Emission by a Charge Moving in a Medium in the Field of an Intense Wave

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The law of motion of a charge moving in an isotropic, transparent medium in the field of an intense electromagnetic wave is found. The dependence of the intensity of Cerenkov radiation and the normal and anomalous scattering cross sections on the intensity of the incident circularly polarized wave is investigated.

THE study of the emission of photons by electrons in a vaccum in the field of an intense electromagnetic wave [1-3] has shown that the spectral and angular characteristics of the emission depend strongly on the value of the field intensity. The presence of a refracting medium changes the character of the interaction of the electrons with the intense field in significant fashion. Actually, even in the interaction of an electron with a weak field (when perturbation theory and the classification of the radiation effects by Feynman diagrams are valid), the possibility of Cerenkov emission and absorption arises, and the effect of next order—the scattering of light by the electron—takes on a number of interesting features, [4-7] associated with the normal and anomalous Doppler effects.

In the interaction of the charge with a sufficiently intense electromagnetic wave, perturbation theory becomes inapplicable and it is necessary to use exact solutions of the equation of motion of the charge in the field of the wave for the calculation of the radiation intensity. Account of this circumstance leads to a dependence of the various physical processes on the field intensity of the wave and allows us in principle to obtain new information on the effect of the optical properties of the medium on the character of the interaction of the particles with the electromagnetic waves. In the present paper we consider the spontaneous emission of electromagnetic waves from an electron moving in an isotropic transparent medium in the field of an intense monochromatic wave, and we study the dependence of the intensity of the Cerenkov radiation and the cross sections of normal and anomalous scattering[6] on the intensity of the incident circularly polarized wave.

For this, we first need to find the solution of the classical equations of motion. The field of a plane electromagnetic monochromatic wave, propagating in a medium along the direction **k**, can be described by the 4 potential $A^i = A^i(\varphi)$, which depends on the coordinates only through the single variable $\varphi = kx$. Here k is a wave 4-vector of nonzero length:

$$k^i = \left(\frac{\omega}{c},\ \mathbf{k}\right),\ |\mathbf{k}| = \frac{n(\omega)\omega}{c},\ k^2 = \frac{\omega^2}{c^2}(1 - n^2(\omega)) \neq 0.$$

It is most convenient to determine the law of motion of the electron by starting out from the Hamilton-Jacobi equation

$$\left(\frac{\partial S}{\partial x_i} + \frac{e}{c}A^i\right)\left(\frac{\partial S}{\partial x^i} + \frac{e}{c}A_i\right) - m^2c^2 = 0, \tag{1}$$

which can be solved exactly in the plane wave case under consideration. The action has the form (see, for example, [8])

$$S = -px + \frac{kp}{k^2}\varphi \pm \frac{1}{k^2}\int R(\varphi)\,d\varphi, \tag{2}$$

$$R(\varphi) = \left\{(kp)^2 - k^2\left[\left(p - \frac{e}{c}A\right)^2 - m^2c^2\right]\right\}^{1/2}, \tag{3}$$

where

$$px = p^0x^0 - \mathbf{px}, \quad kp = k^0p^0 - \mathbf{kp}.$$

The constant 4-vector p is identical with the momentum of the free particle $p^2 = m^2c^2$. The choice of sign in (2) is determined by the condition that as $A^i \to 0$, the action (2) transforms into the expression $S = -px$ for action corresponding to free motion. Therefore, we must choose the plus sign in (2) if the wave is incident on the particle inside the Cerenkov cone ($kp < 0$), and the minus sign in the case of incidence of the wave outside the cone ($kp > 0$). It follows from (3) that the region of motion achieved classically is determined by the condition

$$(kp)^2 \geqslant k^2\left(-\frac{2e}{c}Ap + \frac{e^2}{c^2}A^2\right). \tag{4}$$

Following general rules, we determine the kinetic momentum of the particle

$$q^i = -\frac{\partial S}{\partial x_i} - \frac{e}{c}A^i = p^i - \frac{e}{c}A^i + \frac{k^i}{k^2}(-kp \mp R). \tag{5}$$

Taking (5) into account and the relation $kq = \mp R$ which follows from it, we can write the law of motion ($x^i_{(0)} = -\partial S/\partial p_i$, $x^i_{(0)}$ are constants) in the form

$$x^i = x^i_{(0)} + \int \frac{q^i}{kq}\,d\varphi. \tag{6}$$

Well known expressions are obtained from (2) and (6)[9] for the action and the law of motion of the charge in the field of a plane wave in a vacuum in the limiting transition $n(\omega) \to 1$.

We note that the formula (6) determines the law of motion of the charge in the medium in the field of a plane electromagnetic wave of arbitrary polarization for arbitrary initial conditions, with given parameters $x^i_{(0)}$ and p.

In the following, we shall consider the emission of an electron in a system of coordinates in which the wave is propagated in the direction of the z axis and the electron is at rest in the medium in the xy plane. In the system of coordinates used, (6) takes a very simple form for interaction of the electron with the circularly polarized wave, the potential of which is $A^i = (0, a\cos\varphi, ga\sin\varphi, 0)$. In this case, $kq = \mp R$, the energy cq_0 and the component of the momentum along the z axis are all constant quantities, while the equation of motion has the form

$$x = x_0 + \gamma\frac{mc}{kq}\sin\varphi, \quad y = y_0 - \gamma\frac{mc}{kq}g\cos\varphi,$$
$$z = z_0 + \frac{q_z}{kq}\varphi, \quad \varphi = \frac{c(kq)}{q_0}(t - t_0), \tag{7}$$
$$kq = \frac{\omega q_0}{c}\Delta_0 = \frac{\omega q_0}{c}(1 - n\beta_z).$$

Here $\gamma = ea/mc^2$ is an invariant parameter, and $g = \pm 1$ corresponds to right (left) circular polarization. It is seen from (7) that in this case the electron moves on a helix along the z axis with velocity $v_z = q_z c/q_0$ and radius $\gamma mc/|kq|$. The velocity of the particle here is constant in value. It is not difficult to see that the relation $\overline{q}^2 = m_*^2c^2$ is satisfied for the average momentum $\overline{q}$, where $m_* = (m^2 + m^2\gamma^2)^{1/2}$ plays the role of the effective mass of the electron in the field of the wave.

In addition, we obtain by standard methods[9] the spectral angular distribution of the intensity of radiation of the two principal components of polarization by the electron, the law of motion of which is determined by the formulas (7):

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$$P(\omega, \mathbf{n}) = P_2(\omega, \mathbf{n}) + P_3(\omega, \mathbf{n}), \tag{8}$$
$$P_2(\omega, \mathbf{n}) = \frac{ne^2\omega^2}{2\pi c}\sum_{s=-\infty}^{+\infty}\left(\gamma\frac{mc}{q_0}J_s'(x)\right)^2\delta(\omega\Delta - s\omega_0\Delta_0), \tag{9}$$
$$P_3(\omega, \mathbf{n}) = \frac{ne^2\omega^2}{2\pi c}\sum_{s=-\infty}^{+\infty}\left(\frac{s\omega_0\Delta_0}{\omega n}\operatorname{ctg}\theta - \beta_z\sin\theta\right)^2 J_s^2(x)\,\delta(\omega\Delta - s\omega_0\Delta_0). \tag{10}$$

Here **n** is a unit vector in the direction of emission, $\Delta = 1 - n(\omega)\beta_z\cos\theta$ is the Doppler denominator for the emitted wave, ω_0 the frequency of the emitted wave, $J_s(x)$ and $J_s'(x)$ the Bessel function and its derivative,

$$x = \gamma\frac{mc}{q_0}\frac{n\omega}{\omega_0\Delta_0}\sin\theta.$$

In Eqs. (9) and (10), the s-th term of the sum ($s > 0$) describes the emission of a photon of frequency ω and absorption from the incident wave of s photons of frequency ω_0. The components with $s < 0$ describe the emission of a photon of frequency ω and simultaneous emission by the electron in the incident wave of $|s|$ photons of frequency ω_0.

The total emission of the electron (8) can be divided [5] into Cerenkov (the direction and frequency of which are connected by the relation $\Delta = 0$) and non-Cerenkov ($\Delta \neq 0$) components. Consequently, the spectral angular distribution of the intensity of the Cerenkov emission of the electron in the field of the wave is determined by the component with $s = 0$ in (8). The total intensity of the Cerenkov radiation is

$$P_0 = \int_{n\beta_z > 1}\frac{e^2\omega}{c\beta_z}\left\{\left(\gamma\frac{mc}{q_0}J_0'(x)\right)^2 + (\beta_z\sin\theta J_0(x))^2\right\}d\omega,$$
$$x = \gamma\frac{mc}{q_0}\frac{n\omega}{\omega_0\Delta_0}\left(1 - \frac{1}{n^2\beta_z^2}\right)^{1/2}. \tag{11}$$

If the intensity of the incident wave γ is so small that $x \ll 1$ for all frequencies, then (11) transforms into the well known expression for the spontaneous Cerenkov radiation in the absence of a field.[10] In the opposite case $x \gg 1$, the spectral distribution of the Cerenkov radiation $P_0(\omega)$ has an oscillatory character.

We now consider the non-Cerenkov case. It follows from (9) and (10) that the emission has a discrete spectrum with frequencies determined by the condition

$$\omega = s\omega_0\frac{\Delta_0}{\Delta} = s\omega_0\frac{1 - n(\omega_0)\beta_z}{1 - n(\omega)\beta_z\cos\theta}. \tag{12}$$

Integrating the expression (8) over the frequency and dividing by the energy flux density in the incident wave

$$W = \frac{n_0 e^2\omega_0^2}{4\pi c r_0^2}\gamma^2 \tag{13}$$

($n_0 = n(\omega_0)$ and r_0 is the classical radius of the electron), we get the differential scattering cross section

$$\frac{d\sigma}{do} = \sum_{s=-\infty}^{+\infty}\frac{d\sigma_s}{do},$$

where the partial cross section $d\sigma_s/do$ is equal to

$$\frac{d\sigma}{do} = 2r_0^2\frac{n}{n_0}\left(\frac{s\Delta_0 mc}{\Delta q_0}\right)^2\left[J_s'^2(x) + \left(\frac{\cos\theta - n\beta_z}{\eta\Delta}\right)^2 J_s^2(x)\right] \tag{14}$$
$$\times\left|\Delta - \frac{s\Delta_0}{\Delta}\omega_0\beta_z\cos\theta\frac{dn}{d\omega}\right|^{-1}.$$

Here

$$x = s\eta, \quad \eta = \gamma\frac{mc}{q_0}\frac{n}{\Delta}\sin\theta.$$

We now consider in more detail the case of low intensity of the incident wave. Then $\gamma \ll 1$ and the emission has principally a dipole character. In this case, the components with $s = \pm 1$ in (14) give the principal contribution to the cross section of the normal and anomalous scattering.[6] For superluminal motion ($n\beta_z > 1$) it is expedient to consider the emission inside and outside the Cerenkov zone, which is determined by the condition $n\beta_z\cos\theta_0 = 1$. Then, for $s = 1$, inside the cone ($\theta < \theta_0$), the Doppler effect is anomalous—the frequency ω increases with increase in θ. The spectrum of the anomalous frequencies is complicated and does not have an analogy in the scattering of light by an electron moving in a vacuum. Outside the cone ($\theta > \theta_0$) the Doppler effect is normal, since the frequency ω decreases with increase in θ.

The two cases of light scattering by an electron in a medium, considered above, can be named normal scattering, according to the definition of Frank,[6] since the scattering takes place in the "normal" manner—the photon of the incident wave is absorbed and a photon of the scattered wave is emitted. The conditions $s = -1$, $\Delta_0 > 0$, $\Delta < 0$ and $s = -1$, $\Delta_0 < 0$, $\Delta > 0$ determine the anomalous scattering, which corresponds to the induced emission of a photon of frequency ω_0 in the incident wave and spontaneous emission of frequency ω. It follows from (12) that in the case of anomalous scattering, there is a possibility[6] of obtaining a scattered photon of high frequency in scattering at zero angle:

$$\omega \approx \frac{n_0\beta_z - 1}{1 - \beta_z}, \qquad n(\omega) \approx 1.$$

In a vacuum, the scattering at zero angle takes place without change in frequency. In the case under consideration, of small intensity of the incident wave, the cross section of normal and anomalous scattering has the form

$$\frac{d\sigma_{\pm 1}}{do}=\frac{1}{2}r_0^2\frac{n}{n_0}\left(\frac{\Delta_0 mc}{\Delta q_0}\right)^2\left[1+\left(\frac{\cos\theta-n\beta_z}{\Delta}\right)^2\right]\times\left|\Delta\mp\frac{\Delta_0}{\Delta}\omega_0\beta_z\cos\theta\frac{dn}{d\omega}\right|^{-1}. \quad (15)$$

In the special case of motion in a vacuum, the well known expression for the Thomson scattering cross section from fixed electrons follow from (15) for $\beta_z = 0$.

For scattering in the direction of the Cerenkov cone ($\Delta \to 0$), the cross section (15) has a resonance character. Neglecting dispersion, we find

$$d\sigma_{\pm 1}/do \sim \Delta^{-3}.$$

This result also follows from [5], but contradicts [11]. It should be remarked that Eq. (15) and the resonance behavior found above take place only in the region of applicability of the perturbation theory ($\eta \ll 1$). For analysis of the behavior of the partial cases for arbitrary values of η we use the exact formula (14). Taking into account the well known properties of Bessel functions, we find that as $1-\eta \ll 1$, the partial cross sections increase with increase in $|s|$ in the region $|s| < s_{cr}$ as $|s|^{1/3}$, and for $|s| > s_{cr}$, decrease as $|s| \times \exp(-2|s|/s_{cr}$, where $s_{cr} = (1-\eta^2)^{-1/2}$. In the region $\eta < 1$, one can sum the partial cross sections and obtain the complete cross section of normal scattering[12]

$$\frac{d\sigma}{do}=\sum_{s=1}^{\infty}\frac{d\sigma_s}{do}=\frac{r_0^2}{8|\Delta|}\left(\frac{\Delta_0 mc}{\Delta q_0}\right)^2\times\left[4+3\eta^2+\left(\frac{\cos\theta-n\beta_z}{\Delta}\right)^2\frac{4+\eta^2}{1-\eta^2}\right]\frac{1}{(1-\eta^2)^{5/2}}. \quad (16)$$

For the anomalous scattering cross section, we get an analogous expression. It is seen from (16) that the cross section is large not only near the Cerenkov cone ($\Delta \to 0$) but also for the condition $1-\eta^2 \ll 1$.

In this case, when $\eta \gg 1$, it follows from (14) that

$$\frac{d\sigma_s}{do}\approx 4r_0^2|s|\frac{\Delta_0^2 mc}{\Delta^2 q_0}\frac{1}{\pi\gamma n\sin\theta}\left[\sin^2\left(x-\frac{1}{2}s\pi-\frac{1}{4}\pi\right)+\left(\frac{\cos\theta-n\beta_z}{\gamma\sin\theta}\frac{q_0}{mcn}\right)^2\cos^2\left(x-\frac{1}{2}s\pi-\frac{1}{4}\pi\right)\right]. \quad (17)$$

Thus, in the region $\eta \gg 1$, the resonance behavior of the partial cross sections $d\sigma_s/do \sim \Delta^{-2}$ is considerably different from the corresponding expression found in the region $\eta \ll 1$.[5]

Of course, account of dispersion leads to a smoothing out of the resonance and to a finite expression for the partial scattering cross sections in the direction of the Cerenkov cone. Actually, taking into account the fact that $dn/d\omega \neq 0$, we see from (14) that in the region of the Cerenkov cone ($\eta \gg 1$, $\Delta \to 0$)

$$\frac{d\sigma_s}{do}=4r_0^2\frac{\Delta_0 mc}{q_0 n_0}\left|\pi\beta_z\gamma\sin\theta\cos\theta\omega_0\frac{dn}{d\omega}\right|^{-1}\left[\sin^2\left(x-\frac{1}{2}s\pi-\frac{1}{4}\pi\right)+\left(\frac{\cos\theta-n\beta_z}{\gamma\sin\theta}\frac{q_0}{mcn}\right)^2\cos^2\left(x-\frac{1}{2}s\pi-\frac{1}{4}\pi\right)\right]. \quad (18)$$

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