Vavilov-Cerenkov Effect and Anomalous Doppler Effect in a Medium in which the Wave Phase Velocity Exceeds the Velocity of Light in Vacuum

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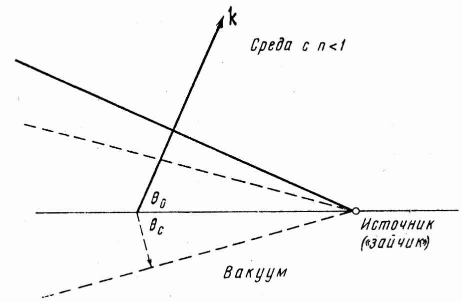
Attention is drawn to the fact that the Vavilov-Cerenkov effect and anomalous Doppler effect may occur for waves whose phase velocity exceeds the velocity of light in vacuum.

THE wave vector **k** of the waves generated in the Vavilov-Cerenkov (V-C) effect makes an angle θ_0 with the velocity **v** of the source (particle), with

$$\cos\theta_0 = \frac{c_{\text{ph}}}{v} = \frac{c}{n(\omega)v}, \quad (1)$$

where $c_{ph} = c/n$ is the phase velocity of the generated waves, $n(\omega)$ is the refractive index corresponding to these waves, and c is the speed of light in vacuum. In the anomalous Doppler effect, the angle θ between **k** and **v** satisfies the condition $\theta < \theta_0$. Thus, the V-C effect and the anomalous Doppler effect are possible only if the source velocity $v \geq c_{ph}$. It is usually assumed that the source velocity $v < c$, and consequently the V-C effect and the anomalous Doppler effect are possible only in a medium with $n > 1$, i.e., at $c_{ph} = c/n < c$. Therefore, in particular, the existence of V-C effects for transverse waves in an isotropic plasma is assumed to be impossible.

The purpose of the present note is to indicate that such a conclusion is incorrect. If we disregard the purely hypothetical tachyons, particles which in all probability cannot exist at all, then the velocity of an individual particle (electron, proton, etc.) is indeed always smaller than c. But the velocity of a source of waves can easily exceed the velocity c of light in vacuum. A well known but by far not the only[1)] example is the "flying spot" produced on a remote screen upon rotation of a collimated source of light or of particles of some sort (for concreteness we shall refer henceforth to electrons). When incident from vacuum on the screen, which in our case is a transparent medium with refractive index $n(\lambda)$, the "spot" produces a perturbation that moves over the surface of the medium with velocity $v = \Omega R$, where Ω is the cyclic frequency of the rotation of the source of light or electrons and R is the distance from this source to the screen. The velocity v of the "spot" can exceed c under real conditions even by many orders of magnitude. Thus, for example, in the case of the pulsar NP0532 in the Crab nebula, the velocity of the "spot" on earth is $v \sim 10^{24}$ cm/sec, since in this case $\Omega \approx 200\ \text{sec}^{-1}$ and $R \approx 5 \times 10^{21}$ cm. If an electron beam or a laser beam is rotated with velocity $\Omega = 10^6\ \text{sec}^{-1}$, then $v > c$ at $R > 3 \times 10^4$ cm.



As is well known, and is immediately evident, the superluminal velocity of the "spot" does not contradict in any way the theory of relativity or the existence of a limiting velocity c for the motion of individual particles. The perturbation ("spot") moving over the screen, regardless of its character (charge, current, polarization) generates waves and for the V-C effect it is necessary to satisfy the condition (1), and for the Doppler effect (in this case the beam must be modulated by a frequency ω_0) the radiated frequency is $\omega = \omega_0 \cdot |1 - (vn/c)\cos\theta|^{-1}$. These relations are thus valid also when $n < 1$.[2)] To be sure, the leading front of the waves in any medium has a velocity c, and consequently in the V-C effect in a medium with $n < 1$ the field differs from zero only inside a cone with angle $\theta_c = \cos^{-1}(c/v)$ (see the figure). We have in mind here the field in the medium; for the field in vacuum, of course, we have $\theta_0 - \theta_c = \cos^{-1}(c/v)$.

To calculate the radiation field itself and the intensity it is necessary, of course, to use Maxwell's equations. It is curious that for a charge moving uniformly with velocity $v > c$, this was first done back in 1904, but for motion in vacuum.[2] Both this calculation and the Tamm-Frank theory for uniform motion of a charge in a medium[3,4] are transferred in general to the case of interest to us. The analysis of the motion not in the interior of the medium but on its surface is the same both for $n > 1$ and for $n < 1$. The fact that we are dealing here not with a point charge but with a "smeared" charge (or a smeared polarization) is also taken into account in the usual manner. What is different is that the determination of the charge density, current, or

[1)]Another example, which is clear from[1], is a light pulse that is narrow along the direction of the wave normal and is incident from vacuum at an angle Ψ on a certain medium. In this case the velocity of the "spot" on the boundary is $v = c/\sin\Psi > c$.

[2)]The role of a medium with $n < 1$ can be played by a waveguide. The possibility of exciting in a waveguide waves with $c_{ph} > c$ with the aid of a moving electron beam (current) intersecting the waveguide was pointed out by B. M. Bolotovskiĭ. I take the opportunity to thank him for this communication as well as for other remarks.

nonlinear polarization produced in the medium by the "flying spot" from an electron gun or laser is an independent problem. Assume that we are dealing, for example, with an electron "beam" with quadratic cross section (electron charge e, electron density N, square with side length d). Let the electrons making up the beam, and hence the beam itself, move perpendicular to the beam axis with a velocity u, and let the angle of incidence of the electrons on the screen be Ψ. Then the velocity of the "spot" is $v = u/\sin\Psi$, and the area of the spot on the screen is $S = d^2/\sin\Psi$ and its charge is $q = eNd^3\cot\Psi$ (it is assumed that the charge falling on the screen is neutralized sufficiently rapidly; allowance for the neutralization time makes the dimensions of the charged "spot" different from the dimensions of the beam). The total intensity (power) of the V-C effect is in this case

$$\frac{dW}{dt}=\frac{q^2v}{c^2}\int F\left(1-\frac{c^2}{n^2(\omega)v^2}\right)\omega\,d\omega, \quad (2)$$

where the integration is carried out over a region of frequencies ω satisfying condition (1), and $F(\omega, d, \Psi, \ldots)$ is a factor that takes into account the motion of the "spot" over and near the surface and the fact that the charged region is not a point; the latter circumstance leads to a decreased intensity for waves whose length $\lambda = 2\pi c/n\omega$ is smaller than the projection of the "spot" dimensions on the direction of $\mathbf{k}$. For a point charge q moving in the interior of the medium, F = 1 and Eq. (2) goes over into the Tamm-Frank formula.[3,4] We note that with respect to the field in vacuum, from which the electron beam is incident, the presented calculation scheme is insufficiently correct and the problem calls for an additional analysis, but we are interested here primarily in the field in the medium. Incidentally, in our problem it is in general more consistent to use a calculation in which the incident electron beam is replaced by a corresponding current. The use of the charge of the "spot" seems, however, to be more illustrative, although its reliable determination still calls for more consistent calculations.[5]

The V-C effect and the anomalous Doppler effect are encountered in a large number of cases and have found different applications. Therefore, as can be thought, both effects at $c_{ph} = c/n > c$ are also not only of methodological but of real physical interest. It suffices to say that in such an important medium as an isotropic plasma, for transverse waves in a well known approximation we have $n = \sqrt{1-\omega_0^2/\omega^2} < 1$ and, as already noted, the V-C effect is usually assumed to be impossible for these waves. Actually, however, the V-C effect and the Doppler effect can be observed both in an isotropic plasma and in a number of cases when $n < 1$. Of course, when $v > c$ these effects take place in media with $n > 1$. We note also that even when $v < c$ the use of a "spot" (a perturbation due to the external source and moving over the surface) can be an effective and convenient method of exciting waves in a large number of cases (excitation of surface waves of different types with the aid of the V-C effect and its analogs or else as a result of transition radiation, excitation of second sound in helium II, etc.)

[1] I. M. Frank, Izv. Akad. Nauk SSSR Ser. Fiz. **6**, 3 (1942).

[2] A. Sommerfeld, Göttingen Nachrichten, pp. 99, 363, 1904; p. 201, 1905.

[3] I. E. Tamm and I. M. Frank, Dokl. Akad. Nauk SSSR **14**, 107 (1937).

[4] B. M. Bolotovskiĭ, Usp. Fiz. Nauk **62**, 201 (1957); Usp. Fiz. Nauk 75, 295 (1961) [Sov. Phys. Usp. 2, 781 (1962)].

[5] B. M. Bolotovskiĭ and V. L. Ginzburg, Usp. Fiz. Nauk **106**, (4), ,(1972) [Sov. Phys. Usp. **15**, (2), ,(1972)].

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23