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Theory of μ^+-Meson Depolarization During Charge Exchange in Transverse Magnetic Fields

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A theory is developed for depolarization of positive muons during charge exchange (or formation of unstable muonium chemical compounds) in transverse fields. Relaxation of the electron spin in muonium is taken into account. Integral equations for depolarization are formulated and solved. An expression is derived for the residual polarization and it is shown that on renormalization of the theory parameters it goes over to an expression for polarization involving a pure muonium depolarization mechanism. For all cases when the time dependence of polarization could be observed, expressions have been obtained and relevant analyses have been carried out. The possibility of obtaining information from experiments in longitudinal and transverse fields is analyzed and various control relations are obtained in a number of cases.

1. The theory of the depolarization of μ^+ mesons with allowance for charge exchange and for magnetic fields parallel to the initial polarization direction was developed in[1]. It turned out that experiments performed in longitudinal fields do not suffice in many cases to determine all the phenomenological parameters of the theory. Additional information can be obtained by investigating depolarization in transverse magnetic fields, and accordingly the theory should be extended to include this case.

Following[2], we introduce the complex polarization

$$\mathbf{P}'(t) = P_x'(t) + iP_z'(t). \tag{1}$$

We assume from now on that the x axis is directed along the polarization of the μ^+ meson at the initial instant of time, and that y is directed along the magnetic field.

Following[1], we assume that the μ^+ meson in matter may turn out to be in one of three states: it can form a diamagnetic chemical compound, it may be free (ionized muonium), and it can form a muonium atom in the ground state. Just as in[1], we mark these states by the respective indices 0, 1, and 2.

As shown in[1], the entire developed formalism also admits of a different interpretation. Namely, there are no charge exchanges, depolarization proceeds in accordance with a "pure muonic mechanism," and the state 1 corresponds to the possibility of formation of an unstable diamagnetic compound that decays via two channels—decay into the initial products and decay with formation of a stable compound.

We introduce the transition probabilities $\alpha_{ik} = 1/\tau_{ik}$, where τ_{ik} is the average time of decay of the state i into the state k. We also define $\mathbf{P}_i(t)$ as the contribution of the μ^+ mesons in the i-th state to the total polarization:

$$\mathbf{P}_i(t) = \mathbf{P}_i'(t) N_i(t) / N(t). \tag{2}$$

The complex polarization $\mathbf{P}_i'(t)$ is determined accordingly by the relation

$$\mathbf{P}_i'(t) = \frac{N_i^+ - N_i^-}{N_i^+ + N_i^-}\cos\varphi_i + i\frac{N_i^+ - N_i^-}{N_i^+ + N_i^-}\sin\varphi_i. \tag{3}$$

Here φ_i is the angle between the direction of the polarization vector of μ^+ mesons in the i-th state and the x axis, and N_i^+ and N_i^- are the numbers of μ^+ mesons whose spins are directed parallel and antiparallel to the polarization vector.

The complete complex polarization of the ensemble is equal to

$$\mathbf{P}(t) = \mathbf{P}_1(t) + \mathbf{P}_2(t) + \mathbf{P}_3(t). \tag{4}$$

For the complex polarization $\mathbf{P}(t)$ we can write a system of kinetic equations perfectly analogous to the system obtained in[1], but with the important difference that now the complex polarizations $\mathbf{P}_1(t)$ and $\mathbf{P}_2(t)$ are altered not only by exchange with other states, as in longitudinal fields, but also by precession of the μ^+-meson spin in the magnetic field. Thus we have

$$\begin{aligned}\frac{d\mathbf{P}_1}{dt} &= -(\alpha_{10}+\alpha_{12})\mathbf{P}_1 + \left(\frac{\partial \mathbf{P}_1}{\partial t}\right)_{N_1/N},\\ \frac{d\mathbf{P}_2}{dt} &= -(\alpha_{20}+\alpha_{21})\mathbf{P}_2 + \alpha_{12}\mathbf{P}_1 + \left(\frac{\partial \mathbf{P}_2}{\partial t}\right)_{N_2/N},\\ \frac{d\mathbf{P}_0}{dt} &= \alpha_{20}\mathbf{P}_2 + \alpha_{10}\mathbf{P}_1 + \left(\frac{\partial \mathbf{P}_0}{\partial t}\right)_{N_0/N}\end{aligned} \tag{5}$$

with initial conditions $\mathbf{P}_2(0) = r$, $\mathbf{P}_0(0) = \beta$, $\mathbf{P}_1(0) = 1 - \beta - r$. Here the initial instant of time obviously corresponds to the instant when the thermalization ends.

Assuming that after entering the diamagnetic state the μ^+ meson precesses with the same frequency as the free meson (i.e., neglecting the chemical shift), we can write

$$\left(\frac{\partial \mathbf{P}_1}{\partial t}\right)_{N_1/N} = i\mathbf{P}_1\omega_\mu, \quad \left(\frac{\partial \mathbf{P}_0}{\partial t}\right)_{N_0/N} = i\mathbf{P}_0\omega_\mu. \tag{6}$$

Here $\omega_\mu = eH/m_\mu c = \zeta x\omega_0$, where $\omega_0 = eH_0/m_e c$ is the frequency of the hyperfine splitting, ζ the ratio of the magnetic moments of the μ^+ meson and the electron, H the transverse magnetic field, and $x = H/H_0$ the dimensionless magnetic field.

In complete analogy with[1] we also have

$$\left(\frac{\partial \mathbf{P}_2}{\partial t}\right)_{N_2/N} = \alpha_{12}\int_0^t \mathbf{P}_1(t')\exp\{-(\alpha_{21}+\alpha_{20})(t-t')\} \times \frac{d\mathbf{q}(t-t')}{dt}dt' + r\exp\{-(\alpha_{21}+\alpha_{20})t\}\frac{d\mathbf{q}(t)}{dt}. \quad (7)$$

The complex function $\mathbf{q}(t)$ determines the polarization change due to the purely muonic mechanism:

$$P_2'(t) = \mathbf{q}(t-t')\mathbf{P}_2'(t'). \quad (8)$$

In accordance with[2], we have $\mathbf{q}(t) = \rho_\mu(t) = \rho_{10}(t) + i\rho_{30}(t)$, where ρ_{10} and ρ_{30} are the μ^+-meson density-matrix components along the axes 1 and 3, respectively.

As already noted in[1], it can be assumed in the analysis of the charge-exchange process that $\alpha_{10} = 0$. The case $\alpha_{10} \neq 0$ may be of interest in the "chemical interpretation" of the formalism. No complication whatever is introduced in the solution of the system when $\alpha_{10} \neq 0$, but we shall not analyze this situation here. The corresponding analysis for both longitudinal and transverse fields will be presented in a different paper.

Integrating the system (5) formally, we obtain a system of integral equations[1)]

$$\mathbf{P}_1(t) = (1-r-\beta)\exp\{(i\omega_\mu - \alpha_{12})t\} + \alpha_{21}\int_0^t \mathbf{P}_2(t')\exp\{(i\omega_\mu-\alpha_{12})(t-t')\}dt', \quad (9)$$

$$\mathbf{P}_2(t) = r\exp\{-(\alpha_{21}+\alpha_{20})t\}\mathbf{q}(t) + \alpha_{12}\int_0^t \mathbf{P}_1(t')\exp\{-(\alpha_{21}+\alpha_{20})(t-t')\}\mathbf{q}(t-t')dt',$$

$$\mathbf{P}_0(t) = \beta\exp\{i\omega_\mu t\} + \alpha_{20}\int_0^t \mathbf{P}_2(t)\exp\{i\omega_\mu(t-t')\}dt'.$$

2. Just as in the case of longitudinal fields[1], the system (9) can easily be solved with the aid of a Laplace transformation, which leads to the algebraic system

$$L(\mathbf{P}_1,\sigma) = \frac{1-r-\beta}{-i\omega_\mu+\alpha_{12}+\sigma} + \frac{\alpha_{21}L(\mathbf{P}_2,\sigma)}{-i\omega_\mu+\alpha_{12}+\sigma} \quad (10)$$

$$L(\mathbf{P}_2,\sigma) = rL(\mathbf{q},\sigma+a_2) + \alpha_{12}L(\mathbf{P}_2,\sigma)L(\mathbf{q},\sigma+a_2),$$

$$L(\mathbf{P}_0,\sigma) = \frac{\beta}{\sigma-i\omega_\mu} + \frac{\alpha_{20}L(\mathbf{P}_2,\sigma)}{\sigma-i\omega_\mu}.$$

Here $a_2 = \alpha_{21} + \alpha_{20}$ is the probability of leaving the state 2. The Fourier transform of the function $\mathbf{q}(t)$ was calculated in[2] and is equal, in the present notation, to

$$L(\mathbf{q},\sigma) = -\frac{2}{\omega_0}\left\{\frac{4\nu}{\omega_0} + \frac{i[A^2B^2-(A+B)^2]}{AB^2-(A+B)}\right\}^{-1} \quad (11)$$

Here

$$A = i\left(\frac{2\sigma}{\omega_0}+\frac{4\nu}{\omega_0}-2i\zeta x\right),\quad B = i\left(\frac{2\sigma}{\omega_0}+\frac{4\nu}{\omega_0}+2ix\right). \quad (12)$$

Solving the system (10), we obtain

$$L(\mathbf{P},\sigma) = \sum_{i=0}^{2} L(\mathbf{P}_i,\sigma) = \frac{\beta}{\mu} + \{(1-\beta)[\mu + \alpha_{12}(\mu+\alpha_{20})L(\mathbf{q},\mu+i\omega_\mu+a_2)] + r\mu[(\mu+a_2)L(\mathbf{q},\mu+i\omega_\mu+a_2) - 1]\}\{\mu[\alpha_{12}+\mu-\alpha_{12}\alpha_{21}L(\mathbf{q},\mu+i\omega_\mu-a_2)]\}^{-1} \quad (13)$$

where $\mu = \sigma - i\omega_\mu$.

In determining the complex residual polarization $\mathbf{P}_\infty$ due to the μ^+ mesons that enter into a stable diamagnetic compound, we shall disregard, in accord with[2], the factor $\exp\{i\omega_\mu\}$, which determines the precession with the μ^+-meson frequency. Then

$$\mathbf{P}_\infty = \lim_{t\to\infty}\mathbf{P}(t)\exp\{-i\omega_\mu(t)\} \quad (14)$$

and

$$\frac{1-\beta}{1-P_\infty} - 1 = \alpha_{20}\left[\frac{1}{f} + \frac{4bf}{(b+f)\omega_0^2}\right]. \quad (15)$$

Here

$$b = \alpha_{20} + 2\nu',\quad f = b + ix_+\omega_0,$$
$$\nu' = \nu + \alpha_{21}/2,\quad x_\pm = (1\pm\zeta)x. \quad (16)$$

We see that, in complete analogy with the results of[1], the residual polarization in the charge-exchange process is determined in perpendicular fields, too, by the same expression as in the "purely muonic" mechanism (see[2]), albeit with a renormalized electron-spin relaxation rate.

Thus, in order to detect the existence of charge exchanges, it is necessary to investigate the time dependence of the polarization. Confining ourselves to an analysis of the residual polarization in longitudinal and transverse fields, we "find ourselves," as it were, in the "purely muonic" case. The corresponding analysis was carried out in[2,3].

3. We proceed now to consider the time dependence of the polarization. To determine $\mathbf{P}(t)$ we must take the inverse Laplace transform of (13). The solution of this problem obviously reduces to a determination of the roots of the equation

$$\alpha_{12} + \mu - \alpha_{12}\alpha_{21}L(\mathbf{q},\ \mu+i\omega_\mu+a_2) = 0. \quad (17)$$

After substitution of (11) and (12), Eq. (17) turns out to be of fifth degree in μ:

$$1+\frac{\omega_0^2}{4}\left(\frac{1}{\mu+f}+\frac{1}{\mu+b}\right)\left(\frac{1}{\mu+f}+\frac{1}{\mu+a_2}\right) - \frac{\alpha_{12}\alpha_{21}}{(\mu+a_2)(\mu+\alpha_{12})}\left[1+\frac{\omega_0^2}{4}\frac{1}{(\mu+f)}\left(\frac{1}{\mu+f}+\frac{1}{\mu+b}\right)\right] = 0. \quad (18)$$

As $\alpha_{12} \to \infty$, Eq. (18) goes over, as it should, into the equation describing the purely muonic mechanism[3,4].

In the analysis of (18), we shall follow the classification assumed in[1]. We note that we are interested only in small roots of (17), $\sigma = \mu + i\omega_\mu \ll \omega_0$, since, as already noted in[1], only in these cases it is possible to observe the time dependence of the polarization at the present level of the experimental technique.

We introduce the notation

$$a = \alpha_{12} + \alpha_{20} + \alpha_{21},\quad b = \alpha_{20} + \alpha_{21} + 2\nu.$$

We begin the analysis with the case $a \gtrsim \omega_0$, $\alpha_{12} \gtrsim \omega_0$, $\omega_0/b \gg 1$. We consider first the case of beats that should be observed in weak fields $x \ll 1$ when $b \lesssim x^2\omega_0$, i.e., very slow processes that lead to relaxation[2,4].

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1) We note that the system (9) can also be obtained directly from physical considerations.

Neglecting in (18) the terms α_{21}/α_{12} and $(\mu/\alpha_{12})^2$ compared with unity, we obtain the old equation for the beats[4]:

$$1+\frac{\omega_0^2}{4}\left(\frac{1}{\mu'+2\nu'+ix_+\omega_0}+\frac{1}{\mu'+2\nu'}\right)\left(\frac{1}{\mu'+2\nu'+ix_+\omega_0}+\frac{1}{\mu'}\right)=0, \quad (19)$$

where $\mu' = \mu + \alpha_{20}$. Thus, in this case the picture of the beats coincides identically with the case of the "purely muonic mechanism"[2]. Recognizing that at the present state of the experiment it is apparently impossible to observe the muonic frequency at "medium" fields $x \sim 1$, we proceed to analyze the case $x \gg 1$, when stopping of the muonium precession should be observed[4]. Then that root of (18) which determines the stopping, calculated accurate to terms of order $(b/\omega_0^2)^2$, is given by

$$\sigma_1 = i\omega_\mu - \nu' - \alpha_{20} - \frac{\omega_0^2\alpha_{21}}{2(4\alpha_{12}^2+\omega_0^2)} - \frac{i\omega_0}{2}\left(1-\frac{2\alpha_{12}\alpha_{21}}{4\alpha_{12}^2+\omega_0^2}\right). \quad (20)$$

Calculating the coefficient A_1, which determines the contribution of this root to the expression for $\mathbf{P}(t)$, we obtain

$$A_1 = \frac{r}{2} + \frac{(1+r-\beta)\alpha_{12}}{2(\alpha_{12}-i\omega_0/2)}. \quad (21)$$

Thus, if the charge exchange has a negligible influence on the magnitude of the root in the case under consideration, then the coefficient is appreciably altered compared with the purely muonic case, namely a noticeable phase appears at $\alpha_{12} \sim \omega_0$ and is connected with the fact that in strong fields the spin of the μ^+ mesons, which were originally in the free state, can turn through a noticeable angle within a time on the order of $1/\alpha_{12}$.

The analysis of the cases $a \gtrsim \omega_0$, $\alpha_{12} \gtrsim \omega_0$, $\omega_0/b \sim 1$ and $a \gtrsim \omega_0$, $\alpha_{12} \gtrsim \omega_0$, $\omega_0/b \ll 1$ is similar to the analysis carried out in parallel fields. When $\omega_0/b \sim 1$ there is no small root (in parallel fields, it could occur only in a strong restoring field). When $\omega_0/b \ll 1$, the small roots appear under the same additional conditions as in the parallel field ($\alpha_{20} \ll \omega_0$ if $\alpha_{12} > \alpha_{21}$, or $\alpha_{20}/\alpha_{21} \ll \omega_0/\alpha_{12}$ if $\alpha_{21} > \alpha_{12}$). Then

$$\mathbf{P}(t) = \mathbf{P}_\infty \exp\{i\omega_\mu t\} + (1-\mathbf{P}_\infty)\exp\left\{i\omega_\mu t - \frac{\alpha_{12}t}{\alpha_{12}+a_2}\left[\alpha_{20}+\frac{\omega_0^2}{4}\left(\frac{1}{f}+\frac{1}{b}\right)\right]\right\}. \quad (22)$$

No essentially new information can be extracted in this case.

A small root appears, however, even in cases when $\alpha_{12} \ll \omega_0$, $b \sim \omega_0$, $a_2 \gtrsim \omega_0$ and $\alpha_{12} \ll \omega_0$, $\omega_0 \ll b$, $a_2 \gtrsim \omega_0$. We then obtain for the polarization $\mathbf{P}(t)$ (at $b \sim \omega_0$)

$$\mathbf{P}(t) = \mathbf{P}_\infty \exp\{i\omega_\mu t\} + \left[1-\mathbf{P}_\infty - r\left(1+\frac{4a_2bf}{\omega_0^2(b+f)}+\frac{a_2}{f}\right)^{-1}\right]\exp\{t(i\omega_\mu+\mu_1)\}. \quad (23)$$

Here

$$\mu_1 = -\alpha_{12}\frac{[\alpha_{20}bf^2+{}^1/_4\omega_0^2(b+f)(f+\alpha_{20})]}{a_2bf^2+{}^1/_4\omega_0^2(b+f)(a_2+f)}. \quad (24)$$

In very strong magnetic fields, formulas (23) and (24) become much simpler:

$$\mathbf{P}(t) = \mathbf{P}_\infty e^{i\omega_\mu t} + \left(1-\mathbf{P}_\infty-\frac{r}{1+4ba^2/\omega_0^2}\right) \times \exp\left\{\left[i\omega_\mu - \alpha_{12}\frac{\alpha_{20}b+\omega_0^2/4}{a_2b+\omega_0^2/4}\right]t\right\}.$$

On the other hand, if $\omega_0 \ll b$, then (23) and (24) can be simplified in obvious fashion.

Finally, we have the trivial case when one small root appears, namely $\nu \sim \omega_0$ and $a \ll \omega_0$. Then

$$\mathbf{P}(t) = \mathbf{P}_\infty \exp\{i\omega_\mu t\} + (1-\mathbf{P}_\infty - r)\exp\{(i\omega_\mu-\alpha_{12})t\}. \quad (25)$$

We note that formula (25) also describes the case $a \ll \omega_0$, $\nu \ll \omega_0$, $x \sim 1$. It should be recognized here that $\mathbf{P}_\infty = \beta$.

As seen from the foregoing relations, experiments performed in perpendicular fields in the case of one small root yield, generally speaking, relatively little additional information compared with experiments in longitudinal fields. The only important exception is the stopping of the precession, when a direct possibility appears of distinguishing between a process with charge exchanges and the purely muonic case, and also the situation described by formulas (23) and (24).

We now consider the case when all the parameters of the problem are small and have one order of magnitude: $a \ll \omega_0$, $\nu \ll \omega_0$, $x \ll 1$, $b \sim x\omega_0 \sim \alpha_{12}$. Then Eq. (18) can be shown to have three small roots, and in the expression for the polarization the coefficient of one of these roots vanishes. The "working" roots are

$$\mu_{2,3} = -\frac{1}{2}\left(\frac{ix_+\omega_0}{2}+\nu+a_2+\alpha_{12}\right) \pm\left[\left(\frac{ix_+\omega_0}{2}+\nu+a_2-\alpha_{12}\right)^2+2\alpha_{12}\alpha_{21}\right]^{1/2}. \quad (26)$$

The corresponding coefficients can easily be calculated from the general formula

$$A_k = (\mu_k^2+a\mu_k+\alpha_{12}\alpha_{20})[(1-r-\beta)\alpha_{12}+r(\mu+\alpha_{12})]Q(\mu_k)\cdot\left[4\mu_k(\mu_k+\alpha_{12})\prod_{l\neq k}(\mu_k-\mu_l)\right]^{-1}, \quad (27)$$

where

$$Q(\mu) = 4(b+\mu)(f+\mu)^2+(b+f+2\mu)\omega_0^2. \quad (28)$$

We shall not present the resultant formulas, since they are quite cumbersome. In this case $\mathbf{P}_\infty$ has a simple form and the sum of the coefficients satisfies the obvious relation

$$A_2+A_3 = 1-\mathbf{P}_\infty-\frac{r}{2} = \frac{(1-\beta)f}{\alpha_{20}+f}-\frac{r}{2}. \quad (29)$$

As seen from (26), charge exchanges change noticeably the picture that would be observed for the purely muonic mechanism. Instead of one muonium frequency there will be observed a complicated two-frequency precession with a very intrincate character of the interference. Unfortunately, the interest in the entire class of cases with $\nu \ll \omega_0$ is greatly decreased, since such cases have low probability in the presence of charge exchange. They can be of interest, however, if the chemical interpretation of the theory is borne in mind.

If we increase the field and go over to the case $a \ll \omega_0$, $\nu \ll \omega_0$, $b \sim x^2\omega_0$, then, just as in the case of

[2]We note that in[2,4], for the sake of simplicity, the roots were written out without the trivial term $\exp(-\alpha_{20}t)$, which determines the damping of the precession amplitude as a result of the entry of the muonium into a chemical compound.

the pure muonium mechanism, beats will be observed. The roots corresponding to the beats can easily be obtained from (18):

$$\mu_{1,3} = -\alpha_{20} - \frac{3}{4}(2\nu + \alpha_{21}) - \frac{ix_+\omega_0}{2}\left[1 - \alpha_{21} \Big/ 4\left(\alpha_{12} - \frac{ix_+\omega_0}{2}\right)\right] \mp \Delta, \tag{30}$$

where

$$\Delta = \frac{1}{4}\left[-x_+^4\omega_0^2 + 4\left(\nu + \frac{\alpha_{12}\alpha_{21}}{2\alpha_{12} - ix_+\omega_0}\right)\right]^{1/2}. \tag{31}$$

The coefficients of these roots can also be determined easily. We note that the coefficient of the third small root is not equal to zero. This root corresponds in this case to root μ_2 in (26), and results in a frequency close to that of the free μ^+ meson. On the whole, the beat picture differs little from the pure muonium case.

We now proceed to the last case in our classification, $a \ll \omega_0$. It is quite realistic in the case of charge exchange, lends itself to simple analysis (there are two small roots), but unfortunately experiments in perpendicular fields give practically no new information compared with experiments in longitudinal fields. We therefore leave out the corresponding formulas.

4. Let us ascertain now what information concerning the depolarization process can be extracted from the aggregate of experiments in longitudinal and transverse fields by using the theory developed in[1] and in the present paper.

We note first that, generally speaking, the differences due to charge exchanges in the depolarization process as compared to the pure muonium process are less pronounced than might be expected beforehand. In particular, as we have shown, the formulas for the residual polarization are identical in structure in both variants, for both longitudinal and transverse fields. Therefore the presence of charge exchange can be revealed only by investigating the time dependence of the polarization.

Let us examine the possible cases, following the assumed classification.

In the case $\alpha_{12} \gtrsim \omega_0$, $b \ll \omega_0$, the rate of departure from the state of the free μ^+ meson is of the same order as or larger than the frequency of the hyperfine splitting, and the rate of departure from the muonium state and the relaxation rate of the electron spin in the muonium atom are small. The main attribute is the presence of the muonium precession frequency. The "hot chemistry" channel can cause precession at the μ^+-meson frequency, but there is no damping of this precession. We note that the pure muonium mechanism corresponds to the limiting case when α_{12} is larger than all the remaining characteristic parameters. As shown by the results of[1] and of the present article, in our case we cannot distinguish the charge-exchange process from the pure muonium mechanism, unless ultrastrong magnetic transverse fields are used. Actually, as seen from (21), the charge-exchange process lead to the appearance of a noticeable phase in the precession-stopping phenomenon, although this phase vanishes when $\alpha_{12} \gg \omega_0$.

We note that in longitudinal fields we obtain in this case a convenient relation between the root and the experimentally observed quantities:

$$\sigma(H) = \frac{1}{b}\frac{P_{\|t=0}(H) - 1}{1 - P_{\|\infty}(H)}. \tag{32}$$

We now proceed to the cases $a \gtrsim \omega_0$, $\alpha_{12} \gtrsim \omega_0$ and $\omega_0/b \sim 1$ or $\omega_0/b \ll 1$. As already noted, experiments in transverse fields will not yield any new information here. An experiment in longitudinal fields at $\omega_0/b \sim 1$ does not make it possible to distinguish the charge-exchange process from the pure muonium mechanism.

When $\omega_0/b \ll 1$ it is also quite difficult to extract any information. We note that in a longitudinal field a relation analogous to (32) should be satisfied:

$$\frac{1}{\sigma(H_1)\left[1 - P_{\|\infty}(H_1)\right]} - \frac{1}{\sigma(H_2)\left[1 - P_{\|\infty}(H_2)\right]} = (H_1^2 - H_2^2)\cdot\text{const.} \tag{33}$$

We proceed now to the case $\alpha_{12} \ll \omega_0$, $b \sim \omega_0$, $a_2 \gtrsim \omega_0$. Its feature is that precession with meson frequency is observed. In this case the aggregate of the experiments in transverse and longitudinal fields makes it possible to determine all the parameters of the theory. We note a curious relation for longitudinal fields:

$$\frac{1}{\sigma(H)} - \frac{1 - P_{\|t=0}(H)}{1 - P_{\|\infty}(H)} = -\frac{r}{\alpha_{12}(1-\beta)}. \tag{34}$$

Formulas convenient for an experimental analysis are obtained in this case also in transverse fields:

$$\sigma(H_\perp) = -\frac{\alpha_{12}}{a_2}\left[\alpha_{20} + \frac{\alpha_{21}}{r}(1 - \mathbf{P}_{t=0}(H_\perp))\right], \tag{35}$$

$$\left(\frac{r}{1 - \mathbf{P}_{t=0}(H_\perp)} - 1\right) \Big/ \left(\frac{1-\beta}{1 - \mathbf{P}_\infty(H_\perp)} - 1\right) - 1 = \frac{\alpha_{21}}{\alpha_{20}}. \tag{36}$$

We see that with the aid of (36) we can determine the parameters r, β, and α_{21}/α_{20}. Further, using (15), we can also obtain ω_0 and b.

Finally, from the formula for the fast losses we can obtain a_2 and ν:

$$\frac{1}{1 - \mathbf{P}_{t=0}(H_\perp)} = \frac{1}{r}\left[\frac{a_2}{f} + 1 + \frac{4bfa_2}{\omega_0^2(b+f)}\right]. \tag{37}$$

Thus, in this case the experiments in transverse fields alone make it possible to determine all the phenomenological parameters.

We proceed now to cases when $a \ll \omega_0$. If at the same time $\nu \sim \omega_0$, then the damping period in longitudinal fields does not depend on the field (with the exception of ultrastrong fields). Experiments in longitudinal fields make it possible to determine r, β, and α_{12}, and thus reveal immediately the presence of charge exchange. Experiments in transverse fields yield no new information. The case $\nu \ll \omega_0$, when two-frequency precession is observed in transverse fields, is quite convenient, as already mentioned, for the extraction of information. Generally speaking, if the damping periods of the two exponentials in the longitudinal field differ greatly, then all the parameters of the theory are determined from experiments in longitudinal fields. It is useful to employ in the analysis the convenient relations

$$1 - P_{\|t=0}(H) = \frac{r}{2(1+x^2)}, \tag{38}$$

$$\sigma_{1\|} + \sigma_{2\|} = -a - \frac{\nu}{1+x^2}, \tag{39}$$

$$\frac{1 - P_{\|\infty}(H)}{1 - P_{\|t=0}(H)}\sigma_{1\|}\sigma_{2\|} = \alpha_{12}b\frac{(1-\beta)}{r}. \tag{40}$$

It is seen from the analysis that experiments in transverse fields make it possible to extract all the information on the parameters of the theory in an independent manner. Finally, if $\nu \gg \omega_0$, then a precession with

μ^+-meson frequency is observed in transverse fields and, as already noted, the experiments in longitudinal fields do not give any new information. We present here only a useful relation which should be satisfied in longitudinal fields in this case:

$$\frac{F(H_1)-F(H_2)}{H_1^2-H_2^2}=\text{const}, \tag{41}$$

$$F(H)=\frac{1}{1-P_{\parallel\infty}(H)}\frac{1}{\sigma_1\sigma_2}. \tag{42}$$

We note that the same relation is satisfied also in the case $a \ll \omega_0$, $\nu \ll \omega_0$. Naturally, if there is no chemical bond at all ($\alpha_{20} = 0$), then the analysis becomes much simpler in all cases. For longitudinal fields, the corresponding formulas were obtained in[1]. We omit here the formulas for the transverse fields, and also certain useful relations, since they can be obtained in trivial fashion from the formulas given in this paper.

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