Slowing-Down of a Dirac Monopole in Metals and Ferromagnetic Substances

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Calculations of the slowing-down of magnetic charges in metals and ferromagnetic substances are presented. It is shown that ferromagnetic substances in large external magnetic fields ($\sim 10^4$ G) are effective traps for magnetic monopoles.

IN connection with the setting-up of an experiment on the 70 GeV proton synchrotron at Serpukhov to search for the Dirac monopole, we have estimated the energy losses of a magnetic charge in metals and ferromagnetic substances for a wide range of velocities of motion—from thermal to ultrarelativistic.

Dirac[1,2] has shown that the value of the magnetic charge may take the values

$$g = (\hbar c / 2e^2) ne = 68{,}5\; ne, \quad n = \pm 1, 2, \ldots$$

A negative result was obtained in all experimental searches.[3-12] In the experiment on the 70 GeV proton accelerator, it is proposed to use the property that the monopoles, created in the reaction $p + N \rightarrow p + N + g^+ + g^-$, accumulate in ferromagnetic foils after first slowing-down in metallic plates.

The estimate of the energy losses of a monopole associated with its passage through metals, which is presented in Sec. 1, has a direct relation to the calculation of the transmission of the experiment. The passage of a monopole through a ferromagnetic substance is treated in Sec. 2. The results obtained permit us to estimate the effectiveness of ferromagnetic foils as traps for magnetic charges.

In the calculations it is assumed that the monopole creates a magnetic field around it according to the equation $\operatorname{div} \mathbf{H} = 4\pi\rho_m$ or $\mathbf{H} = g\mathbf{r}/r^3$.

1. ENERGY LOSSES OF PARTICLES POSSESSING MAGNETIC CHARGE DURING THEIR PASSAGE THROUGH METALS

A. The case of an infinite medium. We shall calculate the monopole energy losses by the method developed by Landau.[13] Let us characterize the medium by the following macroscopic constants: $\epsilon(\omega)$ (the dielectric constant), σ_0 (the electrical conductivity), and $\mu \equiv 1$ (the magnetic permeability). Assuming that the monopole energy losses over macroscopically sufficient distances are small in comparison with its kinetic energy, we regard its motion in the medium to be steady with velocity **u**.

For metals the dielectric constant $\epsilon(\omega)$ can be written in the form (see[14])

$$\varepsilon(\omega) = \frac{i4\pi\sigma(\omega)}{\omega}, \quad \sigma(\omega) = \sigma_0 \frac{1 + i\omega\tau}{1 + \omega^2\tau^2},$$

where τ is the relaxation time which generally depends on the frequency. For small frequencies, τ is equal to the time of free flight of the conduction electrons, that is, $\tau_0 = m\sigma_0/Ne^2$ (m is the electron mass and N is the number of atoms per cubic cm). One can obtain an expression for τ at large frequencies by using the well-known behavior (see[13]) of $\epsilon(\omega)$ as $\omega \rightarrow \infty$: $\tau \rightarrow m\sigma_0/NZe^2$, where NZ is the number of electrons per cubic cm.

The slowing-down of the monopole is produced by the action of the magnetic field of the medium. Omitting the calculations, which are carried out by the standard method (see[13]), we present the final results.

We shall separately distinguish the cases of small, comparable, and large velocities of the monopole motion relative to the velocity of an electron in the atom.

If the monopole is moving more slowly than the electron in an atom, then an effective interaction is possible with the free electrons of the medium, that is, with the conduction electrons. In this case one can derive the following expression for the stopping field:

$$H = 4\pi^2 guNe^2 / v_e mc^2, \tag{1}$$

where $v_e \approx 10^8$ cm/sec is the Fermi velocity of the conduction electrons. In particular, for Al we have $H = (2.3 \times 10^3\; u/10^8)$ Oe.

If the velocity of motion is comparable with or higher than the velocity of the electrons in an atom, the stopping field has the form

$$H = \frac{4\pi NZ^* e^2 g}{mc^2}\left[\ln\frac{2m^2u^2\sigma_0}{NZ^* e^2\hbar} - \frac{1}{2}\right], \tag{2}$$

where Z^* is the effective number of electrons in an atom, which participate in the ionization process. In accordance with the comments with regard to the relaxation time τ, the ratio $Z^*/Z \rightarrow 1$ as $u \rightarrow c$. The region of applicability of this formula is given by

$$(u/c)^2 \leqslant 1/2\pi\sigma_0\tau = NZe^2/2\pi m\sigma_0^2.$$

In the limit of ultrarelativistic velocities we obtain

$$H = \frac{2\pi NZe^2 g}{mc^2}\left[\ln\frac{m^3c^4\gamma^2}{\pi NZe^2\hbar^2} - 1\right], \tag{3}$$

where $\gamma = 1/\sqrt{1 - u^2/c^2}$. The last formula agrees with the corresponding expression for electrically charged particles. The energy loss of a moving monopole is equal to the work done against the retarding force: $|dE/dz| = gH$.

In order to illustrate these results, we present the specific energy losses of a monopole with momentum p = 30 GeV/c in different media, calculated according to formula (3):

Metal:	W	Fe	Cu	Pb	Al
dE/dx in GeV/cm	121	59	65	72	22

B. The case of a medium and a vacuum. The total stopping field acting on a charge moving near the interface between two media consists of two terms. One of these terms corresponds to the field of a charge moving in an infinite medium, and the other term corresponds to the field of the radiation which arises due to the presence of the interface between the media. The field of the first type is determined by the formulas derived above; therefore our problem reduces to only the determination of the additional radiation fields. Similar questions have been considered by Garibyan[15,16] for electrically charged particles.

If arbitrary solutions of the homogeneous equations are added to the solution of the inhomogeneous Maxwell's equations and these solutions are determined from the condition of continuity of the total fields across the interface, then one can derive the following expression for the radiation field at the point where the monopole is located, this expression being valid for small velocities $u \leq v_e$:

$$H' = \frac{\pi g \sigma_0 u}{2c^2 z} e^{-z/z_1}(1 - e^{-z/z_2}), \tag{4}$$

where $z_1 = c^2/8\pi\sigma_0 u$, $z_2 = m\sigma_0 v_e/4\pi Ne^2$, and z is the distance from the monopole to the interface. By combining expressions (1) and (4) we obtain the total stopping field near the surface:

$$H = 4\pi^2 g \frac{uNe^2}{v_e mc^2} - \frac{\pi g \sigma_0 u}{2c^2 z} e^{-z/z_1}(1 - e^{-z/z_2}). \tag{5}$$

As $z \to 0$ the quantity $H \to 2\pi^2 guNe^2/v_e mc^2$, that is, the retarding magnetic field at the surface is one-half as large as the field in an infinite medium. The obtained result seems to be physically reasonable for slow monopoles. In particular, on the surface of an aluminum plate, a retarding field H = 12 Oe will be acting on a Dirac monopole moving with velocity $u = 10^6$ cm/sec.

2. PASSAGE OF A DIRAC MONOPOLE THROUGH A FERROMAGNETIC SUBSTANCE

It is of interest to consider the interaction of magnetic charges with a ferromagnetic medium when the velocity of their motion is appreciably smaller than the velocity of the electrons in an atom. This is due to the fact that the relaxation time for ferromagnetic processes is large in comparison with optical periods; therefore in the region of large velocities of motion of the monopole, the formulas of Sec. 1 are also valid for ferromagnetic substances. On the other hand, for small values of u, because of the purely magnetic nature of the interaction, that is, the absence of the relativistic factor u/c, one can expect the appearance of forces which are so large in magnitude that it will be a good approximation to consider the electrical conductivity of the ferromagnetic substance to be $\sigma_0 = 0$.

Only cases when the ferromagnetic substance is located in an external magnetic field, strong enough to magnetize the sample close to saturation ($H_0 = 10^4$ G), will be considered. Here the "single-domain" representation is valid. And finally, it should be noted that the case of thin ferromagnetic plates, positioned normal to the lines of force of the external magnetic field and to the direction of motion of the monopole, is being investigated.

A. The interaction inside a plate. In the presence of uniform magnetization of the sample, the magnetization vector $\mathbf{M}_0$ will be the same over the entire volume and will be parallel to the external field H_0. The magnetization in the medium varies due to the influence of the magnetic field of the moving monopole, so that $\mathbf{M}(\mathbf{r}, t) = \mathbf{M}_0 + \mathbf{m}(\mathbf{r}, t)$. It is obvious that one can distinguish two regions in the medium: The region of remote distances, $\rho \gg \rho^*$, where $|\mathbf{m}(\mathbf{r}, t)| \ll M_0$, and the region of close distances, $\rho \ll \rho^*$, where $|\mathbf{m}(\mathbf{r}, t)| \approx M_0 \approx M_S$ (M_S denotes the saturation induction). Let us estimate the quantity ρ^* from the condition $\Delta t_{pr} = \Delta t_{eff}$, where Δt_{pr} denotes the precession time of the magnetization vector in the field of the monopole, and Δt_{eff} denotes the effective time of interaction.

$$\Delta t_{pr} \approx 1/\gamma H = \rho^2/\gamma g, \qquad \Delta t_{eff} \approx \rho/u,$$

where γ denotes the gyromagnetic ratio ($\gamma = e/mc$ for an electron); from here it follows that $\rho^* = g\gamma/u$.

Just as in Sec. 1, we characterize the interaction of the monopole with the medium by a field applied to it by the medium. Let us estimate the contribution from the remote region. In order to do this, we jointly consider the Maxwell equations

$$\operatorname{div} \mathbf{b} = 4\pi g \delta(\mathbf{r} - \mathbf{u}t), \qquad \operatorname{rot} \mathbf{h} = 0 \tag{6}$$

and the equation of motion of the magnetization vector in the form given by Landau and Lifshitz:

$$d\mathbf{m}/dt = -\gamma[(\mathbf{M}_0 + \mathbf{m})(\mathbf{H}_{0\,\mathrm{eff}} + \mathbf{h})]. \tag{7}$$

The quantities b, h, and m are connected by the relations $\mathbf{b} = \mathbf{h} + 4\pi\mathbf{m}$ and $\mathbf{m} = \chi\mathbf{h}$, where χ denotes the magnetic susceptibility of the medium; $H_{0\,\mathrm{eff}}$ is the internal effective field, phenomenologically taking the various interactions in a ferromagnetic substance into account. If the energy associated with magnetic anisotropy and the magnetoelastic energy are not taken into account, then one can write $H_{0\,\mathrm{eff}}$ in the form[17-19]

$$\mathbf{H}_{0\,\mathrm{eff}} = \mathbf{H}_i + \mathbf{H}_A + \mathbf{H}_r.$$

Here $\mathbf{H}_i$ is the internal magnetic field with surface demagnetization taken into account; for a thin plate in a perpendicular external field $H_i \approx H_0 - 4\pi M_0$; $\mathbf{H}_A = (2A/M_S^2)\Delta\mathbf{M}$ is the effective field of the exchange forces, A is the constant of the exchange interaction, $\mathbf{H}_r = -\beta\dot{\mathbf{M}}/\gamma M_0$ is the field of the friction forces, and β is the dimensionless attenuation parameter which is related to the experimentally measurable "relaxation frequency" λ by the equation

$$\beta = (\lambda/\gamma M_0)/[1 + (\lambda/\gamma M_0)^2].$$

Using a Fourier expansion and the smallness $|\mathbf{m}(\mathbf{r}, t)| \ll M_0$, from Eqs. (6) and (7) we obtain the following expression for the stopping field

$$H = \frac{ig\omega_a^2}{\pi u^2} \int_0^{\alpha_{max}} \int_{-\infty}^{+\infty} \frac{\alpha x[\Omega^2(x) - x^2]\, d\alpha\, dx}{\alpha^2(\omega_M/\omega_a)\Omega(x) + (\alpha^2 + x^2)[\Omega^2(x) - x^2]}, \tag{8}$$

$$\Omega(x) = \omega_0/\omega_a - i\beta x + \alpha^2 + x^2, \qquad \omega_0 = \gamma H_i, \qquad \omega_M = 4\pi\gamma M_0,$$
$$\omega_a = M_s^2 u^2/2M_0 A\gamma, \quad \alpha = qu/\omega_a, \quad x = \omega/\omega_a,$$

where q denotes the transverse component of the wave vector k in the Fourier expansion. The maximum value α_{max} corresponds to the minimum "impact" parameter, which in the present case we equate to ρ^*, that is $\alpha_{max} = u^2/g\gamma\omega_a$. We carry out the integration over x in the complex plane with the aid of the residues. Here we shall distinguish between the cases $\omega_0/\omega_a \gg 1$ and $\omega_0/\omega_a \ll 1$. We note, without citing the numerical calculations, that $\alpha^2_{max} \ll 1$, $\beta \ll 1$, and $\omega_M/\omega_a \ll 1$ practically up to thermal velocities of motion of the monopole.

The case $\omega_0/\omega_a \gg 1$. This case corresponds to large external fields: $H_0 \geq 2 \times 10^4$ G even for thermal velocities $u = u_{kT}$. Let us present the final result for the stopping field:

$$H = \frac{g\alpha^3_{max}\omega_M\omega_a^3}{3u^2\omega_0^2}(\beta/2 + \alpha^3_{max}\omega_a^2/\omega_0^2). \tag{9}$$

The case $\omega_0/\omega_a \ll 1$. This case is of the greatest practical interest since it corresponds to small, external magnetic fields (up to $\sim 10^4$ G). Carrying out the integration in (8) with the aid of the residues, we obtain

$$H = g\frac{\omega_a^2}{u^2}\left\{\left(\frac{\omega_M\omega_0}{\omega_a^2} + \frac{\beta^2}{2}\right)\ln\frac{\alpha_{max}}{\theta} + \frac{\alpha_{max}\omega_M}{2\omega_a} + \frac{\beta\omega_0}{\theta\omega_a} + \frac{\omega_0^2}{4\omega_a^2\theta^2}\right\} \quad \text{for} \quad \alpha_{max} \gg \theta, \tag{10}$$

where $\theta^2 = \omega_0/\omega_a + \beta^2/4$.

$$H = \frac{g\omega_a^2\alpha^4_{max}}{4u^2}\left\{\frac{1}{\theta^2(1+\beta/2\theta)} \mp \frac{5(\omega_M/\omega_a)}{\theta^2(1+\beta/2\theta)^3} + 0{,}4\frac{\omega_M}{\omega_a}\right\} \tag{11}$$

for $\alpha_{max} \ll \theta$.

As already mentioned above, at small distances $\rho < \rho^*$ the variation of the magnetization is of the order of its magnitude, $|\mathbf{m}(\mathbf{r}, t)| \approx M_S$.

In conducting subsequent estimates of the interaction of a monopole with the medium, we shall start from the assumption that the magnetization vector for each point of the region $\rho < \rho^*$ at any instant of time is directed nearly along the field of the monopole. Such an assumption is based on the fact that in the region close to the monopole, $z \approx \rho^*$, the monopole field is considerably larger than $H_{0\,eff}$, and at larger distances along the z axis inside the region $\rho < \rho^*$ the angle between the direction of the monopole's field and the internal field $\mathbf{H}_i$ ($\mathbf{H}_i \parallel z$) is negligibly small. The distribution of the magnetization in the region $\rho < \rho^*$ specified in this manner enables us to calculate the magnetic field acting on the monopole as an integral over the field of the individual magnetic dipoles. However, we shall use another method, which is simpler in terms of the calculations.

Let us determine the force acting on the magnetic charge from the change of the interaction energy between the magnetization of the medium and the external field

$$F = \left|\frac{\Delta E}{\Delta z}\right| = \int_0^{\rho^*} (\mathbf{M}_0 - \mathbf{M}_z)\mathbf{H}_i 2\pi\rho\, d\rho,$$

where $\mathbf{M}_0$ denotes the initial magnetization at the instant of time $t = -\infty$, $\mathbf{M}_z$ is the magnetization at the same point at the time $t = +\infty$, that is, after the passage of the monopole. According to the assumptions made above, $M_z \approx -M_0$. Thus, in an infinite medium the "short-range" region produces a slowing-down force

$$F = 2\pi M_0 H_i g^2\gamma^2/u^2. \tag{12}$$

B. Slowing-down near the surface of a ferromagnetic substance. If the monopole is near the surface, owing to the large spatial asymmetry, the free energy of the ferromagnetic substance must be a function of the distance z of the monopole from the surface. In analogy to the force of the mirror interaction for electrical charges near the surface of a metal, the force F_{M_S} due to the z-dependence of the interaction energy between the magnetization and the monopole field hinders the escape of the monopole from the medium. The z-dependence of the exchange energy, arising in a ferromagnetic substance because of the radial nature of the monopole field, leads to the appearance of a force F_A which expels the monopole from the medium.

According to[18] the density of the exchange energy has the form $W_A = (A/M_S^2)(\nabla\mathbf{M})^2$. Since it is assumed that $\mathbf{M}$ and the monopole field h are parallel in the region $\rho < \rho^*$, we obtain the following expression for the free energy density of a ferromagnetic substance:

$$W = -\mathbf{hM} + (A/M_s^2)(\nabla\mathbf{M})^2 = -gM_s(1 - 2A/gM_s)r^{-2}$$

We obtain the total energy by integrating the last expression over the volume of a semi-infinite cylinder with radius ρ^*:

$$E = -2\pi gM_s(1 - 2A/gM_s)\{-z\ln\cos\theta^* + \rho^*(\pi - \theta^*) - a\}, \tag{13}$$

where $\tan\theta^* = \rho^*/z$ and a is the interatomic distance in the medium.

In order to determine the forces acting on the monopole near the surface, it is necessary to differentiate (13) with respect to z:

$$F = F_{M_s} + F_A = -\pi M_s g(1 - 2A/gM_s)\ln(1 + \rho^{*2}/z^2). \tag{14}$$

The minus sign indicates that the force is directed against the motion associated with the escape of the monopole from the medium. If the internal field $H_i > g/\rho^{*2}$, where $\rho^* = g\gamma/u$, then in formula (14) one should assume $\rho^* = \sqrt{g/H_i}$.

Formula (14) enables us to determine the minimum value of the magnetic field H_{min}, which is necessary in order to remove the monopole from the surface of a ferromagnetic substance:

$$\frac{H_{min}}{\pi M_s(1 - 2A/gM_s)} = \ln(1 + 4g/a^2H_{min}). \tag{15}$$

The solution of Eq. (15) for the following types of ferromagnetic substances is shown in Fig. 1: Permalloy 79NM ($4\pi M_S = 7500$ G,[22] $A = 1 \times 10^{-6}$ erg/cm[20]), Permalloy 50N ($4\pi M_S = 15{,}000$ G, $A = 1 \times 10^{-6}$ erg/cm), and Permendur Fe-Co ($4\pi M_S = 22{,}400$ G, $A = 1.9 \times 10^{-6}$ erg/cm[21]).

C. Use of ferromagnetic substances as traps for monopoles. The results permit us to calculate the slowing-down of magnetic charges in ferromagnetic foils which are located in an external magnetic field.

In a ferromagnetic substance the equation of motion of a monopole has the form

$$m_g d^2\mathbf{z}/dt^2 = g\mathbf{B} + \Sigma\mathbf{F}_i,$$

where $\mathbf{B}$ is the magnetic induction in the medium, and $\Sigma\mathbf{F}_i$ is determined according to formulas (10), (11), (12), and (14).

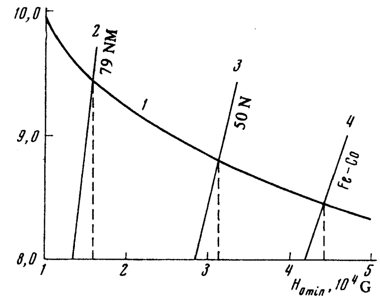

FIG. 1. Minimum value of the magnetic field, capable of extracting a monopole from ferromagnetic substances. Curve 1 corresponds to ln $(1 + g/z^2_{min}/H_{0\,min})$, and curves 2, 3, and 4 correspond to $H_{0\,min}/\pi M_S$ $(1-2A/gM_S)$.

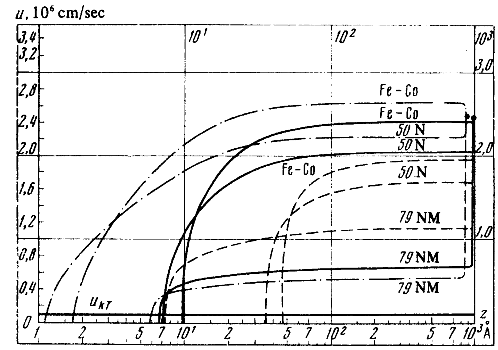

FIG. 2. Slowing-down of a monopole near the surface of a ferromagnetic foil. The dot-dashed curves are for g = 51.4e, the solid curves are for g = 68.5e, and the dashed curves are for g = 137e.

The slowing down of monopoles of various charges to thermal velocities in ferromagnetic foils (Permendur, Permalloy 50N, and Permalloy 79NM) placed in an external magnetic field of 12,000 G perpendicular to the surface of the foil, is shown in Fig. 2. It follows from the figure that ferromagnetic substances are an effective trap for monopoles even in the presence of large external magnetic fields.

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