

# Investigation of Spontaneous Parametric Radiation in the Biaxial $\alpha$ -HIO<sub>3</sub> Crystal

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Spontaneous parametric radiation (SPR) excited in the nonlinear biaxial crystal  $\alpha$ -HIO<sub>3</sub> by radiation from an argon laser ( $\lambda_3 = 4880 \text{ \AA}$  and  $5145 \text{ \AA}$ ) is investigated. The altered angular, electrooptical, and temperature curves are obtained. On variation of the polar angle, concurrent with alteration of the SPR frequencies, an alteration of frequencies in the given biaxial crystal was observed. The distribution of the SPR intensity over the frequency spectrum is investigated. The optimal parameters of a resonator for an  $\alpha$ -HIO<sub>3</sub> crystal parametric generator involving interaction of the  $e=0+e$  type are calculated.

## INTRODUCTION

THE biaxial crystal  $\alpha$ -HIO<sub>3</sub> belongs to class 222 of the orthorhombic system, is transparent in the region 0.4-1.4  $\mu$ , and has large components of the nonlinear-susceptibility tensor.<sup>[1-3]</sup> The crystal grows readily from an aqueous solution and can reach considerable dimensions (up to 10-15 cm). In addition, no optical inhomogeneities of the refractive index are produced in this crystal by optical radiation of high power density.<sup>[1,2]</sup> These properties of the crystal have attracted attention to it from the point of view of producing harmonic generators<sup>[2-4]</sup> and tunable parametric generators of light in the visible and near infrared bands.<sup>[5,6]</sup> An investigation of spontaneous parametric radiation (SPR)<sup>[7-11]</sup> can be used to obtain angular, electrooptical, temperature, elasto-optical tuning curves, to study the dispersion of the nonlinear optical constants in a wide spectral interval, including the region of strong absorption of one of the parametric frequencies, and also to measure the dispersion of the refractive indices in the infrared band.

The present paper is devoted to an investigation of SPR and to a calculation of the optimal resonator parameters for a parametric generator using the crystal  $\alpha$ -HIO<sub>3</sub>.<sup>1)</sup>

## TUNING CURVES

As is well known, the process of spontaneous parametric radiation proceeds quite effectively when the condition

$$\omega_3 = \omega_1 + \omega_2, \quad \mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2, \quad (1)$$

is satisfied, where  $\omega_3, \omega_2, \omega_1, \mathbf{k}_3, \mathbf{k}_2, \mathbf{k}_1$  are the frequencies and wave vectors of the pump and parametric waves, respectively. For biaxial crystals such as  $\alpha$ -HIO<sub>3</sub> the condition (1) in the case of collinearly propagating waves can be written in the form

$$\omega_3 n(\omega_3, \theta, \Phi) = \omega_1 n(\omega_1, \theta, \Phi) + (\omega_3 - \omega_1) n(\omega_2, \theta, \Phi), \quad (2)$$

where  $n(\omega_i, \theta, \Phi)$  is the refractive index at the frequency  $\omega_i$  in the propagation of a light wave with wave vector  $\mathbf{k}_i(\theta, \Phi)$ . Unlike uniaxial crystals, the parametric-radiation frequencies in biaxial crystals depend not only on the polar angle  $\theta$  but also on the azimuthal angle  $\Phi$ .

The refractive index  $n(\omega_i, \theta, \Phi)$  can be expressed in terms of the principal values  $N_X(\omega_i), N_Y(\omega_i), N_Z(\omega_i)$  of the refractive indices of the crystal with the aid of the Fresnel equation<sup>[13]</sup>

$$\frac{\sin^2 \theta \cos^2 \Phi}{[n(\omega_i, \theta, \Phi)]^{-2} - [N_X(\omega_i)]^{-2}} + \frac{\sin^2 \theta \sin^2 \Phi}{[n(\omega_i, \theta, \Phi)]^{-2} + [N_Y(\omega_i)]^{-2}} + \frac{\cos^2 \theta}{[n(\omega_i, \theta, \Phi)]^{-2} - [N_Z(\omega_i)]^{-2}} = 0. \quad (3)$$

The crystallographic axes of  $\alpha$ -HIO<sub>3</sub> are designated in accordance with the IRE standards<sup>[14]</sup> ( $c < a < b$ ), X is directed along a, Y along b, and Z along c, with  $N_X \geq N_Y > N_Z$ .

Let us consider first the satisfaction of the synchronism condition (2) for the propagation of light with a wave vector lying in one of the principal planes of the crystal. An analysis of the dispersion properties of the refractive indices of the  $\alpha$ -HIO<sub>3</sub> crystal<sup>[2]</sup> shows that the condition (2) is satisfied at  $\lambda \approx 0.5 \mu$  for an interaction of the type  $e = o + o$ ,<sup>2)</sup> and for an interaction of the type  $e = o + e$ , for the propagation of a light wave with a pump wave vector lying in the principal plane (XZ) or (YZ). The  $\alpha$ -HIO<sub>3</sub> crystal has three nonzero linearly-independent coefficients  $d_{14}, d_{25},$  and  $d_{36}$ , which, when account is taken of the Kleinman symmetry,<sup>[15]</sup> are equal to each other. The effective nonlinear coefficient for the interaction  $e = o + e$  in the principal planes (XZ) and (YZ) is equal to  $d_{\text{eff}} = 2d_{14} \sin 2\theta$  and  $d_{\text{eff}} = 0$  for the interaction of the type  $e = o + o$  in all the principal planes. However, when the pump wave vector goes outside the principal plane,  $d_{\text{eff}}$  becomes different from zero also in the case of the  $e = o + o$  interaction.

In our experiment the pump was a cw argon laser at wavelengths  $\lambda_3 = 4880 \text{ \AA}$  and  $\lambda_3 = 5145 \text{ \AA}$ , with output power up to 1 W for each of the wavelengths. The  $\alpha$ -HIO<sub>3</sub> crystals measured 1-2 cm. The crystals were oriented along the natural faces or with the aid of x-rays. The orientation was made more precise by using the interference pattern in converging beams in polarized light. Figure 1 shows plots of the angular tuning curves in the  $\alpha$ -HIO<sub>3</sub> crystal pumped at wavelengths  $\lambda_3 = 4880 \text{ \AA}$  (curves 1) and  $\lambda_3 = 5145 \text{ \AA}$  (curves 2). The solid curves in Fig. 1 are the results of the calculation for the case of one-dimensional parametric interaction on the basis of (2) and (3) for the (XZ) and (YZ) planes,

<sup>1)</sup>Initial results from observation of SPR in the  $\alpha$ -HIO<sub>3</sub> crystal were reported earlier [12].

<sup>2)</sup>Here e stands for a wave polarized in the principal plane of the crystal in which the wave vector is located, and o for a wave polarized perpendicular to this plane.

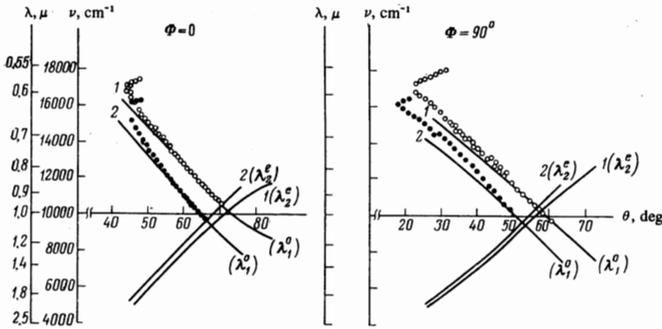


FIG. 1. Angular tuning curves for one-dimensional parametric interaction  $e = o + e$  at a pump  $\lambda_3 = 4880\text{\AA}$  (curves 1) and  $\lambda_3 = 5145\text{\AA}$  (curves 2).  $1/\lambda_3^e = 1/\lambda_1^o + 1/\lambda_2^e$ .

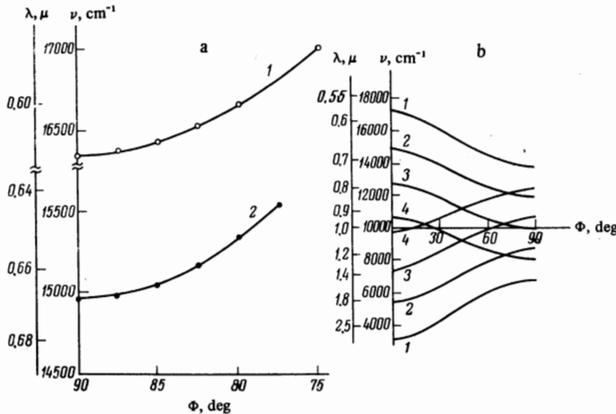


FIG. 2. Tuning curves relative to the azimuthal angle  $\Phi$ : a—experimental tuning curves for  $\lambda_3 = 4880\text{\AA}$  (curve 1) and  $\lambda_3 = 5145\text{\AA}$  (curve 2) at  $\theta = 26^\circ$ ; b—calculated tuning curves for  $\lambda_3 = 4880\text{\AA}$ ; 1— $\theta = 40^\circ$ , 2— $\theta = 50^\circ$ , 3— $\theta = 60^\circ$ , 4— $\theta = 70^\circ$ .

corresponding to the cases  $\Phi = 0^\circ$  and  $\Phi = 90^\circ$ . The calculation was based on the data on the dispersion of the  $\alpha$ - $\text{HIO}_3$  crystal, measured up to  $\lambda = 1.2\mu$  by Kurtz et al.<sup>[2]</sup> For the spectral region with  $\lambda > 1.2\mu$ , the dispersion of the refractive indices was approximated with the Cauchy formulas. The results of the experiment are marked by circles for  $\lambda = 4880\text{\AA}$  and by dots for  $\lambda_3 = 5145\text{\AA}$ . It is seen from Fig. 1 that when the additional frequency  $\omega_2$  approaches the absorption band, the experimental tuning curves deviate from the calculated ones, since the Cauchy formulas no longer hold. The deviation begins earlier for the principal plane (YZ) than for the principal plane (XZ), which agrees with the polarization anisotropy of the infrared absorption edge for the  $\alpha$ - $\text{HIO}_3$  crystal.<sup>[2]</sup> In addition, the tuning curves exhibit anomalies which we attribute to the anomaly of the dispersion of the refractive index in the absorption region of the additional frequency  $\omega_2$ .

Figure 2a shows the experimental angular tuning curves with respect to the azimuthal angle  $\Phi$  for  $\theta = 26^\circ$ ,  $\lambda_3 = 4880\text{\AA}$  (curve 1), and  $\lambda_3 = 4145\text{\AA}$  (curve 2). Figure 2b shows a family of calculated angular tuning curves relative to the azimuthal angle  $\Phi$  for different values of the polar angle  $\theta$  at  $\lambda_3 = 4880\text{\AA}$ . It should be noted that the experimental angular tuning curves relative to the azimuthal angle were obtained in a region close to the absorption of the additional frequency, where the Cauchy

FIG. 3. Experimental temperature-tuning curve for  $\lambda_3 = 4880\text{\AA}$  at  $\Phi = 0^\circ$  and  $\theta = 50^\circ$ .

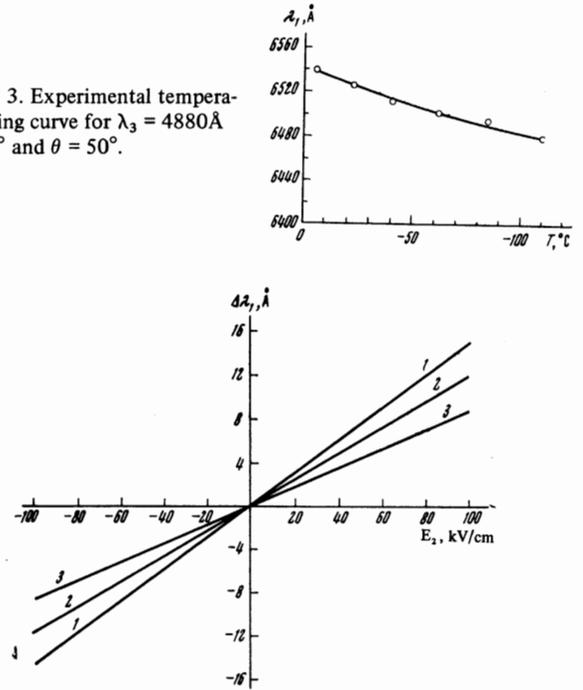


FIG. 4. Calculated electrooptical tuning curves for  $\lambda_3 = 4880\text{\AA}$  at  $\Phi = 0$ : 1— $\theta = 45^\circ$ , 2— $\theta = 55^\circ$ , 3— $\theta = 65^\circ$ .

formulas for the dispersion of the refractive indices no longer hold. From the point of view of choosing the geometry of the crystal, however, it is difficult to obtain experimental curves in the interval of the polar angles  $\theta$  where the Cauchy formulas hold.

The dependence of the refractive indices of the crystal on the temperature can lead to a variation of the SPR frequency with temperature. Owing to the absence of published data on the temperature dependence of the refractive indices for the  $\alpha$ - $\text{HIO}_3$  crystal, we were unable to calculate temperature-tuning curves and confined ourselves to an experimental investigation of the temperature variation of the SPR signal frequency. The results of the experiment for  $\lambda_3 = 4880\text{\AA}$ ,  $\Phi = 0$  and  $\theta = 50^\circ$  are shown in Fig. 3. It follows from the experimental data of Fig. 3 that  $d\lambda/dT \approx 0.55\text{\AA}/^\circ\text{C}$  when the temperature ranges from  $-110$  to  $0^\circ\text{C}$ .

The presence of an electro-optical effect in the  $\alpha$ - $\text{HIO}_3$  crystal<sup>[16]</sup> makes it possible to vary the parametric frequencies by means of an electric field applied to the crystal. Let us consider the case when the pump wave vector lies in the (XZ) plane and can change its direction in this plane, while the external electric field is applied along the Y axis ( $E = (0E_20)$ ). In this case the change of the refractive index for the e-beam has in accordance with<sup>[16]</sup> the form

$$\Delta n(\omega_i, \theta, 0, E_2) = 1/2 n^3(\omega_i, \theta, 0, 0) r_{52} E_2 \sin 2\theta, \quad (4)$$

where  $r_{52}$  is the electrooptical coefficient. The refractive index for the o-beam in this case is independent of the field (in the approximation linear in the field). Consequently, the condition (2) can now be rewritten as follows:

$$\omega_3 [n(\omega_3, \theta, 0, 0) + 1/2 n^3(\omega_3, \theta, 0, 0) r_{52} E_2 \sin 2\theta] = \omega_1 N_y(\omega_1) + (\omega_3 - \omega_1) [n(\omega_2, \theta, 0, 0) + 1/2 n^3(\omega_2, \theta, 0, 0) r_{52} E_2 \sin 2\theta]. \quad (5)$$

We have calculated the tuning curves on the basis of (5), using the electrooptical coefficient  $r_{52} = 7.6 \times 10^{-10}$  cm/V measured at the wavelength  $\lambda = 6328 \text{ \AA}$ .<sup>[16]</sup> The results of the calculation are shown in Fig. 4. The curves on Fig. 4 represent the dependence of the variation of the SPR wavelength

$$\Delta\lambda_1(E_2) = \lambda_1(\theta, 0, 0) - \lambda_1(\theta, 0, E_2)$$

on the intensity of the external electric field applied to the crystal at different values of the angle  $\theta$ . As follows from Fig. 4, the weak dependence of the wavelength on the external field does not yield an appreciable parametric-frequency tuning range, but this effect can be used, for example, to adjust the frequency of the resonator in a parametric light generator.

#### SPR INTENSITY IN THE CASE OF ABSORPTION OF ONE OF THE PARAMETRIC FREQUENCIES

Figure 5 shows the results of the experimental investigation of the dependence of the spectral density  $I_1$  of the SPR on the wavelength  $\lambda_1$ , when the wavelength of the additional radiation  $\lambda_2$  changes near the infrared absorption edge of the  $\alpha$ -HIO<sub>3</sub> crystal. The SPR wavelength was varied by rotating the crystal. The spectral densities were measured with an ISP-51 spectrograph by photographic photometry. The density scale was produced with a calibrated SI-8 tungsten incandescent lamp. The accuracy of the relative measurements was 25%. It follows from Fig. 5 that as the wavelength  $\lambda_1$  approaches the region of the anomaly in the tuning curves (Fig. 1), which we relate to the anomalous dispersion of the refractive index at the wavelength  $\lambda_2$  near the infrared absorption edge of the crystal, a decrease is observed in the SPR spectral density, followed by an increase. We note that the "dip" in the SPR spectral density is observed in the region of the spectrum ahead of the start of the anomalies in the angular tuning curves (Fig. 1), i.e., while  $\lambda_2$  is still not in the region of maximal absorption. With further approach of the wavelength  $\lambda_2$  to the strong absorption, an appreciable broadening of the SPR spectrum at wavelength  $\lambda_1$  is observed, owing to the character of the anomalies in the tuning curves and owing to the strong absorption at the wavelength  $\lambda_2$ .

The observed "dip" in the SPR spectral density can be attributed to the dispersion of the nonlinear susceptibility  $d(\omega_1 = \omega_3 - \omega_2)$ , when the parametric frequency  $\omega_2$  lies near the infrared absorption edge of the crystal. In this case the nonlinear susceptibility can be represented in the form of the electronic and lattice (Placzek) nonlinearities<sup>[17,18]</sup>

$$d(\omega_1 = \omega_3 - \omega_2) = d_e \left[ 1 + \sum_n C_n \left( 1 - \frac{\omega_2^2}{\omega_n^2} - i \frac{\omega_2 \Gamma_n}{\omega_n^2} \right)^{-1} \right], \quad (6)$$

where  $C_n$  is a constant,  $\omega_n$  and  $\Gamma_n$  are the resonant frequency and damping constant of the lattice vibrations, and  $d_e$  is the nonlinear electronic susceptibility. Depending on the sign of  $C_n$ , the quantity  $d(\omega_1 = \omega_3 - \omega_2)$  can either increase or decrease as  $\omega_2$  approaches  $\omega_n$ . Thus, the dip in the SPR spectral density can be attributed to the fact that the electronic and lattice nonlinearities at the dip frequency cancel each other.<sup>[18]</sup> The dip observed by us cannot be interpreted as being due to the influence of the absorption of the additional wave,<sup>[19]</sup>

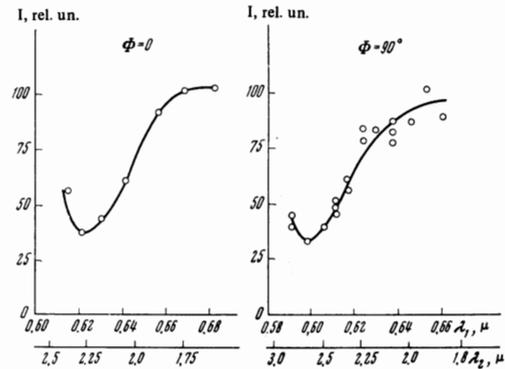


FIG. 5. Experimental plot of the SPR spectral density  $I_1$  against the wavelength for  $\lambda_3 = 4880 \text{ \AA}$ .

since the dip is followed by an increase in the SPR spectral density, whereas the absorption coefficient at the wavelength  $\lambda_2$  continues to increase. The different positions of the dip on Figs. 5a and 5b can be attributed to dichroism in the infrared absorption of the  $\alpha$ -HIO<sub>3</sub> crystal<sup>[22]</sup> and to the anisotropy of the constant  $C_n$ .

#### OPTIMUM FOCUSING OF WAVES IN A PARAMETRIC LIGHT GENERATOR IN THE $e = o + e$ INTERACTION

When estimating the threshold of parametric generation in the  $\alpha$ -HIO<sub>3</sub> crystal, it is necessary to take into account the birefringence of light, which takes place in directions along which the phase synchronism condition (1) is satisfied. If the given biaxial crystal is oriented in such a way that the light rays propagate in one of its principal planes (XZ) or (YZ), then it is similar in its birefringent properties to a uniaxial negative crystal with Z axis as the "optical axis." The ray whose electric vector lies in the principal plane then behaves like the extraordinary ray, and the ray with perpendicular polarization like the ordinary ray.

In comparison with the case of the  $e = e + o$  interaction, which was considered in detail by Boyd and Kleinman,<sup>[20]</sup> the determination of the optimal conditions of parametric generation and the calculation of the threshold pump power in the  $e = o + e$  interaction, which is observed in the principal planes (XZ) and (YZ) of  $\alpha$ -HIO<sub>3</sub>, are made complicated by the fact that in this case, owing to birefringence, the parametric waves diverge in the crystal, and the divergence angle depends on the geometry of the resonator, which effects the feedback at both parametric frequencies  $\omega_1^o$  and  $\omega_2^e$ . By way of an example, let us consider a resonator made up of a uniaxial negative crystal of length  $l$  with spherical surfaces having a radius of curvature  $R$ .<sup>3)</sup> As seen in Fig. 6, the mirror on the right-hand surface of the crystal is displaced relative to the left-hand mirror in such a way that the extraordinary ray with wave vector  $\mathbf{k}_2$  directed along the z axis, on emerging from the "center" of the left-hand mirror (i.e., from the point at which the normal to the mirror is parallel to the z axis), falls on the "center" of the right-hand mirror. In this case the extraordinary ray returns along the previous path after

<sup>3)</sup>The question of optimal focusing of the pump for a parametric generator with a resonator made up of flat mirrors was considered in [21].

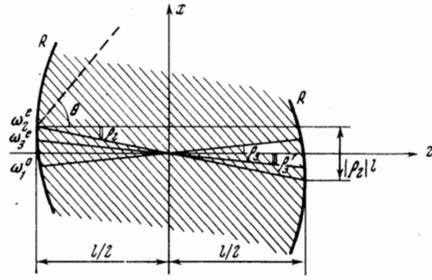


FIG. 6. Ray paths in resonator made up of a uniaxial negative crystal in parametric  $e = o + e$  excitation. The dashed line shows the orientation of the optical axis of the crystal.

each reflection, and accordingly the fundamental "extraordinary" mode of the given resonator in the stability region  $0 < l/R < 2$  becomes a superposition of two opposing waves of the type<sup>[20]</sup>

$$E_2^e = \frac{|E_{02}^e|}{1 + 2iz/b} \exp \left[ \pm ik_2 z - k_2 \frac{(x - \rho_2 z)^2 + y^2}{b + 2iz} \right] \quad (7)$$

where  $b = l[2R/l - 1]^{1/2}$  and the birefringence angle at the frequency  $\omega_2^e$  is

$$\rho_2 = \frac{(N_{2z}^2 - N_{2y}^2) \sin 2\theta}{(N_{2z}^2 + N_{2y}^2) + (N_{2z}^2 - N_{2y}^2) \cos 2\theta} < 0 \quad (8)$$

for the principal plane YZ.

As to the behavior of the ordinary waves in this resonator, it follows from<sup>[22]</sup> that the centers of the ordinary transverse modes that are established in this resonator are shifted on the surfaces of the left (and right) mirror downward (and upward) by an amount  $|\rho_2|l/(2 - l/R)$  from the center of the corresponding mirror. Consequently, in the cw regime the ordinary parametric beam (together with its wave vector  $\mathbf{k}_1$ ) is deflected from the z axis through the angle

$$\rho_1 = -\rho_2 \frac{l/R}{2 - l/R} = -\rho_2 \xi^2 > 0, \quad \xi = \frac{l}{b} = \left( \frac{2R}{l} - 1 \right)^{1/2}, \quad (9)$$

and crosses the extraordinary parametric beam in the middle of the crystal (Fig. 6). Under the natural assumption that the parametric generation occurs at the fundamental transverse mode, the ordinary wave traveling in the direction of the pump wave should be of the form

$$E_1^o = \frac{|E_{01}^o|}{1 + 2iz/b} \exp \left[ ik_1(z \cos \rho_1 + \rho_1 x) - k_1 \frac{(x - \rho_1 z)^2 + y^2}{b + 2iz} \right]. \quad (10)$$

The angle  $\rho_1$  is assumed here to be small, just as the angle  $\rho_2$  in (7). It should be noted that both parametric waves are focused at the point of their intersection (and the latter is located at the center of the crystal because of the symmetry of the chosen resonator).

Since the vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$  of the ordinary and extraordinary waves diverge at an angle  $\rho_1$ , it is easy to prove that when the synchronism condition is satisfied the pump wave vector  $\mathbf{k}_3$  should be inclined to the z axis at an angle

$$\varphi_3 = k_{1\rho_1}/k_3 = -k_{1\rho_2}\xi^2/k_3 > 0 \quad (11)$$

and accordingly the angle between the ordinary pump ray and the axis is

$$\rho_3' = \varphi_3 + \rho_3. \quad (12)$$

The angle introduced here is calculated from formula (8), in which the principal values  $N_{2i}^2(\omega_2^e)$  are replaced by the values  $N_{3i}^2(\omega_3^e)$ . The dispersion of the birefringence in the  $\alpha$ -HIO<sub>3</sub> crystal is appreciable. Assuming further that the pumping is by means of a laser with a stable resonator in the fundamental transverse mode, and that this wave is focused at the point of intersection of the parametric waves, the field of the extraordinary pump wave inside the crystal can be described by the expression

$$E_3^e = \frac{|E_{03}^e|}{1 + 2iz/b_3} \exp \left[ ik_3(z \cos \varphi_3 + \varphi_3 x) + i\psi - k_3 \frac{(x - \rho_3' z)^2 + y^2}{b_3 + 2iz} \right]. \quad (13)$$

The constant phase shift  $\psi$  and the parameter  $b_3$  are determined from the condition that the pump threshold power be a minimum.

In analogy with the  $e = o + o$  interaction<sup>[20]</sup> we can use (7), (10), and (13) to calculate the increase of the radiation power  $P_{1(2)}$  at the parametric frequencies  $\omega_1^o$  and  $\omega_2^e$  for the  $e = o + e$  interaction:

$$\begin{aligned} \Delta P_{1(2)} &= -\frac{\omega_{1(2)}}{2} \operatorname{Im} \left\{ \int_{-\infty}^{\infty} dx dy \int_{-l/2}^{l/2} dz d_{\text{eff}} E_1^o E_2^e E_3^e \right\} \quad (14) \\ &= -\frac{\omega_{1(2)}}{2} d_{\text{eff}} |E_{01}^o E_{02}^e E_{03}^e| \frac{\pi b^2 b_3}{2k_3(b + b_3)} \\ &\times \operatorname{Im} \left\{ e^{i\varphi} \int_{-1}^1 \frac{d\tau}{1 - i\tau} \exp \left[ -i\tau \frac{b}{2} \left( k_1 + k_2 - k_3 - \frac{k_1 k_2 \rho_1^2}{2k_3} \right) \right. \right. \\ &\left. \left. - \frac{\tau^2 b^2 (k_2 \rho_2 - k_3 \rho_3)^2}{4k_3(b + b_3)} - \frac{\tau^2 k_1 k_2 b (\rho_1 - \rho_2)^2}{4k_3(1 - i\tau)} \right] \right\} \end{aligned}$$

where  $\tau = 2z/b$ . Since the integral in the right-hand side of (14) is positive, the maximum of  $\Delta P_{1(2)}$  is reached at  $\psi = -\pi/2$  (we note that on going over to the  $e = o + o$  interaction it is necessary to put in (14), (7), (10), and (13)  $\rho_2 = \rho_1 = \varphi_3 = 0$ ; the last term in the square brackets of (14) then vanishes).

Recognizing that in the stationary generation regime the gain  $P_{1(2)}/P_{1(2)}$  cancels out the radiation loss  $2\delta_{1(2)}$  occurring when the wave  $E_1^o(E_2^e)$  passes through the resonator in the forward and backward directions, and expressing the amplitudes of the waves (7), (10), and (13) in terms of the corresponding radiation powers, we obtain an equation for the threshold pump power:

$$\delta_1 \delta_2 = \frac{\Delta P_1}{P_1} \frac{\Delta P_2}{P_2} = \frac{64\pi^2 \omega_1^2 \omega_2^2 \omega_3^2 d_{\text{eff}}^2}{c^2 k_3^2} P_3 M, \quad (15)$$

where the quantity  $M$ , whose maximum corresponds to the minimum of the generation threshold, is given by

$$M = \frac{\alpha}{(1 + \alpha)^2 \xi}, \quad J = \int_{-1}^1 \frac{d\tau}{1 + i\tau} \exp \left( i\sigma\tau - \beta^2 \tau^2 - \frac{\gamma \tau^3}{1 + i\tau} \right). \quad (16)$$

We have introduced here the notation

$$\begin{aligned} \alpha &= b_3/b, \quad \sigma = 1/2 b (k_1 + k_2 - k_3 - k_1 k_2 \rho_1^2 / 2k_3), \\ \beta^2 &= \beta^2 / (1 + \alpha) \xi, \quad \beta^2 = l(k_2 \rho_2 - k_3 \rho_3) / 4k_3, \\ \gamma &= k_1 k_2 b (\rho_1 - \rho_2)^2 / 4k_3 = \bar{\gamma} (1 + \xi^2) / \xi, \quad \bar{\gamma} = k_1 k_2 l \rho_2^2 / 4k_3. \end{aligned} \quad (17)$$

The varied parameters  $\alpha$ ,  $\xi$ ,  $\sigma$  have been separated in explicit form in (16) and (17). The remaining quantities  $\bar{\beta}^2$  and  $\bar{\gamma}$  remain constant when the function  $M(\alpha, \xi, \sigma)$  is maximized. We recall that throughout the paper of Boyd and Kleinman<sup>[20]</sup> the parameter  $\alpha$  was assumed equal to unity on the basis of one particular deduction for the case when there is no birefringence (and consequently

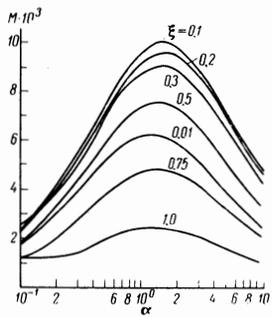


FIG. 7. Dependence of the function  $M(\alpha, \xi)$ , which characterizes the threshold of parametric generation in the  $\alpha$ -HIO<sub>3</sub> crystal, on the parameters  $\xi$  and  $\alpha$ .

the indirect dependence of  $M$  on  $\alpha$  via the integral  $J$  disappears:  $\beta^2 = \gamma = 0$ ). It is easy to see, however, that in the presence of noticeable birefringence the dependence of the integral  $J$  on the parameters  $\alpha$  and  $\xi$  is, generally speaking, equally significant. The results of numerical calculation, shown in Fig. 7, confirm these considerations.

From the results of the numerical calculation shown in Fig. 7 for the principal plane (YZ) and  $\theta_m = 58^\circ$  (corresponding to the degenerate case at  $\lambda_3 = 4880 \text{ \AA}$ ) it follows that the minimum threshold pump power  $P_3$  for a parametric generator using an  $\alpha$ -HIO<sub>3</sub> crystal is reached at the parameter values  $\alpha = 1.5$  and  $\xi = 0.1$ . We note that variation of the parameter  $\sigma$  in the interval from zero to two does not lead to a noticeable change of  $M$ . The results in Fig. 7 pertain to the case  $\sigma = 0$ . Thus, according to (15), the minimal threshold pump power for the degenerate case in the (YZ) plane at  $\lambda_3 = 4880 \text{ \AA}$ ,  $\theta_m = 58^\circ$ ,  $d_{14} \approx 2.25 \times 10^{-8}$  cgs esu ( $d_{14}(\text{HIO}_3) = 1.5d_{31}(\text{LiNbO}_3) = 11d_{36}(\text{KDP})$ ),<sup>[2,23]</sup> where  $d_{36}(\text{KDP}) = d_{36}(\text{ADP}) = (1.36 \pm 0.16) \times 10^{-9}$  cgs esu<sup>[24,25]</sup>,  $l = 1-2$  cm,  $M = 10^{-2}$ , and  $\delta_1 = \delta_2 = 0.01$  amounts to  $P_3 = 1$  W.

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- <sup>1</sup>S. K. Kurtz and T. T. Perry, J. Appl. Phys. **39**, 3798 (1968).
- <sup>2</sup>S. K. Kurtz, T. T. Perry, and J. G. Bergman, Jr., Appl. Phys. Lett. **12**, 186 (1968).
- <sup>3</sup>J. E. Bjorkholm, IEEE QE-4, 970 (1968) (revised in IEEE QE-5, 260 (1969)).
- <sup>4</sup>H. D. Edmonds and A. W. Smith, IEEE QE-6, 356, 1970.
- <sup>5</sup>A. I. Izrailenko, A. I. Kovrigin, and P. V. Nikles, Zh. Eksp. Teor. Fiz. Pis'ma Red. **12**, 475 (1970) [JETP Lett. **12**, 331 (1970)].
- <sup>6</sup>A. I. Kovrigin and P. V. Nikles, Zh. Eksp. Teor. Fiz. Pis'ma Red. **13**, 440 (1971) [JETP Lett. **13**, 313 (1971)].
- <sup>7</sup>D. N. Kryshko, Zh. Eksp. Teor. Fiz. Pis'ma Red. **6**, 490 (1967) [JETP Lett. **6**, 23 (1967)].
- <sup>8</sup>S. A. Akhmanov, V. V. Fadeev, R. V. Khokhlov, and O. N. Chunaev, Zh. Eksp. Teor. Fiz. Pis'ma Red. **6**, 575 (1967) [JETP Lett. **6**, 85 (1967)].
- <sup>9</sup>D. N. Kryshko and D. P. Krindach, Zh. Eksp. Teor. Fiz. **54**, 697 (1968) [Sov. Phys.-JETP **27**, 371 (1968)].
- <sup>10</sup>D. Magde and H. Marh, Phys. Rev. Lett. **18**, 905 (1967).
- <sup>11</sup>S. E. Harris, M. K. Oshman, and R. L. Byer, Phys. Rev. Lett. **18**, 732 (1967).
- <sup>12</sup>G. F. Dobrzanskiĭ, F. V. Kitaeva, L. A. Kulevskii, Yu. N. Polivanov, S. N. Poluĭktov, A. M. Prokhorov, and N. N. Sobolev, Zh. Eksp. Teor. Fiz. Pis'ma Red. **12**, 505 (1970) [JETP Lett. **12**, 353 (1970)].
- <sup>13</sup>M. Born and E. Wolf, Principles of Optics, Pergamon, 1966.
- <sup>14</sup>Standards on piezoelectric crystals, Proc. IRE **37**, 1378 (1949).
- <sup>15</sup>D. A. Kleinman, Phys. Rev. **126**, 1977 (1962).
- <sup>16</sup>G. F. Dobrzanskiĭ, L. A. Kulevskii, Yu. N. Polivanov, A. M. Prokhorov, and V. V. Smirnov, Kratkie soobshcheniya po fizike (Brief Communications in Physics), FIAN, Vol. 8, p. 61, 1970.
- <sup>17</sup>W. L. Faust and C. H. Henry, Phys. Rev. Lett. **17**, 1265 (1966).
- <sup>18</sup>D. N. Kryshko, A. N. Penin, and B. F. Polkovnikov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **11**, 11 (1970) [JETP Lett. **11**, 5 (1970)].
- <sup>19</sup>T. G. Giallorenzi and C. G. Tang, Phys. Rev. **184**, 353 (1969).
- <sup>20</sup>G. D. Boyd and D. A. Kleinman, J. Appl. Phys. **39**, 3597 (1968).
- <sup>21</sup>M. M. Sushchik and G. I. Frėidman, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **14**, 1176 (1971).
- <sup>22</sup>V. A. Kiselev, Zh. Prikl. Spektrosk. **4**, 37 (1966).
- <sup>23</sup>G. D. Boyd, R. C. Miller, K. Nassau, W. L. Bond, and A. Savage, Appl. Phys. Lett. **5**, 234 (1964).
- <sup>24</sup>R. C. Miller, Appl. Phys. Lett. **5**, 17 (1964).
- <sup>25</sup>G. E. Francois, Phys. Rev. **143**, 597 (1966).

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