Theory of the Electric Conductivity of a Turbulent Plasma

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The effect of ion-acoustic oscillations on the electric conductivity of a plasma is investigated in the case of a weak external electric field. It is shown that if the intensity of the ion-acoustic oscillations considerably exceeds the thermal level then the plasma electric conductivity should be considerably lower than the electric conductivity of a quiescent plasma at the same electron temperature.

1. The behavior of a plasma with ion-acoustic turbulence in an external electric field was investigated in [1-5]. The field was assumed to exceed the critical electron-runaway field.

In the present paper we investigate the influence of ion-acoustic oscillations on the electric conductivity of a plasma in the case when the electric field is weaker than the critical electron-runaway field.

We show that if the intensity of the ion-acoustic oscillations greatly exceeds the thermal level, then the electric conductivity of the plasma in weak fields will be much smaller than the electric conductivity of a quiescent plasma at the same electron temperature.

2. We start with the kinetic equation for the electron distribution function $F \equiv F(\mathbf{v}, \mathbf{r}, t)$

$$\frac{\partial F}{\partial t}+\mathbf{v}\frac{\partial F}{\partial \mathbf{r}}+\frac{e}{m}\mathbf{E}\frac{\partial F}{\partial \mathbf{v}}=J\{E\} \tag{1}$$

with a collision integral $J\{F\}$ that describes both the interaction of the electrons with the electrons and the ions of the plasma and their interaction with the ion-acoustic oscillations,

$$J\{F\}=J_{ee}\{F\}+J_{ei}\{F\}+J_{ew}\{F\}.$$

The collision integrals J_{ie} and J_{ee}, which describe the electron-ion and electron-electron interactions, are determined by Landau's well known expression,[6] and the collision integral J_{ew}, which describes the interaction of the electrons with the ion-acoustic oscillations (it can be called the electron-wave collision integral) is given by

$$J_{ew}\{F\}=\frac{1}{2}\frac{\partial}{\partial v_i}\left[D_{ij}(v)\frac{\partial F}{\partial v_j}\right], \tag{2}$$

where $D_{ij}(v)$ is the diffusion-coefficient tensor

$$D_{ij}(v)=\frac{v_iv_j}{v^2}D_{\parallel}(v)+\left(\delta_{ij}-\frac{v_iv_j}{v^2}\right)D_{\perp}(v),$$
$$D_{\parallel}(v)=\frac{e^2}{8\pi^3m^2v^2}\iint d\omega\, d^3\mathbf{k}\frac{\omega^2}{k^2}\langle|\mathbf{E}|^2\rangle_{\mathbf{k},\omega}\delta(\omega-\mathbf{kv}), \tag{3}$$
$$D_{\perp}(v)=\frac{e^2}{16\pi^3m^2}\iint d\omega\, d^3\mathbf{k}\left(1-\frac{\omega^2}{k^2v^2}\right)\langle|\mathbf{E}|^2\rangle_{\mathbf{k},\omega}\delta(\omega-\mathbf{kv})$$

and $\langle|\mathbf{E}|^2\rangle_{\mathbf{k},\omega}$ is the spectral density of the random electric field produced following the ion-acoustic collisions,

$$\langle|\mathbf{E}|^2\rangle_{\mathbf{k},\omega}=\frac{4\pi a_e^2}{1+a_e^2k^2}T_w(\mathbf{k})\{\delta(\omega-\omega_\mathbf{k})+\delta(\omega+\omega_\mathbf{k})\}. \tag{4}$$

Here $\omega_k = kv_s/\sqrt{1+a_e^2k^2}$ is the frequency of the ion sound (a_e is the electronic Debye radius, $v_s = (T_e/m)^{1/2}$ is the velocity of the ion sound) and $T_w(k)$ is a certain function of the wave vector **k** and can be called the temperature of the waves; for an equilibrium plasma it coincides with the electron temperature T_e.

We shall consider the case of isotropic turbulence and assume that T_w is a function of only the absolute magnitude of the wave vector **k**.

3. We take the electric field to be sufficiently weak and assume that the distribution function F takes the form

$$F=F_0+\frac{\mathbf{v}}{v}\mathbf{F}_1, \tag{5}$$

where the functions F_0 and $\mathbf{F}_1$ depend on the absolute value of the velocity (they can depend also on the coordinates and on the time).

Substituting (5) in (1), we obtain the following equations for F_0 and $\mathbf{F}_1$:

$$\frac{\partial F_0}{\partial t}+\frac{v}{3}\operatorname{div}\mathbf{F}_1+\frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{F}_1)=J_{ee}^0+J_{ei}^0+J_{ew}^0, \tag{6}$$
$$\frac{\partial \mathbf{F}_1}{\partial t}+v\operatorname{grad}F_0+\frac{e}{m}\mathbf{E}\frac{\partial F_0}{\partial v}=\mathbf{J}_{ee}^1+\mathbf{J}_{ei}^1+\mathbf{J}_{ew}^1, \tag{7}$$

where

$$J^0=\int\frac{do}{4\pi}J\{F\},\qquad \mathbf{J}^1=\int\frac{do}{4\pi}\frac{\mathbf{v}}{v}J\{F\}. \tag{8}$$

We shall show subsequently that in the case of a sufficiently weak electric field (the criterion for its smallness will be established later on) the distribution F_0 is Maxwellian. In addition, we also assume the ion distribution to be Maxwellian. In this case, which is the only one we consider from now on, the collision integrals J_{ee}^0, J_{ei}^0, J_{ee}^1, and J_{ei}^1 have the simple form[7]

$$J_{ee}^0=\frac{1}{\gamma v^2}\frac{\partial}{\partial v}\left\{g(v)\left[F_0+\frac{T_e}{mv}\frac{\partial F_0}{\partial v}\right]\right\}, \tag{9}$$
$$J_{ei}^0=\frac{1}{\gamma v^2}\frac{\partial}{\partial v}\left[\frac{m}{M}F_0+\frac{T_i}{Mv}\frac{\partial F_0}{\partial v}\right], \tag{10}$$
$$\mathbf{J}_{ee}^1+\mathbf{J}_{ei}^1=-\mathbf{F}_1/\gamma v^3, \tag{11}$$

where

$$g(v)\equiv g(x)=\Phi(x)-x\Phi'(x),$$
$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2}d\xi,\qquad x=\frac{mv^2}{2T_e},$$
$$\gamma=[4\pi n(e^2/m)^2\Lambda]^{-1};$$

n is the density of the electronic and ionic plasma components, Λ is the Coulomb logarithm, T_i and T_e are the temperatures of the ions and of the electrons, and M is the ion mass. (In the derivation of the collision integral (11) we disregarded the contribution of the electron-electron collisions, since its influence is negligible and leads to the appearance of a factor on the order of unity in the expression for the electric conductivity of the plasma.[8])

Substituting (2)–(4) in (8), we obtain the expressions for the collision integrals J_{ew}^0 and J_{ew}^1:

$$J_{ew}{}^0 = \frac{T_\parallel}{M\gamma\Lambda}\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial F_0}{\partial v}\right),$$

$$\mathbf{J}_{ew}{}^1 = -\frac{T_\perp}{3T_e\Lambda}\frac{\mathbf{F}_1}{\gamma v^3}, \tag{12}$$

where

$$T_\parallel = \int \frac{T_w(k)\,(a_e k)^2 d(a_e k)}{[1+(a_e k)^2]^2},$$

$$T_\perp = \int \frac{T_w(k)\,(a_e k)^2 d(a_e k)}{1+(a_e k)^2}.$$

The integration in the expressions for $T_\parallel$ and $T_\perp$ is carried out over all the values of the wave vector k for which the ion-acoustic oscillations are weakly-damped ($k \ll a_i^{-1}$, a_i is the ionic Debye radius). Since $a_i \ll a_e$ for a plasma with hot electrons, it follows that $T_\perp/T_\parallel \sim T_e/T_i \gg 1$.

The system of equations that determines the distribution function of the electrons in a plasma with a high level of ion-acoustic oscillations thus takes the form

$$\frac{\partial F_0}{\partial t} + \frac{v}{3}\operatorname{div}\mathbf{F}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{F}_1) = \frac{1}{\gamma v^2}\frac{\partial}{\partial v}\left\{\left[g(v)+\frac{m}{M}\right]F_0 + \frac{1}{Mv}\left[\frac{T_\parallel}{\Lambda}+T_i+\frac{M}{m}g(v)T_e\right]\frac{\partial F_0}{\partial v}\right\},$$

$$\frac{\partial \mathbf{F}_1}{\partial t} + v\operatorname{grad}F_0 + \frac{e}{m}\mathbf{E}\frac{\partial F_0}{\partial v} = -\frac{\mathbf{F}_1}{\gamma v_3}\left(1+\frac{T_\perp}{3T_e\Lambda}\right). \tag{13}$$

We are interested in a stationary homogeneous state of the plasma. The general formal solution of the system (13) takes in this case the form

$$F_0 = C\exp\left\{-\int 3m^2\left[g(v)+\frac{m}{M}\right]\left(1+\frac{T_\perp}{3T_e\Lambda}\right)v\,dv\left[(eE\gamma)^2v^6 + 3m\left(1+\frac{T_\perp}{3T_e\Lambda}\right)\left[g(v)T_e+\frac{m}{M}\left(T_i+\frac{T_\parallel}{\Lambda}\right)\right]\right]^{-1}\right\}, \tag{14}$$

$$\mathbf{F}_1 = -\frac{e}{m}\mathbf{E}\frac{\gamma v^3}{1+T_\perp/3T_e\Lambda}\frac{\partial F_0}{\partial v}.$$

The distribution F_0 goes over into a Maxwellian distribution if $E \ll E_0$, where

$$E_0 = \left[\frac{3g(1)}{8}\right]^{1/2}\frac{e\Lambda}{a_e^2}\left(1+\frac{T_\perp}{3T_e\Lambda}\right)^{1/2}. \tag{15}$$

In this case

$$F_0(v) = n\left(\frac{m}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{mv^2}{2T_e}\right),$$

$$\mathbf{F}_1(v) = \mathbf{E}\frac{n(m/2\pi T_e)^{5/2}}{2e^3\Lambda}\left(1+\frac{T_\perp}{3T_e\Lambda}\right)^{-1}v^4\exp\left(-\frac{mv^2}{2T_e}\right), \tag{16}$$

where the electron temperature is given by

$$T_e = T_\parallel/\Lambda + T_i. \tag{17}$$

We call attention to the fact that at a sufficiently high level of the ion-acoustic oscillations this quantity can greatly exceed the ion temperature.[1)]

5. We now calculate the plasma electric-conductivity coefficient σ in the case of weak fields $E \ll E_0$

$$\sigma = \frac{4\pi e}{3E}\int \mathbf{F}_1(v)v^3\,dv.$$

Substituting in place of F_1 the expression (16), we get

$$\sigma = \sigma_0(T_e)\,(1+T_\perp/3T_e\Lambda)^{-1}, \tag{18}$$

where $\sigma_0(T_e)$ is the coefficient of electric conductivity of the quiescent plasma,

$$\sigma_0(T_e) = \frac{2m}{e^2\Lambda}\left(\frac{2T_e}{\pi m}\right)^{3/2}.$$

Since $T_\perp/T_e \sim T_\perp/T_\parallel \gg 1$, the electric conductivity of the turbulent plasma can be much lower than the electric conductivity of the quiescent plasma at the same electron temperature.

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