} \operatorname{ch}\frac{2\pi\Delta}{d}\right), \quad -d/2 \leqslant \Delta \leqslant d/2. \tag{14}$$

The form of the function $W(\Delta)$ is shown in Fig. 3. From symmetry considerations it is obvious that this function can be continued periodically, with a period d along the entire 0x axis. These are indeed the potential wells or magnetic "traps" in which the vortices are captured. The pinning force per unit length of the vortex is equal to the maximum value of $|dW/d\Delta|$:

$$p_c = |dW/d\Delta|_{\Delta=\Delta_i},$$

where Δ_i the point of inflection of the function $W(\Delta)$ (14). Simple calculations yield

$$p_c = 6.9/\varkappa^2 d.$$

Changing over to absolute Gaussian units and equating the pinning force to the Lorentz force $j_c\Phi_0/c$ exerted on the vortex by the transport current, we obtain the critical current density:

$$j_c = 0.062\ cH_{cm}/\varkappa d. \tag{15}$$

The critical current given by this formula can be quite large. Indeed, if $H_{cm} \sim 10^3$ Oe, $d \sim 10^{-5}$ cm, and $\kappa \sim 100$, then $j_c \sim 6 \times 10^5$ A/cm^2.

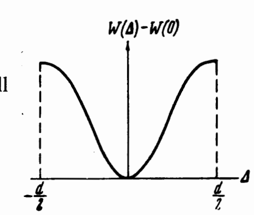
We assume from the very beginning that $d \gg r \gg \xi(T)$. It is of interest to consider at least the order of magnitude of the critical current j_c at $d \sim r \sim \xi(T)$. Substituting then $d \sim \xi$ in (15) and omitting the numerical coefficient, we have $j_c \sim cH_{cm}/\delta_0$, i.e., we obtain the order of magnitude of the pair-breaking critical current. This means that a sufficiently dense network of sufficiently thin channels is capable of ensuring the so-called rigid pinning of the vortices. The idea of a rigidly pinned vortex lattice was first advanced and experimentally demonstrated in[5].

We proceed now to the case of a strong field. By strong field we mean one for which there are many vortices in each unit cell. We assume as before that the distances between the vortices is $a \gg r$. Just as in the case of a wall of channels, we assume that the vortex density is constant, and that the channels occupy certain points of a regular triangular vortex lattice.[1)] If we let transport current flow perpendicular to the channels in the directions of one of the sides of the unit cell of channels, then the Lorentz force will shift the vortices relative to the channels. Repeating verbatim all the arguments of the preceding section, we arrive at the same definition of the critical displacement, $\Delta_c = b/2$, where $b = a\sqrt{3}/2$. We already know the energy (5) (which depends on the displacement Δ) of interaction between one channel and all the vortices. We know likewise the force exerted on one channel by the entire vortex system $(8\pi^2\Delta/\kappa^2 ab)$). Then the force exerted by the entire vortex system on a unit volume of the superconductor is equal to the density of the pinning force $f_p = 8\pi^2\Delta N_2/\kappa^2 ab$, where $N_2 = d^{-2}$ is the total length of all the channels per unit volume of the superconductor. Substituting here $\Delta = \Delta_c = b/2$, changing over to absolute Gaussian units, and equating the density of the Lorentz force $j_c B/c$ to the density of the pinning force f_p, we obtain the critical current density:

$$j_c = \frac{3^{1/4}}{4} \frac{cH_{cm}}{\varkappa} \left(\frac{\Phi_0}{B}\right)^{1/2} N_2. \tag{16}$$

The expression obtained by Coffey[6] for the critical current goes over into (16), apart from a numerical coefficient, when $H_0 \ll H_{c2}$.

FIG. 3. Energy of a vortex in a unit cell of channels containing only one vortex.



[1)]Strictly speaking, in this case the channel lattice cannot be quadratic and should be rectangular. Indeed, in the case of a strong field the sides of the unit cell of channels should be 2Lb and Ma, where L and M are large integers such that $2Lb \approx Ma$. This refinement does not affect in any way the arguments that follow.

If we assume $H_{cm} \sim 10^3$ Oe, $\kappa \sim 100$, and $N_2 \sim 10^{10}$ cm^{-2}, then we obtain at $j_c \sim 10^6$ A/cm^2 $B \sim 10^4$ G.

Just as in the case of a channel wall, it may likewise turn out in this model that if the vortex density is high enough the vortices will produce, even before they are displaced to the critical distance $\Delta_c = b/2$, so large a pressure on the channel system that the channels cannot retain the captured flux quanta, and the magnetic "traps" break down. Repeating the arguments given in Sec. 2, we obtain the critical current density for this case in the form

$$j_c = \alpha/B, \quad \alpha = {}^1/_2 c H_{cm}^2 \xi N_2. \tag{17}$$

We see thus that the dependence of the critical current on the field satisfies in this case Anderson's well known law[17].

Equating the right-hand sides of (16) and (17) and solving the obtained equation with respect to B, we obtain the value of the induction B_1 at which the dependence (16) is replaced by (17). Again, just as in the case of a channel wall, we obtain $B_1 = H_{c2}/\sqrt{3}\pi$. Thus, at $B < B_1$ the critical current is determined by (16), and at $B > B_1$ by (17).

4. PINNING BY A SYSTEM OF SPHERICAL CAVITIES

We consider now a somewhat more realistic model of a heterogeneous type-II superconductor. Assume that an ideally homogeneous type-II superconductor contains a system of spherical cavities (pores) forming a regular cubic lattice with period d. The radius r of the pore satisfies the inequality $\xi \ll r \ll d \ll \delta_0(T)$. Let the induction in the superconductor be $B \geq \Phi_0/d^2$. This means that one vortex filament passes through each spherical pore. These vortex filaments form a system pinned on the pores, which plays in our case the role of the channels in the preceding model. Indeed, so long as the vortices pinned on the spherical pores do not break away from these pinning centers, they form magnetic "traps" for the unpinned vortices located inside the unit cells made up by the spherical pores. This results in the same two mechanisms for upsetting the stability of the vortex structure which were considered in the preceding (channel) model. If the pinned vortices remain pinned, and the unpinned ones negotiate the "passes," then the critical current is determined by (16). However, even in this model it is possible for the unpinned vortices to exert sufficient pressure to detach the pinned vortices from the pinning centers before the unpinned vortices are displaced to the critical distance $b/2$ ($b = a\sqrt{3}/2$, where a is the side of the elementary triangle of the vortex lattice). We then arrive again at Anderson's formula, in which now $\alpha = cf_c N_3$. Here f_c is the force pinning the vortex to one spherical pore, and N_3 is the number of pores per unit volume. Assuming for f_c the estimate of[3], $f_c = H_{cm}^2 \xi r$, we obtain ultimately

$$j_c = \alpha/B, \quad \alpha = cH_{cm}^2 \xi r N_3. \tag{18}$$

Equating the right-hand sides of (16) and (18), we can find the induction B_1 starting with which j_c is given by formula (18):

$$B_1 = \frac{4}{\sqrt{3}\pi}\frac{r^2}{d^2}H_{c2}.$$

This result was obtained for a regular cubic lattice of spherical pores. If the pinning centers have a random distribution, the concepts of pinned and unpinned vortices becomes artificial. Vortices pinned in a given cross section (whose plane is perpendicular to the vortices) will be unpinned in another cross section (and vice versa). The critical current must therefore be defined only as the current sufficient to detach the vortices from the pinning centers, i.e., by formula (18).

We can attempt to compare this result with the experimental data. According to[8], neutron bombardment produces in Nb_3Sn inhomogeneities of diameter ~100 Å and density $N_3 \sim 5 \times 10^{15}$ cm^{-3}; these inhomogeneities can be seen in an electron microscope. Let us assume, in accord with[9], $H_{cm} = 2000$ Oe, $H_{c2} = 180$ kOe, and then $\xi = (\Phi_0/2\pi H_{c2})^{1/2} = 4.2 \times 10^{-7}$ cm. Substituting these values in (18), we obtain $\alpha_{theor} = 4.2 \times 10^{10}$ at. un./cm^2. According to[8], the experimental value is $\alpha_{exp} = 2.41 \times 10^{10}$ at. un./cm^2. The agreement is perfectly satisfactory.

5. DISCUSSION OF RESULTS

We consider first the channel-wall model. We found that a set of parallel channels whose centers lie on one straight line can serve as a path for transport current in a type-II superconductor, in a direction parallel to the channels.

If we take several such walls, parallel to one another and spaced apart a distance larger than $2\delta_0$ (double the penetration depth), then these walls will carry the transport current independently of one another; of course, the current will flow in a region $\sim 2\delta_0$ near each of the walls (see Fig. 1).

Our channels constitute a model of different nonsuperconducting inclusions or pores in a real superconductor. Thus, for example, the niobium-titanium alloys employed in applications are aging alloys. If aging annealing occurs after plastic deformation (say the drawing of a wire), then the non-superconducting inclusions fall out mainly along the slip bands produced by the cold deformation. We can therefore state that the superconducting transport current will flow in such an alloy precisely through such a film-filament network filling the entire cross section of the superconducting wire. In a certain sense this means a return to the Mendelsson-Gorter idea of a superconducting "sponge"[10,11].

We turn now to the regular lattice of channels. When we considered the case of a strong field we did not use anywhere the fact that the channel lattice is a regular one. This means that the result, namely formulas (16) and (17), has a wider range of applicability. These formulas are valid for an almost random distribution of parallel channels. By almost random channel distribution we mean one in which they occupy certain arbitrary points of the regular triangular vortex lattice. We require here that the distance s between any two nearest channels satisfy the inequality $a \ll s \ll \delta_0$. A completely random channel distribution should not lead to any pinning at all in the case of the undeformed vortex lattice assumed by us[12].

In conclusion, let us summarize the results.

We have considered several pinning models: a wall of empty channels, a lattice of channels, and a lattice of spherical pores. In the case of a wall of channels, the transport current flows along the wall (perpendicular to the channels) through a region of thickness $\sim 2\delta_0$. The critical current is determined by formula (9). In the case of a regular channel lattice ($d \times d$) and a weak field ($B < 2\Phi_0/d^2$) the critical-current density is given by (15). In the case of a strong field ($B \gg 2\Phi_0/d^2$) the critical-current density is determined by (16) if $B < H_{c2}/\sqrt{3}\pi$ and by (17) if $B > H_{c2}/\sqrt{3}\pi$. Finally, for a regular cubic lattice of spherical pores we have formula (16) when $B < 4r^2H_{c2}/(\sqrt{3}\pi d^2)$ and (18) when $B > 4r^2H_{c2}/(\sqrt{3}\pi d^2)$.

[1]V. V. Shmidt, Zh. Eksp. Teor. Fiz. **57**, 2095 (1969) [Sov. Phys.-JETP **30**, 1137 (1970)].
[2]V. V. Shmidt, Zh. Eksp. Teor. Fiz. **61**, 398 (1971) [Sov. Phys.-JETP **34**, 211 (1972)].
[3]G. S. Mkrtchyan and V. V. Shmidt, Zh. Eksp. Teor. Fiz. **61**, 367 (1971) [Sov. Phys.-JETP **34**, 195 (1972)].
[4]A. I. Rusinov and G. S. Mkrtchyan, Zh. Eksp. Teor. Fiz. **61**, 773 (1971) [Sov. Phys.-JETP **34**, 413 (1972)].
[5]Yu. F. Bychkov, V. G. Vereshchagin, V. R. Karasik, G. B. Kurganov, and V. A. Mal'tsev, Zh. Eksp. Teor. Fiz. **56**, 505 (1969) [Sov. Phys.-JETP **29**, 276 (1969)].
[6]H. T. Coffey, Phys. Rev. **166**, 447 (1968).
[7]P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962).
[8]G. W. Cullen, Proc. 1968 Summer Study Supercond. Devices and Accelerators, N. Y., 1969, p. 437.
[9]R. Hecht, translation in: Sverkhprovodyashchee soedinenie niobiǐ-olovo (The Superconducting Niobium-tin Alloy), Metallurgiya, 1970, p. 139.
[10]K. Mendelsson, Proc. R. Soc. A **152**, 34 (1935).
[11]C. J. Gorter, Physica (Utr.) **2**, 449 (1935).
[12]W. W. Webb, J. Appl. Phys. **42**, 107 (1971).

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ERRATA

Article by I. S. Shikin, "Analogs of Anisotropic Homogeneous Models of General Relativity in Newtonian Cosmology," Vol. 34, No. 2, 236 (1972):

1. In formula (2.3c) on p. 237 in the last round bracket read δ_a^g.
2. In formula (2.3d) on p. 237 there should be a plus sign in front of $(\kappa_a^a)^2$.
3. In formula (2.13) on p. 238 read $K_{\delta\beta}$ in place of $K_{\gamma\beta}$.
4. In formula (3.5) on p. 238 read R^3 instead of B^3.

Article by E. G. Brovman, Yu. Kagan, and A. Kholas, "Properties of Metallic Hydrogen under Pressure," Vol 35, No. 2, 783 (1972):

The legend on the ordinate axis of Fig. 6 on p. 786 should read

E_{vib}, 10^{-2} Ry/atom.