

Effect of Joule Heat on Destruction of Superconductivity by a Current

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The effect of the heat evolved in a sample on destruction of superconductivity by a current is considered for a cylindrical superconductor of the first kind. The distribution of temperature in the region occupied by the intermediate state is determined. The volt-ampere characteristics of the sample are determined. The resistance jump on transition to the intermediate state exceeds the London value which is 0.5. At current values $I \geq I_2$ the superconductivity is completely destroyed as a result of heating of the sample to the critical temperature T_c . The effect of the Kapitza temperature discontinuity at the boundary between the sample and liquid helium is considered in greater detail than in^[14]. In particular, the cause of the converse transition to the superconducting state on decrease of the current is explained. It is found that in the presence of a Kapitza discontinuity the sample cannot exist in a stationary state in a certain temperature range (T_0, T_1) near the temperature of helium, T_0 .

AS shown by Andreev and the author^[1], thermal effects can noticeably influence the electro-dynamics of the intermediate state of type-I superconductors. This influence is particularly strong in dirty metals and in type-I alloys. In this case, as shown earlier^[1], the dynamics of the intermediate state does not break up into purely electrodynamic and purely thermal parts, but constitutes a self-consistent problem in which both electromagnetic and thermal quantities must be determined. In the same paper^[1], we obtained a complete system of equations describing the dynamics of the intermediate state. In view of the nonlinearity of these equations, they cannot be solved in the general case. In certain particular cases, however, the problem becomes simpler and can be solved. In the present paper, such a problem is solved for a cylindrical current-carrying sample. This is all the more interesting, because the experimental data on the destruction of superconductivity by a current^[2-9] disagree with the classical London solution^[10]. The reasons for this disparity are the finiteness of the structure of the intermediate state^[4,11-13] and the heat release in regions occupied by the normal method. The first attempt to determine the influence exerted on the current-induced destruction of superconductivity by Joule heat and by the temperature discontinuity at a boundary with liquid helium was made by Berkovich and Lapir^[14], who obtained good agreement with experiment. They did not specify, however, the conditions under which their solution is valid (in particular, the temperature was assumed constant over the sample cross section), nor did they explain the reason for the inverse transition into the superconducting current when the current through the sample is decreased. Since we have obtained^[1] equations with which to determine the temperature distribution in the intermediate state, the effect of the heat released in the normal phase can be taken into account in the general case. We note that deviations from the London behavior, due to the already mentioned causes, can be of equal order of magnitude, and to determine which of them plays the principal role in each concrete case it is necessary to know the properties of the metal, its purity, and the temperature at which the experiment is performed.

1. We consider a superconductor of the first kind in

the form of a cylinder of radius r_2 , placed in liquid helium having a temperature T_0 . If the current I through the sample is lower than the critical values $I_c(T_0)$, then the sample is in the superconducting state. The electric resistance of the sample is then $R = 0$ and the temperature throughout its interior remains unchanged at $T = T_0$ so long as $I < I_c(T_0)$. When the current reaches the critical value, the magnetic field on the surface becomes critical, $H(r_2) = H_c(T_0)$, and the sample goes over into the intermediate state. At the first instant, when the temperature is still constant and equal to T_0 , the resistance increases jumpwise to a value $R = 1/2$, and the intermediate state occupies the entire volume of the sample. The heat released in the normal regions raises the sample temperature until a stationary regime is established, in which heat balance is established at each point inside the sample and on the surface.

If, as follows from the condition¹⁾

$$\operatorname{div} \mathbf{q} = \mathbf{jE} = \frac{cE_z}{4\pi}(\operatorname{rot} \mathbf{H})_z,$$

the radial component of the heat flux density $q = cE_z H / 4\pi$, then the heat-balance condition determines the stationary temperature gradient ∇T at each point of the sample:

$$q = cE_z H / 4\pi = -\kappa(r) \nabla T(r), \quad (1)$$

where $\kappa(r)$ is the thermal conductivity at the point r . The Kapitza temperature jump is observed on the surface in the general case. For simplicity, using the theoretical expression for the jump^{[15] 2)} at an arbitrary value of the latter, we can write down the boundary condition on the sample surface in the form

$$T^+(r_2) = T_0^+ + Kq(r_2), \quad (2)$$

where $K = 15D(hs_t)^3 / 4\pi^5 \rho c k^4 F$ (see^[15]). Thus, heating of the sample at a current $I = I_c(T_0)$ establishes a certain temperature distribution in it, and furthermore,

¹⁾Since we are interested in a stationary solution with axial symmetry, $\partial/\partial t = \partial/\partial z = \partial/\partial A = 0$. It follows from the condition $\operatorname{curl} \mathbf{E} = 0$ that $E_z = \text{const}$. Neglecting the Hall effect and taking the symmetry into account, we have $H_r = H_z = 0$ and $H_\phi = H$.

²⁾A comparison of the experimental data on the value of the Kapitza jump with the theory can be found in^[16,17].

in the presence of a Kapitza jump, the temperature on the sample surface increases to $T(r_2) > T_0$. As will be explained later on, this leads to an increase of the sample resistance to a value $R(I_C(T_0)) > 1/2$. The intermediate state, if a Kapitza jump exists, will occupy as a result of the heating a volume smaller than the volume of the entire sample.

Assume that at a certain current I the intermediate state occupies a certain volume whose boundary is denoted by r_1 . Then the metal in the region $r_1 \leq r \leq r_2$ is in the normal state. We introduce the dimensionless quantities

$$\begin{aligned} x &= r/r_2, & z &= R/R_n = \pi\sigma r_2^2 R, & \tau &= (T/T_0)^2, \\ h &= H(r)/H(r_2), & i &= I/I_0, \end{aligned} \quad (3)$$

where $R_n = 1/\pi\sigma r_2^2$ is the resistance of the entire sample in the normal state, σ is the electric conductivity of the normal metal, T_C is the critical temperature of the superconductor, and I_0 is defined by the relation $2I_0/cr_2 = H_C(T=0) \equiv H_0$. Using Maxwell's equations and the boundary condition $H(r_2) = 2I/cr_2$, we obtain the magnetic field in the region occupied by the normal metal:

$$h(x) = zx + (1-z)/x. \quad (4)$$

From (1), in which $\kappa(r)$ is the thermal conductivity of the normal metal, we obtain, using the Wiedemann-Franz law³⁾ $\kappa_n(T) = \kappa_n(T_C)T/T_C$ for the temperature dependence of the thermal conductivity as well as the boundary condition (2), the temperature distribution in the layer of normal metal, i.e., at $x_1 \leq x \leq 1$ ($x_1 = r_1/r_2$):

$$\begin{aligned} \tau(x) &= (\tau_0^2 + 2\delta_1 z i^2)^{1/2} + \delta_1 z i^2 [z(1-x^2) - (1-z) \ln x^2], \\ \delta_1 &= \frac{c^2 H_0^2}{8\pi^2 \sigma \kappa_n(T_C) T_C}, & \delta_2 &= \frac{c^2 H_0^2 K}{16\pi^2 \sigma r_2 T_C^4} \end{aligned} \quad (5)$$

The electric field $E_Z = RI = R_n I_0 z i$ is constant over the entire cross section of the sample, in view of the condition $\text{curl } \mathbf{E} = 0$ and the continuity at $x = x_1$.

We consider now the region $0 \leq x \leq x_1$ occupied by the intermediate state. As indicated earlier^[1], in the general case, in the macroscopic description of the intermediate state it is necessary to determine, in addition to the macroscopic quantities, also (in a self-consistent manner) the parameters of its structure, i.e., the normal n to the layer boundaries and the velocity V of the structure (for layered structures). Since we are interested in sufficiently dirty metals (in pure metals the temperature is constant over the sample cross section—see below), we can assume that a static London structure is realized: $n_Z = 1$, $V = 0$. Neglecting the effect of the magnetic field on the electric and thermal conductivities of the metal, we obtain the densities of the electric current $j_Z = \sigma E_Z/x_n$ and of the heat flux $q = -\kappa_{||} \nabla T$, where x_n is the concentration of the normal phase, $\kappa_{||} = x_n \kappa_n + x_S \kappa_S$, $x_S = 1 - x_n$, and κ_S is the thermal conductivity of the superconducting phase. Thus, the temperature distribution in the region occupied by the intermediate state is determined by Eq. (1), in which $\chi(r) = \kappa_{||}$ and $H(r)$ is

the magnetic field in the intermediate state, i.e., the critical field at the temperature at the given point r .

We denote the dimensionless temperature and magnetic field in this region by $\tilde{\tau}$ and \tilde{h} . Then $\tilde{h}(x) = H_C(\tilde{\tau}(x))/iH_0$ or, using the simple parabolic temperature dependence of the critical field, we have

$$i\tilde{h}(x) = 1 - \tilde{\tau}(x). \quad (6)$$

The concentration of the normal phase is expressed in terms of the temperature by the formula

$$x_n = \frac{\sigma E_z}{j_z} = \frac{4\pi\sigma E_z}{c(\text{rot } \mathbf{H})_z} = \frac{2zix}{1 - \tilde{\tau} - x d\tilde{\tau}/dx}. \quad (7)$$

Equation (1) takes consequently the form

$$k(\tilde{\tau}, x) \frac{x d\tilde{\tau}/dx}{1 - \tilde{\tau} - x d\tilde{\tau}/dx} + \delta_1 (1 - \tilde{\tau}) = 0, \quad k = 1 + \frac{x_n}{x} \frac{x_n}{\nu}. \quad (8)$$

For comparison, we write down Eq. (1) for a normal metal:

$$\frac{d\tau}{dx} + 2\delta_1 z i^2 \left(zx + \frac{1-z}{x} \right) = 0.$$

Without solving (8) in the general case, we consider three particular cases: low temperatures, high temperatures, and a pure metal. At low temperatures $\tau \ll 1$ the thermal conductivity of the superconducting phase can be neglected in comparison with the thermal conductivity of the normal phase, $\kappa_S \ll \kappa_n$. The coefficient k is then close to unity. Solving (8), we obtain the temperature distribution for this case:

$$x(\tilde{\tau}) = \frac{\text{const}}{1 - \tilde{\tau}} \exp \left[-\frac{1}{\delta_1 (1 - \tilde{\tau})} \right]. \quad (9)$$

At high temperatures, $\tau \lesssim 1$, the thermal conductivities of the normal and of the superconducting phases differ little, $\kappa_S \approx \kappa_n$. In this case $kx_n \approx 1$ and Eq. (8) takes the form

$$\frac{d\tilde{\tau}}{dx} + 2\delta_1 z i (1 - \tilde{\tau}) = 0, \quad (10)$$

and its solution

$$\tilde{\tau}(x) = 1 - \text{const} \cdot \exp(2\delta_1 z i x)$$

depends very little on the coordinate.

In pure metals the parameter $\delta_1 \sim (\kappa_{G-L} \xi_0 / l)^2$ (where κ_{G-L} is the parameter of the Ginzburg-Landau theory) is negligibly small and, as seen from (8), the temperature is constant in the intermediate state (and analogously when $x_1 \leq x \leq 1$). In the absence of a Kapitza jump, the temperature in the sample is equal to the helium temperature, $\tilde{\tau} = \tau = \tau_0$. In our case we have the London destruction of superconductivity by a current. Taking δ_1 into account in the first order of smallness, we obtain the temperature distribution in the intermediate state in metals that are not too dirty:

$$\begin{aligned} \tilde{\tau}(x) &= \text{const} - \frac{\delta_1 i_1^2}{1 - x_n/x_n} \ln \left[\frac{x_n x_n}{x_n} + \left(1 - \frac{x_n}{x_n} \right) x \right], \\ i &= I_C(T_0)/I_0 = 1 - \tau_0, \quad x_n = i_1/2zli, \quad z_n = 1/2 \{ 1 + [1 - (i_1/i)^2]^{1/2} \} \end{aligned} \quad (11)$$

is the London resistance, and κ_S and κ_n are taken at the temperature τ_0 . As seen from a comparison of Eqs. (9)–(11), the largest non-uniformity of the heating is observed at low temperatures in dirty samples. Figure 1 shows the dependence of the square of the relative temperature on the radius in the absence of a

³⁾This is possible, since we are interested in sufficiently dirty metals (see below) at low temperatures, when the electrons are scattered mainly by impurities.

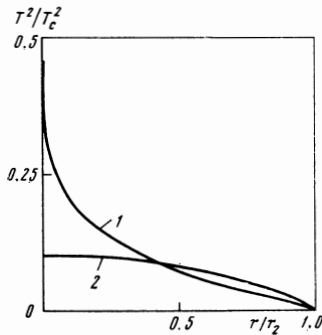


FIG. 1

Kapitza jump at zero temperature of the helium (see (9), $T_0 = 0$) and at the minimum current $i = i_1$, i.e., in the case when the entire volume of the sample is in the intermediate state (curve 1). For the parameter we have $\delta_1 = 0.1$. For comparison, the same figure shows the distribution of the temperature in the non-superconducting metal (curve 2) at the same values of the parameters i_1 , T_0 , and δ_1 .

2. Now, to complete the analysis of the problem, it remains to use the boundary conditions on the internal surface $x = x_1 = r_1/r_2$. They reduce to a continuity of the magnetic field and of the temperature and to equality of the normal-phase concentration to unity. Taking (4), (6), and (7) into account, we can write

$$zx_1 + \frac{1-z}{x_1} = \frac{1-\bar{\tau}(x_1)}{i}, \quad \bar{\tau}(x_1) = \tau(x_1), \quad (12)$$

$$2zx_1 = 1 - \bar{\tau}(x_1) - x_1 \left. \frac{d\bar{\tau}}{dx} \right|_{x_1}.$$

These equations determine the dependence of the unknown quantities x_1 , z , and const (which is contained in $\bar{\tau}(x)$) of the total current i through the sample. From the continuity of the heat flux at $x = x_1$ and from the equation $x_{n1}(x_1) = 1$ follows the continuity of the derivative of the temperature: $d\bar{\tau}/dx = d\tau/dx$ at $x = x_1$. Taking this circumstance into account, we obtain a system of two equations for the determination of the sample resistance z and the radius x_1 of the region occupied by the intermediate state:

$$zx_1 + \frac{1-z}{x_1} = \frac{1-\tau(x_1)}{i}, \quad 2zx_1 = 1 - \tau(x_1) - x_1 \left. \frac{d\tau}{dx} \right|_{x_1}, \quad (13)$$

where $\tau(x)$ is defined in (5).

Assume at first that there is no Kapitza jump ($\delta_2 = 0$). Then at we obviously have $x_1 = 1$. From (13) we obtain the value of the observed resistance jump

$$z_1 = \frac{1}{2} \frac{1}{1 - \delta_1 i_1}. \quad (14)$$

In pure metals $\delta_1 \ll 1$ and at high temperatures $\tau_0 \leq 1$ the current is $i_1 \ll 1$ and $z_1 \approx 1/2$. In dirty metals and in type-I alloys, on the other hand, we have $\delta_1 \lesssim 1$, so that at not too high values of T the jump can exceed 0.5 noticeably. Further, since the temperature in the sample approaches T_c with increasing current, and at the same time, as seen from the first equation of (13), we have $x_1 \rightarrow 0$ and $z \rightarrow 1$, it follows that at a certain current i_2 the sample becomes entirely normal: $z_1 = 0$, $z = 1$. In this case the radial temperature distribution is given by

$$\tau(x) = \tau_0 + \delta_1 i_2^2 (1 - x^2).$$

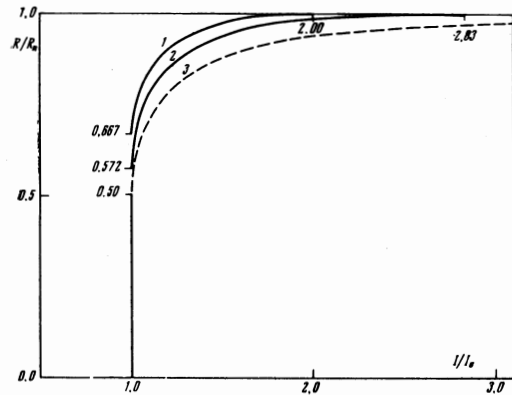


FIG. 2. Curve 1— $\delta_1 = 0.33$ and $(T_0/T_c)^2 = 0.25$; curve 2— $\delta_1 = 0.5$ and $(T_0/T_c)^2 = 0.75$; curve 3—London's theory.

From the condition $\tau(0) = 1$ we obtain the current i_2 :

$$i_2 = (i_1 / \delta_1)^{1/2}. \quad (15)$$

We note that, unlike the London model, the value of this current is finite. Thus, the intermediate state exists at current $i_1 \leq i \leq i_2$.

From (13), in the first approximation in terms of the actually small parameter δ_1 , we obtain z and x_1 ⁴⁾:

$$z(i/i_1) = 1/2 \{ 1 + [1 - (i/i_1)^2 \Phi(i/i_1)]^{1/2} \}, \quad (16)$$

$$x_1 \left(\frac{i}{i_1} \right) = \frac{i_1}{i} \frac{1 + \delta_1 i_1 (1 - f_1)}{2z(i/i_1)}$$

where

$$f_1 \left(\frac{i}{i_1} \right) = \left(\frac{i}{i_1} \right)^2 z_1 \left(2z_1 - 1 + \frac{i_1}{i} x_{n1} \ln \frac{1}{x_{n1}} \right),$$

$$\Phi \left(\frac{i}{i_1} \right) = 1 - 2\delta_1 i_1 f_1 - (\delta_1 i_1)^2 (1 - f_1^2).$$

Figure 2 shows plots of the resistance $z(i/i_1)$ plotted in accordance with these formulas (solid curves). The deviation from the London curve (dashed) is most noticeable in dirty samples at low temperatures. We note that this deviation does not depend on the sample radius. Unfortunately, all the known experiments were performed on sufficiently pure samples of Sn and In (for which $\delta_1 \ll 1$). Therefore the deviation from the London plot in these experiments is due to the influence of the structure of the intermediate state and to the presence of the Kapitza jump. It is desirable to perform experiments with dirty metals and alloys (where κ_n and σ are low) at low T in large-diameter samples (where the influence of the structure and of the Kapitza jump is smallest).

3. We now take into account the influence of the Kapitza temperature jump on the boundary of the sample with the liquid helium. For the sake of clarity, we assume the temperature to be constant over the cross section of the sample (this is true at high temperatures $\tau_0 \lesssim 1$, and at low temperatures in pure metals $\delta_1 \ll 1$

⁴⁾The exact solution of (13) reduces in the general case to a determination of z from the following transcendental equation:

$$zx_1 + \frac{1-z}{x_1} = \frac{i_1}{i} \left\{ 1 - \delta_1 i_1 \left(\frac{i}{i_1} \right)^2 z \left[z(1 - x_1^2) + (1-z) \ln \frac{1}{x_1^2} \right] \right\},$$

$$x_1(x) = \frac{1}{3\alpha} \{ 1 - (1-k)^{1/2} (\cos u - 3^{1/2} \sin u) \}, \quad u = \frac{1}{3} \arccos \frac{1-6k}{(1-k)^{3/2}},$$

$$k = 3\alpha^2(1-z)/z, \quad \alpha = 2\delta_1 i_1.$$

at not too large currents, $i \ll i_2$). In this case we obtain from (13) and (5)

$$z = \frac{1}{2} \left\{ 1 + \left[1 - \frac{I_c^2(T)}{I^2} \right]^{1/2} \right\} = \frac{1}{2} \left\{ 1 + \left[1 - \left(\frac{1-\tau}{i} \right)^2 \right]^{1/2} \right\},$$

$$x_1 = (1-z)^{1/2} / z^{1/2}, \quad \tau = (\tau_0^2 + 2\delta_2 i^2 z)^{1/2}. \quad (17)$$

The solution of this equation takes in the general case the form⁵⁾

$$z(i) = 1 - f_1^{-1}(i), \quad f_1(u) = \frac{1}{2} (1 - \tau_0^2) / (1 - u)^{1/2} \cdot$$

$$\times \left\{ u^{1/2} + \tau_0 \left[u + \frac{\delta_2 (1 - \tau_0^2)}{2\tau_0^2} \right]^{1/2} \right\}, \quad (18)$$

$$\tau(i) = f_2^{-1}(i),$$

$$f_2(u) = (u^2 - \tau_0^2) \left\{ 2\delta_2 \left[\left(1 - \frac{\delta_2}{2} \right) u^2 + \delta_2 u - \left(\tau_0^2 + \frac{\delta_2}{2} \right) \right] \right\}^{-1/2}.$$

In the case of a weak jump $\delta_2 \ll 1$, at sufficiently high temperatures $\tau_0 \lesssim 1$, we have $\tau \approx \tau_0 + \delta_2 i^2 z / \tau_0$, so that we obtain from (17)

$$z(i) = \frac{1}{2} \frac{1 + \Delta/2 + [1 + \Delta - (i/\tau_0)^2]^{1/2}}{1 + (\Delta/2i)^2}, \quad (19)$$

where $\Delta = \delta_2 i_1 / \tau_0 = \delta_2 (1 - \tau_0) / \tau_0$. This formula coincides with that obtained by Berkovich and Lapir^[14], who cite also a fairly good agreement with experiment. At low temperatures $\tau_0 \ll 1$ we can assume $\tau \approx (2\delta_2 i^2 z)^{1/2}$ in sufficiently pure metals. From (17) we have

$$z(i) = f_3^{-1}(i), \quad f_3(u) = 1/u^{1/2} [2(1-u)^{1/2} + (2\delta_2)^{1/2}],$$

$$\tau(i) = f_4^{-1}(i), \quad f_4(u) = u^2 \left\{ 2\delta_2 \left[\left(1 - \frac{\delta_2}{2} \right) u^2 + \delta_2 u - \frac{\delta_2}{2} \right] \right\}^{-1/2}. \quad (20)$$

Since the destruction of superconductivity in the presence of a Kapitza jump has certain distinguishing features not noted earlier^[14], we shall discuss in greater detail the kinematics of the transitions in this case. We consider for convenience a weak jump. Let the superconducting sample, without current, be placed in helium at $T_0 < T_C$. We start to increase the current i through the sample. It is obvious that at $i = i_1 = 1 - \tau_0$ a transition from the superconducting state to the intermediate one will set in. The heat flux released in the sample is

$$q_1 = I^2 R / 2\pi r_2 = ai^2 z, \quad a = I_0^2 / 2\pi^2 \sigma r_2^3.$$

On the other hand, the heat flux that can be transferred from the sample to the helium is (see (2))

$$q_2 = \frac{1}{2} T_c h \tau_0^{-1/2} (\tau - \tau_0) = b(\tau - \tau_0), \quad b = \frac{1}{2} T_c h \tau_0^{-1/2},$$

$$h = h(T_0) = 4T_0^3 / K,$$

where h is the thermal resistance of the boundary. Let us plot these fluxes in the (τq) plane (see Fig. 3). At the initial instant at $i = i_1$ we have $z = 1/2$ and $x_1 = 1$, and the sample will become heated, since $q_1 = ai_1^2/2 > q_2 = 0$. We move along the $q_1(\tau)$ curve from the point A to the point B, where $q_1 B = q_2 B$, i.e., the stationary temperature $\tau_B = \tau(i_1)$ is reached. The observed resistance jump $z_1 = z(i_1)$ is consequently (see (19))

$$z_1 = \frac{1}{2} \frac{1 + \Delta/2 + \Delta^{1/2}}{1 + (\Delta/2)^2} > \frac{1}{2}.$$

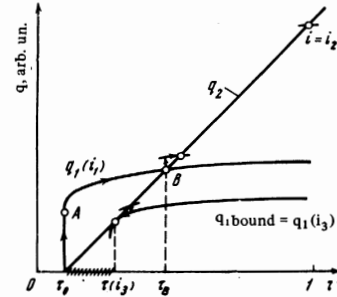


FIG. 3

We note that the region occupied by the intermediate state has become smaller as a result of the heating, $x_1 < 1$. (In the case of an arbitrary jump (Eqs. (18) and (20)), z_1 is obtained by solving fourth-degree equations.) With increasing current ($i > i_1$) we move upward along the $q_2(\tau)$ curve until, at a certain current i_2 , the stationary temperature reaches the critical value T_C , i.e., $\tau_B = 1$. From (17) we obtain in the general case

$$i_2 = \left(\frac{1 - \tau_0^2}{2\delta_2} \right)^{1/2}, \quad z_2 = 1, \quad x_1(i_2) = 0. \quad (21)$$

When $i \geq i_2$, the sample is in the normal state. If we decrease the current ($i < i_2$) then moving downward along the $q_2(\tau)$ curve, as shown in Fig. 3, we reach the boundary curve $q_{1\text{bound}} = q_1(i_3)$ at a certain current i_3 . With further decrease of the current, the heat fluxes q_1 and q_2 can no longer be equal. Since $q_1 < q_2$ the sample will be cooled, the current becomes less than critical, and superconductivity is restored. The condition for the absence of a solution of the equation⁶⁾ $q_1 = q_2$ determines the value of the current i_3 :

$$i_3 = \frac{i_1}{(1 + \Delta)^{1/2}}, \quad z(i_3) = \frac{1}{2} \frac{1 + \Delta}{1 + \Delta/2} > \frac{1}{2}, \quad (22)$$

$$\tau(i_3) = \tau_0 + \frac{\Delta i_1}{2 + \Delta}, \quad x_1(i_3) = \frac{1}{(1 + \Delta)^{1/2}} = \frac{i_2}{i_1}.$$

At low temperatures we obtain from (20) (in the general case, a cubic equation is obtained for i_3):

$$i_3 = z_3^{-1/2} \left[3 \left(\frac{\delta_2}{8} \right)^{1/2} + \left(2 + \frac{\delta_2}{8} \right)^{1/2} \right]^{-1/2},$$

$$z_3 = \frac{1}{2} \left\{ 1 - \frac{\delta_2}{8} + \left[\frac{\delta_2}{4} + \left(\frac{\delta_2}{8} \right)^2 \right]^{1/2} \right\} > \frac{1}{2}, \quad (23)$$

$$\tau(i_3) = (2\delta_2)^{1/2} \left[3 \left(\frac{\delta_2}{8} \right)^{1/2} + \left(2 + \frac{\delta_2}{8} \right)^{1/2} \right]^{-1/2}, \quad x_1 = \left(\frac{1 - z_3}{z_3} \right)^{1/2} < 1.$$

It is interesting to note that in addition to deviations from the London behavior and the presence of hysteresis in the current dependence of the sample resistance $z(i)$, there is in this case a very pronounced singularity⁷⁾, in that the temperature region from τ_0 to $\tau(i_3)$ becomes inaccessible. Namely, at such values of the current the sample cannot be in the stationary state at these temperatures. We note also that in the presence of a Kapitza jump the intermediate state never "climbs out" to the sample surface: $\max x_1 = x_1(i_3) < 1$. This must be taken into account in experiments aimed at observing the structure of the intermediate state.

⁶⁾It can be shown that in the general case this condition is equivalent to $dz(i)/di = \infty$ at $i = i_3$.

⁷⁾This fact was pointed out by A. F. Andreev.

⁵⁾ f_1^{-1} to f_4^{-1} in (18) and (20) are functions inverse to f_1 to f_4 .

In conclusion, I am grateful to A. F. Andreev for valuable consultations and for a useful discussion of the work.

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