Surface Waves in Fermi Liquids

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Submitted June 28, 1971

Zh. Eksp. Teor. Fiz. 61, 2562–2570 (December, 1971)

It is shown that weakly damped surface waves should exist in a Fermi liquid in which the longitudinal and transverse zero-sound velocities are great. These waves would have a linear dispersion law and be similar to Rayleigh waves in solids. However, because of strong damping, such waves will not propagate in helium 3.

THE character of surface wave propagation, as well as any other oscillations in a Fermi liquid, depends on the relation between the frequency of oscillation ω and the characteristic time between the collisions of the quasiparticles, τ. At low frequencies ($\omega\tau \ll 1$), the laws of ordinary hydrodynamics are applicable, from which follows the existence of capillary-gravitational waves with the dispersion law $\omega = \sigma k^3/\rho + gk$, where k is the wave vector, σ the surface tension, ρ the liquid density, g the acceleration due to gravity. The specifics of a Fermi liquid appear in the high-frequency region ($\omega\tau \gg 1$), when so-called "zero sounds" become possible types of oscillations in the bulk of the Fermi liquid.[1] Here, transverse vibrations can also exist in principle along with longitudinal vibrations, transverse vibrations can also exist in principle along with longitudinal vibrations. In a solid, the presence of two types of vibrations leads to the possibility of propagation of surface waves with a linear dispersion law (Rayleigh waves[2]). In this connection, it is interesting to consider the question of the possibility of propagation of high-frequency oscillations of the Rayleigh-wave type in a Fermi liquid. In the present work, this problem is considered for a Fermi liquid in which the function $F(\theta)$, which describes the interaction of quasiparticles in the Landau theory,[3] contains only two spherical harmonics: $F(\theta) = F_0 + F_1 \times \cos\theta$. We shall also assume that F_0 and F_1 are such that there exist longitudinal and transverse modes of the zero sound.

1. STATEMENT OF THE PROBLEM

At low frequencies, the distribution function of quasiparticles satisfies the kinetic equation without the collision integral:

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial \mathbf{r}}\frac{\partial \varepsilon}{\partial \mathbf{p}}-\frac{\partial n}{\partial \mathbf{p}}\frac{\partial \varepsilon}{\partial \mathbf{r}}=0. \quad (1)$$

Setting $n = n_0 + \nu(\mathbf{l})\partial n_0/\partial\epsilon$, where n_0 is the equilibrium Fermi function and $\mathbf{l}$ is the unit vector in the direction of the momentum of the quasiparticle, we linearize Eq. (1); as a result, we get

$$\frac{\partial \nu}{\partial t}+(\mathbf{v}\nabla)\varphi=0. \quad (2)$$

Here $\mathbf{v}$ is the Fermi velocity, and φ is a function connected with ν by the relation

$$\varphi(\mathbf{l}) = \nu + F_0\bar{\nu} + F_1 l_i\overline{\nu l_i} \equiv (1+\hat{F})\nu. \quad (3)$$

The bar above denotes averaging over the solid angle. It is convenient to introduce the additional operator $\hat{G}$, such that

$$\nu = (1-\hat{G})\varphi \equiv \varphi - G_0\bar{\varphi} - G_1 l_i\overline{\varphi l_i}. \quad (3')$$

It is easy to see that $\hat{F}\nu = \hat{G}\varphi$, $G = F_0/(1+F_0)$ and $G_1 = F_1/(1+F_1/3)$. Substituting (3′) in (2), we get the equation for φ:

$$(1-\hat{G})\frac{\partial\varphi}{\partial t}+(\mathbf{v}\nabla)\varphi=0. \quad (4)$$

We choose the boundary of the liquid as the xy plane and direct the z axis into the liquid. Solutions of Eq. (4) which are proportional to $\exp[i(k_x x-\omega t)]$, which fall off as $z\to\infty$ and satisfy definite boundary conditions at $z=0$, correspond to surface waves. We shall write out these conditions. The reflection of the quasiparticles on the free surface of the liquid can be assumed to be specular,[4] when the condition

$$\varphi(l_z) = \varphi(-l_z) - 2p_0 u l_z \quad \text{for} \quad z=0; \quad (5)$$

is obtained for waves of small amplitude. Here p_0 is the Fermi momentum and u is the z component of the velocity of the boundary.

In addition, it is necessary to require continuity of the momentum flux through the boundary. We shall assume that the vapor pressure is equal to zero; then the conditions for the z component of the momentum flux tensor are written in the form

$$\Pi_{xz}=\Pi_{yz}=0, \qquad \Pi_{zz}=\sigma\frac{\partial^2\zeta}{\partial x^2}, \quad (6)$$

where $\zeta(x)$ is the displacement of the surface from its equilibrium position. In a Fermi liquid $\Pi_{ik} = -3N\overline{\varphi l_i l_k} \equiv -3N\varphi_{ik}$, N is the number of particles in a unit volume. Vanishing of the xz and yz components follows from the condition (5) and the equation for Π_{zz} is an additional restriction:

$$\varphi_{zz} = -\frac{\sigma}{3N}\frac{\partial^2\zeta}{\partial x^2} \quad \text{for} \quad z=0. \quad (6')$$

The effect of the surface tension on the surface waves in the case under consideration can be neglected: their contribution is small in comparison with the contribution of the remaining terms in Π_{zz} in the high-frequency region, and is measured by the ratio of the interatomic distance to the wavelength; therefore, we can replace (6′) by the condition

$$\varphi_{zz}(z=0)=0. \tag{7}$$

Thus, the desired solution of Eq. (4) should satisfy the conditions (5) and (7) for z = 0.

2. DERIVATION OF THE DISPERSION EQUATION

The problem stated is analogous in many ways to the problem solved by Bekarevich and Khalatnikov [5] in connection with the calculation of the thermal discontinuity at the boundary of liquid helium 3 with a solid. The difference is that we are dealing with the dependence of the distribution function on two spatial variables instead of one; however, the method of solution is not changed in this case.

We introduce the unknown function χ according to the equation

$$\varphi=\chi+\gamma,\ \gamma=\begin{cases}-2p_0ul_ze^{iqz} & \text{for } l_z>0,\\ 0 & \text{for } l_z<0,\end{cases} \tag{8}$$

while q is so chosen that the function γ satisfies the equation

$$-i\omega\gamma+(\mathbf{v}\nabla)\gamma=0,$$

i.e., $v(l_xk_x+l_zq)=\omega$; then the specular condition for the function χ is written in the form

$$\chi(l_z)=\chi(-l_z)\quad\text{for}\quad z=0. \tag{9}$$

Substituting (8) in (4) and keeping it in mind that the desired solution is proportional to $\exp[i(k_xx-\omega t)]$, we obtain an inhomogeneous integrodifferential equation for χ:

$$-i\omega\chi+i\omega\hat{G}(\chi+\gamma)+ik_xv_x\chi+v_z\frac{\partial\chi}{\partial z}=0. \tag{10}$$

This equation is conveniently solved by the Fourier method, extending provisionally the quantity $\hat{G}\gamma$ to negative values of z, so that the condition (9) is satisfied. As will be seen from the answer, it is sufficient to continue $\overline{\gamma}$ and $\overline{\gamma l_x}$ in even fashion and $\overline{\gamma l_z}$ in odd. We now transform in (10) to the Fourier components in z:

$$\chi(z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ik_zz}\chi(k_z)\,dk_z.$$

Similarly, for $\overline{\gamma}$, $\overline{\gamma l_\lambda}$ (λ = x, z), we obtain as a result the equation for the Fourier components

$$-s\chi+\nu_\lambda l_\lambda\chi+G_0(\chi_0+\gamma_0)+sG_1l_\lambda(\chi_\lambda+\gamma_\lambda)=0. \tag{11}$$

In Eq. (11), the following notation is introduced: $s=\omega/kv$, ν_λ the unit vector in the direction $\mathbf{k}=(k_x, 0, k_z)$ and also $\chi_0=\overline{\chi}$, $\gamma_0=\overline{\gamma}$, $\chi_\lambda=\overline{\chi l_\lambda}$, $\gamma_\lambda=\overline{\gamma l_\lambda}$, and summation is carried out over repeated indices. The Fourier transforms are denoted by the same letters as the original functions and the argument will be given only in those cases in which confusion is possible. We then solve (11) relative to χ:

$$\chi=\frac{s}{s-l_\lambda\nu_\lambda}[G_0(\chi_0+\gamma_0)+G_1l_\lambda(\chi_\lambda+\gamma_\lambda)]. \tag{12}$$

Averaging (11) over the angle, we find $\chi_0+\gamma_0$:

$$\chi_0+\gamma_0=\frac{s\gamma_0+\chi_{\parallel}}{s(1-G_0)}, \tag{13}$$

where $\gamma_{\parallel}=\chi_\lambda\nu_\lambda$.

We then multiply (12) by l_m and average over the angle:

$$\chi_m=\frac{\overline{l_m}}{s-l_\lambda\nu_\lambda}sG_0(\chi_0+\gamma_0)+\frac{\overline{l_ml_k}}{s-l_\lambda\nu_\lambda}sG_1(\chi_k+\gamma_k). \tag{14}$$

The quantities $\overline{l_m/(s-l_\lambda\nu_\lambda)}$, $\overline{l_kl_m/(s-l_\lambda\nu_\lambda)}$ and also $\overline{l_il_kl_m/(s-l_\lambda\nu_\lambda)}$ are calculated in the Appendix. Using the notation introduced there, we get

$$\chi_m=b_1\nu_msG_0(\chi_0+\gamma_0)+a_2sG_1(\chi_m+\gamma_m)+b_2\nu_msG_1(\chi_{\parallel}+\gamma_{\parallel}). \tag{15}$$

We multiply (15) by ν_m, sum over m and express $\chi_{\parallel}+\gamma_{\parallel}$ in terms of the resultant equation:

$$\chi_{\parallel}+\gamma_{\parallel}=\frac{(1+F_1/3)}{\Delta_{\parallel}}[wF_0(s\gamma_0-\gamma_{\parallel})+\gamma_{\parallel}], \tag{16}$$

where

$$\Delta_{\parallel}=(1-wF_0)(1+F_1/3)-s^2wF_1,$$
$$w\equiv w(s)=\frac{s}{2}\ln\frac{s+1}{s-1}-1. \tag{17}$$

Substituting the values of χ_0 and χ_m found from (13), (15), (16) in (12), we get

$$\chi=\frac{1}{s-l_\lambda\nu_\lambda}\Big\{\Big[sG_0+\frac{sb_1(l_k\nu_k)G_0G_L}{1-a_3sG_1}\Big](\chi_0+\gamma_0)$$
$$+\frac{s^2G_1{}^2b_2(l_k\nu_k)}{1-a_2sG_1}\frac{(1+F_1/3)}{\Delta_{\parallel}}[wsF_0\gamma_0+(1-wF_0)\gamma_0]\Big\}. \tag{18}$$

We can now convince ourselves that in the chosen method of continuation of γ_0, γ_x and γ_z in negative values of z, the quantity χ satisfies the condition (9). Actually, for the simultaneous substitution $l_z\to-l_z$ and $\nu_z\to-\nu_z$, (18) does not change, i.e.,

$$\chi(l_z,k_z)=\chi(-l_z,-k_z).$$

Then

$$\chi(l_z,z=0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(l_z,k_z)\,dk_z=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(-l_z,-k_z)\,dk_z$$
$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(-l_z,k_z)\,dk_z=\chi(-l_z,z=0),$$

i.e., the condition (9) is satisfied.

The connection between the frequency of the surface waves ω and their wave vector k_x is found from the condition (7). This condition contains the quantity χ_{zz}, for the calculation of which we multiply (12) by l_z^2 and average over the angle. After a number of simplifications, we obtain

$$\chi_{zz}=(a_2+b_2\nu_z{}^2)sG_0(\chi_0+\gamma_0)+[(a_3+b_3\nu_z{}^2)(\chi_{\parallel}+\gamma_{\parallel})$$
$$+2a_3\nu_z(\chi_z+\gamma_z)]sG_1.$$

It is convenient to express $\chi_z+\gamma_z$ in terms of $\gamma_{\parallel}+\chi_{\parallel}$ and the new quantity $\chi_\perp+\gamma_\perp\equiv\chi_\lambda\tau_\lambda+\gamma_\lambda\tau_\lambda$, where τ_λ is a two-dimensional vector with components $\tau_x=-\nu_z$, $\tau_z=\nu_x$; it is orthogonal to the vector ν_λ. We find from (15)

$$\chi_\perp+\gamma_\perp=\frac{(1+F_1/3)}{\Delta_\perp}\gamma_\perp, \tag{20}$$

where

$$\Delta_\perp=1-\frac{F_1}{6}[1+3(1-s^2)w]. \tag{21}$$

Keeping in mind the definitions of $\chi_\perp + \gamma_\perp$ and $\chi_\parallel + \gamma_\parallel$, we get

$$\chi_z + \gamma_z = \nu_z(\chi_\parallel + \gamma_\parallel) + \nu_x(\chi_\perp + \gamma_\perp). \qquad (22)$$

We substitute (22) in (19); then

$$\chi_{zz} = \nu_z^2 s\left[\frac{wF_0(s\gamma_0 - \gamma_\parallel) + \gamma_\parallel}{\Delta_\parallel} - \gamma_\parallel\right] + 2\nu_x\nu_z s\gamma_\perp\left(\frac{1}{\Delta_\perp} - 1\right)$$
$$+ \nu_x^2\left\{\frac{F_0(s\gamma_0 - \gamma_\parallel) + (s^2-1)\gamma_\parallel}{2s} + \frac{s}{2\Delta_\parallel}[wF_0(s\gamma_0 - \gamma_\parallel) + \gamma_\parallel]\right.$$
$$\left.\times\left[\frac{(1+F_1/3)(1+F_0)}{s^2} - 1\right]\right\}. \qquad (23)$$

It is convenient to make the substitution $\gamma = -2p_0 uif/k$. With the help of the definition of γ (8), we easily establish the fact that

$$f_0 = \overline{l_z^2/(s - \nu_\lambda l_\lambda)}, \qquad f_m = \overline{l_m l_z^2/(s - \nu_\lambda l_\lambda)}.$$

The value of the indices in f corresponds to their value in γ.

The condition (7) has the form

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi_{zz}dk_z = -\gamma_{zz}(0) = \frac{p_0 u}{4}. \qquad (24)$$

Substituting (23) in (24), we obtain

$$-2i\int_{-\infty}^{+\infty}d\varkappa\frac{s^2}{c}\left\{\frac{2c^2 + [(1+F_0)(1+F_1/3) - 3s^2]}{2c^2\Delta_\parallel}\left(\frac{wF_0}{3} + f_\parallel\right)\right.$$
$$-\left(1 - \frac{s^2}{c^2}\right)f_\parallel + \frac{4}{F_1}\frac{s^2}{c^2}\left(1 - \frac{s^2}{c^2}\right)\frac{1}{\Delta_\perp} - 2\left[\frac{2}{F_1} + \frac{1}{3}\right.$$
$$\left.\left.+ (1-s^2)w\right]\frac{s^2}{c^2}\left(1 - \frac{s^2}{c^2}\right) + \frac{F_0 + 3(s^2-1)f_\parallel}{6c^2}\right\} = \frac{\pi}{2}, \qquad (25)$$

where $c = \omega/k_x v$. Integration on the left side of (25) is carried out over the new variable $\kappa = cvk_z/\omega$ along the straight line which passes through the origin of the coordinates at an angle $\alpha = \arg c$ to the real axis. The functions w, $\Delta_\parallel$, $\Delta_\perp$ and $f_\parallel$ depend on s, and $s^2 = c^2/(1+\kappa^2)$. In the transformation, we used the relation $sf_0 = f_\parallel = 1/3$, and also the fact that

$$f_\parallel = s^2 w - \frac{1}{3} - \frac{s^2}{2c^2}[(3s^2-1)w - 1].$$

The singularities of the integrand are the poles for κ which correspond to the vanishing of $\Delta_\parallel$ and $\Delta_\perp$, and also the branch for $s = \pm 1$. The root of the equation $\Delta_\parallel(s)$ (which we denote by $s_\parallel$) is the velocity of propagation of the longitudinal mode of zero sound in Fermi units, $s_\perp$ is the analogous quantity for the transverse mode. The drawing shows the poles $\kappa_\parallel$ and $\kappa_\perp$ and the branch for $\kappa = \pm\sqrt{c^2-1}$ for the case in which c is real and satisfies the inequalities $c > 1$, $c < s_\parallel$ and $c < s_\perp$. Integration in this case is carried out along the real axis. The cuts must be circled as shown in the drawing. This rule of circling is obtained with account of the small collision integral, i.e., with the substitution of $\omega + i\delta$ for ω, where δ is a small positive quantity. On the upper edge of the right-hand cut,

$$w(s) = \frac{s}{2}\ln\left|\frac{s+1}{s-1}\right| - 1 + \frac{i\pi}{2}s \equiv w_0 + \frac{i\pi}{2}s. \qquad (26)$$

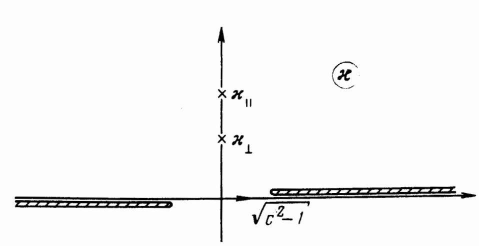
We close the contour of integration in the upper half-plane κ. After calculation of the residues, we obtain

$$-\frac{4\pi c}{s_\parallel\varkappa_\parallel}\frac{1}{(d\Delta_\parallel/ds_\parallel)}\left[1 + \frac{(1+F_0)(1+F_1/3) - 3s_\parallel^2}{2c^2}\right]$$
$$\times\left[\frac{w(s_\parallel)F_0}{3} + f_\parallel\right] + \frac{32\pi}{c}\frac{s_\perp^2}{\varkappa_\perp F_1^2}\left(1 - \frac{s_\perp^2}{c^2}\right)\frac{1}{1 + (1-3s_\perp^2)w(s_\perp)} -$$
$$-2\pi c\int_0^1\frac{s^3 ds}{\sqrt{c^2-s^2}}\left\{\frac{2c^2 + (1+F_0)(1+F_1/3) - 3s^2}{2c^2|\Delta_\parallel|^2}\right.\cdot$$
$$\left.\times\left[1 + \frac{(1+F_0)(1+F_1/3)}{2c^2}\right] + \frac{2}{c^2}\left(1 - \frac{s^2}{c^2}\right)\frac{1-s^2}{|\Delta_\perp|^2} + \frac{(1-s^2)^2}{8c^4}\right\} = 0. \qquad (27)$$

in $|\Delta_\parallel|^2$ and $|\Delta_\perp|^2$ we must substitute the value of w according to (26).

Equation (27) is the dispersion equation for the surface vibrations of the Fermi liquid in the high-frequency limit; its roots c determine the dispersion law of these vibrations $\omega = cvk_x$. Equation (27) remains correct also for complex values of c, inasmuch as the character of the location of the singularities of the integrand in (25) relative to the contour of integration does not change in this case. The real roots of Eq. (27) correspond to undamped surface waves, satisfying the inequalities $c > 1$ and $c < s_\perp$, $s_\parallel$. Real roots $c < 1$ cannot exist, inasmuch as any collective mode with a propagation velocity less than the Fermi velocity experiences Landau damping. If, for example, $c > s_\perp$, then the pole corresponding to $s_\perp$ is shifted on the real axis and this means, by virtue of the properties of the Fourier transformation, that as $z \to \infty$, the quantity χ_{zz} will behave as $\exp(ik_\perp z)$, where $k_\perp = (\omega/cv)\sqrt{c^2/s_\perp^2 - 1}$, i.e., it will not decay and the vibration will not be a surface vibration. We note that the contribution of the cut falls off as $z \to \infty$ according to the law $1/z^2$ (see, for example, [6]).

However, real roots satisfying the inequalities given above for Eq. (27) do not exist, inasmuch as for real $c < s_\perp$, $s_\parallel$, the first two terms in Eq. (27) are purely imaginary, and the third term is real; it is generally impossible to satisfy Eq. (27) with real c in this case. The absence of real roots means the absence of undamped surface waves. The physical reason for the damping of the waves is the scattering of quasiparticles by the vibrating surface of the liquid. In order to establish this fact, we note that the left side of Eq. (27) is equal to the Π_{zz} component of the momentum flux tensor on the boundary of the liquid, with accuracy to within a real factor. The contribution of the collective modes to the momentum flux is shifted in phase by the imaginary unit through a quarter period relative to the surface velocity, and therefore does not lead to dissipation. The contribution of the cut itself, i.e., the contribution of the quasiparticles scattered by the vibrating surface of the liquid, is out of phase with the vibrations of the liquid; this part of the momentum flux also produces energy dissipation.



3. CASE OF LARGE F_0, F_1

Equation (27) can only be solved numerically for arbitrary values of F_0 and F_1. We shall not make use of this, but shall consider the asymptotic behavior of x as F_0 and $F_1 \to \infty$. This permits us to understand the general character of the change of c with change in F_0 and F_1.

We shall first assume that only F_1 is large. With increasing F_1, $s_{\parallel}$ and $s_{\perp}$ also increase. Limiting ourselves to the terms that do not decay as $F_1 \to \infty$, we obtain for them

$$s_{\parallel}^2 = \frac{1}{3}\left(\frac{9}{5}+F_0\right)(1+F_1/3), \qquad s_{\perp}^2 = \frac{F_1}{15}+\frac{3}{7}. \tag{28}$$

The principal term of the imaginary part of (27) will be the following:

$$\frac{2\pi i s_{\perp}}{\xi^3 F_1}\left[\frac{(\xi^2-2)^2}{\sqrt{1-\varepsilon\xi^2}} - 4\sqrt{1-\xi^2}\right]. \tag{29}$$

Here we have introduced a new unknown ξ from the formula $c = \xi s_{\perp}$, and also the notation $\epsilon = (s_{\perp}/s_{\parallel})^2$. The given expression tends to zero as $1/s_{\perp}$ for $s_{\perp} \to \infty$. It is easy to establish the fact that the real part of Eq. (27) falls off as $1/s^4$; therefore, in first approximation, the integral term in (27) can be omitted and the imaginary part substituted in (29).

The equation that is thus obtained can be reduced to the form

$$\xi^6 - 8\xi^4 + 8\xi^2(3-2\varepsilon) - 16(1-\varepsilon) = 0. \tag{30}$$

The resultant equation is identical with the equation for the dimensionless velocity of Rayleigh waves in a solid; it has a real root $\xi < 1$ which changes from 0.96 to 0.87 when ϵ changes from 0 to $\frac{1}{2}$.[2]

In order to estimate the damping of the surface waves, we assume that $F_0 \to \infty$; then $s_{\perp}^2/s_{\parallel}^2 \sim 3/5F_0 \to 0$. Neglecting this ratio, we obtain the result that the principal term of the real part of Eq. (27) is equal to

$$-\frac{\pi}{2c^4}\left[\int_0^1 \frac{s^3 ds}{w_0^2+\pi^2 s^2/4} + \frac{1}{2}\int_0^1 s^3(1-s^2)^2 ds\right]. \tag{31}$$

The contribution of the term with $|\bar{\Delta}_{\perp}|^2$ falls off as c^{-6} and can be neglected. The first of the two integrals entering into (3.1) was found numerically and is equal to 0.19; the second is computed exactly and amounts to $\frac{1}{24}$. Using (29) for $\epsilon = 0$ and (31), we obtain the equation

$$(\xi^2-2)^2 - 4\sqrt{1-\xi^2} \approx -\frac{0{,}79}{\xi s_{\perp}^3} i. \tag{32}$$

We then substitute $\xi = \eta + i\zeta$, $\zeta \ll \eta$. From the equation

$$(\eta^2-2)^2 - 4\sqrt{1-\eta^2} = 0$$

we obtain $\eta \approx 0.96$.

For the imaginary part, we have

$$4\eta\left[(\eta^2-2)+\frac{4}{(\eta^2-2)^2}\right]\zeta \approx -\frac{0{,}79}{\eta s_{\perp}^3},$$

whence

$$\zeta/\eta \approx 1/10 s_{\perp}^3. \tag{33}$$

Thus, in a Fermi liquid for which $s_{\perp}^2 \gg 1$, there are weakly damped, high-frequency surface waves with a linear dispersion law. The formulas obtained in this section cannot, unfortunately, be applied to liquid helium 3, for which $F_1 = 6.25$ and $s_{\perp} = 1.006$; however, Eq. (33) indicates that for $s_{\perp} \sim 1$, $\zeta \sim 1$, i.e., the damping is large. It is clear from physical considerations that upon decrease in F_0 and F_1, the contribution of the collective modes to the momentum flux tensor decreases in favor of the contribution of incoherent quasiparticles. On the basis of the results obtained, we can draw a conclusion on the absence in liquid helium 3 of weakly damped surface waves at absolute zero.

The author is grateful to A. F. Andreev for useful discussions.

APPENDIX

It is clear from symmetry considerations that

$$\overline{l_m/(s-l_\lambda\nu_\lambda)} = b_1\nu_m, \tag{A.1}$$

$$\overline{l_k l_m/(s-l_\lambda\nu_\lambda)} = a_2\delta_{km} + b_2\nu_k\nu_m, \tag{A.2}$$

$$\overline{l_i l_k l_m/(s-l_\lambda\nu_\lambda)} = a_3(\delta_{ik}\nu_m + \delta_{im}\nu_k + \delta_{km}\nu_i) + b_3\nu_k\nu_i\nu_m. \tag{A.3}$$

Here $a_{2,3}$ and $B_{1,2,3}$ are functions of s; for their determination, we must set $\nu_x = 0$, $\nu_z = 1$ and compute several components of the described tensors by the number of unknown functions. We shall show how this is done, for example, for a_2 and b_2. Setting $l = m = z$ in (A.2), we obtain

$$a_2 + b_2 = \int_{-1}^{+1} \frac{t^2}{s-t}\frac{dt}{2} = sw(s):$$

Now taking the convolution, we get

$$3a_2 + b_2 = \int_{-1}^{+1} \frac{1}{s-t}\frac{dt}{2} = \frac{w+1}{s}.$$

We can find a_2 and b_2 from these considerations. For reference, we write down the expressions for a and b and useful relations between them:

$$b_1 = w,$$

$$a_2 = \frac{1}{2s}[(1-s^2)w+1], \qquad b_2 = \frac{3}{2}sw - \frac{w+1}{2s},$$

$$a_3 = {}^1/_6[1+3(1-s^2)w], \qquad b_3 = {}^5/_2 s^2 w - {}^7/_3,$$

$$3a_3 + b_3 = s^2 w - {}^1/_3, \quad 2a_3 + b_3 = sb_2,$$

$$a_2 - a_3/s = 1/3s.$$

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Translated by R. T. Beyer
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