EFFECT OF DIELECTRIC COATING ON THE CRITICAL TEMPERATURE OF THIN SUPERCONDUCTING FILMS

G. F. ZHARKOV and Yu. A. USPENSKIĬ

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted June 4, 1971

Zh. Eksp. Teor. Fiz. 61, 2123–2132 (November, 1971)

The role of a dielectric coating in the critical temperature T_c of thin superconducting films is considered within the framework of a model description of a superconductor ("jellium" model). For extremely thin films with almost two-dimensional electron motion, a formula is derived which qualitatively describes variation of the films T_c with the permittivity of the coating $\epsilon(\omega)$. It is found that for metallic films the dielectric coating alters the transition temperature comparatively weakly.

1. The problem of obtaining superconducting materials with high transition temperature has recently been attracting increasing attention (see, for example, the reviews[1,2] and the literature cited therein). With the exception of the special case of metallic hydrogen, which apparently can exist only at very high pressures (or in a metastable state), and where phonon exchange is significant, our main hope of obtaining high critical temperatures lies in the exciton mechanism of superconductivity. The attraction between electrons is due in the latter case mainly to exchange of excitons, i.e., excitations of the electronic type, and not of phonons. According to the BCS theory[3], one can expect for such materials $T_c \sim \omega^{(e)} e^{-1/g}$, where $\omega^{(e)}$ is the characteristic exciton frequency, with $\omega_D \ll \omega^{(e)} \ll \epsilon_F$, and g is of the same order of magnitude as in superconductors with the phonon mechanism. Thus, owing to the large preexponential factor, we can expect an appreciable increase in the critical temperatures. More accurate calculations[4] in accordance with the generalized "jellium" model confirm the correctness of such an estimate.

The possibility of realizing the exciton mechanism of superconductivity was discussed in the literature many times (see, for example,[1,2]), and several variants of concrete superconducting systems in which the exciton exchange plays the principle role have been proposed.

In the present paper we discuss one such system, namely a sandwich consisting of a thin superconducting film coated with a dielectric. In this case, an increase of the critical temperature should result from the interaction between the electrons in the film and the high-frequency excitations in the dielectric. It is clear that inasmuch as this interaction is screened by the conduction electrons in the film, the exciton mechanism of superconductivity can be effectively realized only in a layer having a thickness on the order of the screening length, near the metal-dielectric boundary. In the remaining metal the superconductivity mechanism remains of the phonon type as before. In order for exciton exchange to play the principal role in the production of the superconducting pairs, it is therefore important to make the film sufficiently thin, not thicker than 5–10 Å. The possibility of a phase transition in such a nearly two-dimensional system will not be discussed here[5,6]; we confine ourselves to calculation of the superconducting properties in the spirit of the BCS theory, where $T_c \neq 0$ even for two-dimensional systems[7].

We consider below such a metallic film, and calculate the change in its critical temperature if the film is covered on both sides with a thick dielectric layer having a permittivity $\epsilon(\omega)$. We shall thus estimate the effectiveness of the exciton mechanism for sandwiches.

2. To solve our problem, we must know, first, the nature of the electron interaction in such a system and, second, how T_c is affected by a change in the electron interaction as compared with the case $\epsilon(\omega) = 1$.

Let us answer the first question first. We cannot, of course, determine the exact interaction in such a complicated system, and we confine ourselves to a simplified model description.

We use for the superconducting film the "jellium" model in which the metal is described as a continuous medium (see[8]), and the Fourier transform of the electric-field potential with respect to time

$$\Phi_\omega(\mathbf{r}) = \int dt\, e^{i\omega t} \Phi(\mathbf{r}, t)$$

is determined, as can easily be shown, from the equation

$$\Delta\Phi_\omega(\mathbf{r}) - \varkappa_D^2 \alpha(\omega)\Phi_\omega(\mathbf{r}) = -4\pi e \alpha(\omega)\rho_{\text{ext}}(\mathbf{r}, \omega), \qquad (1)$$

where $\alpha(\omega) = \omega^2/(\omega^2 - \omega_i^2)$; ω_i is the ion plasma frequency, $1/\kappa_D$ is the Debye screening radius, and $\rho_{\text{ext}}(\mathbf{r}, \omega)$ is the density of the external charges.

The "jellium" model[9-11] reflects such characteristic metal properties as the screening of the electric field by the conduction electrons and the existence of acoustic excitations. It describes quite well the behavior of the effective permittivity in metals at $\omega \ll kv_F$, which is most important for superconductivity, and gives reasonable estimates for T_c. As shown in[12], if we neglect the influence of the periodicity of the lattice, such a description will be all the more exact if the metal is compressed, when $a_B k_F \gg 1$.

3. Let us apply this qualitative model to our problem. Let the interaction potential satisfy Eq. (1) in the metallic film (region II in Fig. 1), and the Laplace equation $\Delta\Phi_\omega(\mathbf{r}) = 0$ in the dielectric (regions I and III).

We introduce the permittivity of the dielectric (we neglect the spatial dispersion in the dielectric and stipu-

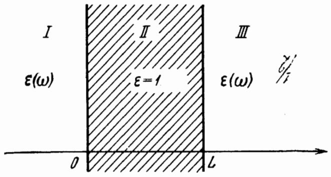

FIG. 1

late the usual field and potential continuity conditions on the boundaries

$$\Phi_\omega(\mathbf{r})|_{z\to-0}=\Phi_\omega(\mathbf{r})|_{z\to+0},\quad \Phi_\omega(\mathbf{r})|_{z\to L-0}=\Phi_\omega(\mathbf{r})|_{z\to l+0},$$
$$\varepsilon(\omega)\frac{\partial}{\partial z}\Phi_\omega(\mathbf{r})\Big|_{z\to-0}=\frac{\partial}{\partial z}\Phi_\omega(\mathbf{r})\Big|_{z\to+0},\tag{2}$$
$$\frac{\partial}{\partial z}\Phi_\omega(\mathbf{r})\Big|_{z\to L-0}=\varepsilon(\omega)\frac{\partial}{\partial z}\Phi_\omega(\mathbf{r})\Big|_{z\to L+0}.$$

In the boundary conditions we put $\epsilon = 1$ for the metal, since the effects of polarization of the medium have already been taken into account by the very form of Eq. (1).

We use Eq. (1) with the foregoing boundary conditions (2) to obtain the interaction energy of two electrons located at points $\mathbf{r}$ and $\mathbf{r}'$ inside the metallic film. To this end, we put in (1) $\rho_{ext}(\mathbf{r}, \omega) = \delta(\mathbf{r} - \mathbf{r}')$ and calculate

$$V_\omega(\varkappa, z, z')=e\int d\rho\,\Phi_\omega(\mathbf{r})e^{-i\varkappa\rho},$$

where $\rho = (x, y)$. As a result we obtain

$$V_\omega(\varkappa,z,z')=4\pi e^2 i\alpha(\omega)\Big(\frac{\exp\{iq|z-z'|\}}{q}+\frac{2[\varepsilon(\omega)\varkappa+iq]}{\Delta(\varkappa,\omega)}\{[\varepsilon(\omega)\varkappa$$
$$+iq]e^{iqL}\cos(q|z-z'|)-[\varepsilon(\omega)\varkappa-iq]\cos((z+z'-L)q)\}\Big),$$
$$\Delta(\varkappa,\omega)=e^{-iqL}[\varepsilon(\omega)\varkappa-iq]^2-e^{iqL}[\varepsilon(\omega)\varkappa+iq]^2,$$
$$q=[-\varkappa^2-\varkappa_D{}^2\alpha(\omega)]^{1/2}.\tag{3}$$

We assume here and below that $\sqrt{x} = \sqrt{|x|}\exp(\tfrac{1}{2}i\arg x)$, with $-\pi < \arg x \le \pi$.

As $L \to 0$ we have $V_\omega(\kappa, z, z') \to 4\pi e^2\alpha(\omega)/\kappa\epsilon(\omega)$, i.e., as expected, the frequency dependence of the dielectric has a very strong effect, and the Coulomb potential is spatially unscreened.

We present also an expression for $V_\omega(\kappa, z, z')$ for $L \to \infty$, small z and z', and $\omega \gg \omega_i$. In this case we have

$$V_\omega(\varkappa,z,z')=\frac{4\pi e^2}{(\varkappa^2+\varkappa_D{}^2)^{1/2}}\Big[\mathscr{E}(|z-z'|)+\frac{\varepsilon(\omega)\varkappa-(\varkappa^2+\varkappa_D{}^2)^{1/2}}{\varepsilon(\omega)\varkappa+(\varkappa^2+\varkappa_D{}^2)^{1/2}}\mathscr{E}(z+z')\Big],$$

where $\mathscr{E}(x) = \exp\{-x(\kappa^2 + \kappa_D^2)^{1/2}\}$. In coordinate space this corresponds to

$$V_\omega(\rho=0,z,z')=e^2\Big\{\frac{\exp\{-\varkappa_D|z-z'|\}}{|z-z'|}$$
$$+\frac{4\pi}{(2\pi)^2}\int\frac{d\varkappa}{(\varkappa^2+\varkappa_D{}^2)^{1/2}}\frac{\varepsilon(\omega)\varkappa-(\varkappa^2+\varkappa_D{}^2)^{1/2}}{\varepsilon(\omega)\varkappa+(\varkappa^2+\varkappa_D{}^2)^{1/2}}\mathscr{E}(z+z')\Big\}.$$

According to the calculations, that part of the interaction which depends on z + z', attenuates like $\exp\{-\kappa_D(z + z')\}$ with increasing distance from the boundary.

The macroscopic description used here for the interaction is suitable only for scales greatly exceeding the interatomic distances. On the other hand, in the case of interest to us, when $L \lesssim 10$ Å, effects of periodicity of the lattice should be significant and such an approach, of course, will not give an exact description. We can hope, however, that the most essential features of the phenomenon still fit the model description introduced above, as is evidenced also by the two limiting cases for V_ω as $L \to 0$ and $L \to \infty$.

4. We have obtained above a certain model description for the interaction of electrons in a film surrounded by a dielectric. We shall subsequently be interested in a calculation of the temperature at which such a film becomes superconducting. As the zeroth-approximation Hamiltonian H_0 we choose the Hamiltonian of free electrons in a film. Owing to the finite thickness of the film, the projection of the momentum on the z axis will run through a discrete set of values with $\Delta p_z \approx \pi/L$ and $\psi_{\kappa i}(\rho, z) = e^{i\kappa\rho}\psi_i(z)$, where i is the number of the discrete value p_z. Then $\hat{H}_{int}(t)$, which corresponds to the model interaction of the electrons (3) introduced above, will have in the interaction representation the form

$$\hat{H}_{int}(t)=\int dt'\sum a^+_{\varkappa_1 i_1 s_1}(t)a^+_{\varkappa_2 i_2 s_2}(t')V\tag{4}$$
$$(t-t',\varkappa,i_1,i_2,i_3,i_4)a_{\varkappa_2-\varkappa,i_3,s_2}(t')a_{\varkappa_1-\varkappa,i_4,s_1}(t),$$

where s_1 and s_2 are the spin indices, the operators $a^+_{\kappa,i,s}(t)$ etc. are taken in the interaction representation, and the summation in (4) is carried out over i_1, i_2, i_3, i_4, κ_1, κ_2; κ, s_1, s_2. The interaction-energy matrix element is

$$V(t-t',\varkappa,i_1,i_2,i_3,i_4)=\int_0^L dz\int_0^L dz'\psi_{i_1}{}^*(z)\psi_{i_2}{}^*(z')V(t-t',\varkappa,z,z')\psi_{i_3}(z')\psi_{i_4}(z),$$

where $V(t, \kappa, z, z')$ is obtained from (3) by taking the inverse Fourier transform.

The calculation of the transition temperature with $\hat{H}_{int}(t)$ in the form (4) at an arbitrary film thickness is quite complicated, owing to the spatial inhomogeneity of the problem. We confine ourselves to the simpler case when the film thickness $L < \pi/k_F$ and the electron motion in it can be regarded as two-dimensional. In this case all that remains of the sum over i in (4) is the one term corresponding to $p_z = 0$, and $\hat{H}_{int}(t)$ takes the form

$$\hat{H}_{int}(t)=\int dt'\sum a^+_{\varkappa_1 s_1}(t)a^+_{\varkappa_2 s_2}(t')V_{eff}(t-t',\varkappa)a_{\varkappa_2-\varkappa,s_2}(t')a_{\varkappa_1-\varkappa,s_1}(t),\tag{5}$$

where the summation is over κ_1, κ_2, κ, s_1 and s_2, and where

$$V_{eff}(t-t',\varkappa)=\int_0^L\int_0^L dz\,dz'\psi^*(z)\psi^*(z')V(t-t',\varkappa,z,z')\psi(z')\psi(z).$$

In very thin conducting films it is natural to assume that $\psi(z)$ is constant[1)] and then, using (3), we obtain

$$V_{eff}(\omega,\varkappa)=\int d\tau\,e^{-i\omega\tau}V_{eff}(\omega,\tau)=\frac{1}{L^2}\int_0^L dz\int_0^L dz'V_\omega(\varkappa,z,z')$$
$$=\frac{4\pi e^2}{\mathscr{L}^2(\varkappa^2+\varkappa_D{}^2)^{1/2}}\frac{\omega^2}{\omega^2-\Omega^2}\begin{cases}\mathscr{L}-2/Q\left(\operatorname{ctg}\dfrac{\mathscr{L}Q}{2}-\dfrac{Q}{\varepsilon(\omega)\Omega}\right), & Q^2>0\\ \mathscr{L}-2/|Q|\left(\operatorname{ctg}\dfrac{\mathscr{L}|Q|}{2}+\dfrac{|Q|}{\varepsilon(\omega)\Omega}\right), & Q^2<0\end{cases};\tag{6}$$
$$\mathscr{L}=L(\varkappa^2+\varkappa_D{}^2)^{1/2},\quad Q=\left(\frac{\omega^2-\Omega^2}{\omega_i{}^2-\omega^2}\right)^{1/2},\quad \Omega=\frac{\varkappa}{(\varkappa^2+\varkappa_D{}^2)^{1/2}}.\tag{6}$$

Here, as everywhere else in the article, we use the causal function of the effective electron interaction, and

[1)]The boundary condition $\psi|_{z = \pm L} = 0$ leads in the limit as $L \to 0$ to vanishing of the conducting properties, $\psi(z) \to 0$. Our assumption $\psi(z) = \text{const}$ corresponds to the boundary condition $d\psi/dz|_{z = \pm L} = 0$. In the latter case the influence of the boundary turns out to be the largest.

stipulate accordingly the rule for going around the poles.

For further analysis of the system with the effective interaction calculated above we need to specify concretely the permittivity $\epsilon(\omega)$ of the dielectric. To understand the influence of the dielectric coating on the superconducting properties of the film, it suffices to use for $\epsilon(\omega)$ the simple expression

$$\varepsilon(\omega)=1+\frac{(\varepsilon_0-1)\,\omega_1^2}{\omega_1^2-\omega^2-i\delta},\quad \varepsilon(0)=\varepsilon_0, \tag{7}$$

where δ is a positive infinitesimally small quantity. Such a choice of $\epsilon(\omega)$ corresponds to a medium without absorption, having a natural oscillation frequency $\omega^{(e)}=\omega_1\sqrt{\epsilon_0}$. In most cases one can assume for electron excitations that $\omega_1\gg\omega_i$. In addition, in order for the exciton exchange to be more effective it is necessary that the exciton transition be sufficiently strong, i.e., the quantity proportional to the transition oscillator strength should be large: $\sqrt{\epsilon_0^2-1}\gg 1$. We shall therefore assume from now on that the parameters in (7) satisfy the following inequalities:

$$\omega_i\ll\omega_1\ll\omega_1\sqrt{\varepsilon_0}\ll\varepsilon_F. \tag{8}$$

5. In a system with interaction of the type (6) there can exist excitations of different types.

A. Phonon excitations at $\omega_i>\omega_n^{(ph)}(\kappa)>\omega_i\Omega$. They are determined from the equation

$$\operatorname{ctg}\frac{\mathscr{L}Q}{2}=\frac{Q}{\varepsilon(\omega)\Omega}\approx\frac{Q}{\varepsilon_0\Omega}. \tag{9}$$

This equation has an infinite set of solutions (see Fig. 2):

$$Q_n(\varkappa)=\frac{2\pi n}{\mathscr{L}}+\frac{\pi}{\mathscr{L}}\xi_n(\varkappa),$$

where n = 0, 1, 2, ..., and $0<\xi_n(\kappa)<1$. The number ξ_n decreases with increasing n and $\xi_n\to 0$ as $n\to\infty$. These solutions correspond to an infinite number of phonon oscillations, which can propagate across the film. We recall that the "jellium" model describes a metal as a continuous medium and therefore the phonon momenta can be arbitrarily large.

The values of importance for superconductivity are $\Omega\sim 1$. If $\mathscr{L}\gg 1/\epsilon_0$, then $Q_0\approx\pi/\mathscr{L}$. Expanding $\cot(\mathscr{L}Q/2)$ in the vicinity of the point $Q=\pi/\mathscr{L}$, we readily find that

$$Q_0(\varkappa)\approx\frac{\pi}{\mathscr{L}}\left(1-\frac{2}{\mathscr{L}\varepsilon_0\Omega}\right). \tag{10}$$

When $\mathscr{L}\ll 1/\epsilon_0$, as seen from Fig. 2, we have $Q\mathscr{L}\ll 1$. Expanding in this case $\cot(\mathscr{L}Q/2)$ near the point Q = 0, we obtain

$$Q_0(\varkappa)\approx\left[\frac{2}{\mathscr{L}}\Big/\left(\frac{1}{\varepsilon_0\Omega}+\frac{\mathscr{L}}{6}\right)\right]^{1/2}. \tag{10'}$$

Phonon excitations with frequencies $\omega_n^{(ph)}(\kappa)$ correspond to coupling constants

$$a_n^{(ph)}(\varkappa)=\lim\,[V_{\text{eff}}\,(\omega,\varkappa)(\omega-\omega_n^{(ph)}(\varkappa))]\,\frac{1}{\omega_n^{(ph)}(\varkappa)}$$
$$=\frac{4\pi e^2}{\mathscr{L}^2(\varkappa^2+\varkappa_D^2)^{1/2}}\frac{2}{A(Q_n)}\quad\text{as}\quad\omega\to\omega_n^{(ph)}(\varkappa),$$
$$A(Q_n)=Q_n^2(1+Q_n^2)\left[\frac{1}{\varepsilon_0\Omega}+\frac{\mathscr{L}}{2}\left(1+\frac{Q_n^2}{\varepsilon_0^2\Omega_0}\right)\right]. \tag{11}$$

We shall subsequently need the quantity $\sum_n a_n^{(ph)}(\kappa)$ at $\kappa\sim 2k_F$. To calculate this quantity we put $Q_n=\pi(2n+1)/\mathscr{L}$ if $\mathscr{L}\gg 1/\epsilon_0$, which results, as can be readily seen from Fig. 2 and formula (10), in a relative error on the order of $1/\epsilon_0\Omega\mathscr{L}\ll 1$, which can be neglected. To sum the obtained series, we rewrite it in the form of an integral

$$\sum_n a_n^{(ph)}(\varkappa)=\int_C\frac{dQ}{\mathscr{L}^2}\frac{4\pi e^2}{(\varkappa^2+\varkappa_D^2)^{1/2}}\frac{2}{A(Q)}\frac{1}{4\pi\left[\exp(iQ\mathscr{L}-i\pi)-1\right]}. \tag{12}$$

The contour C is shown by the continuous line in Fig. 3; A(Q) is defined in (11).

The integrand of (12) has, in addition to simple poles, a multiple pole on the real axis at the point Q = 0 and simple poles at the points $Q=\pm i$ and $Q=\pm iq$, where $q^2=\epsilon_0^2\Omega^2(1+2/\epsilon_0\Omega\mathscr{L})\gg 1$. By deforming the contour, as shown by the dashed lines in Fig. 3, we reduce the integral to residues at the points Q = 0, $Q=\pm i$, and $Q=\pm iq$. As a result we obtain

$$\sum_n a_n^{(ph)}(\varkappa)\approx\frac{4\pi e^2}{\mathscr{L}^2(\varkappa^2+\varkappa_D^2)^{1/2}}\frac{\mathscr{L}-2\,\text{th}(\mathscr{L}/2)}{2}. \tag{13}$$

If $\mathscr{L}\ll 1/\epsilon_0$, then in the calculation of the quantity

$$\sum_{n=0}^{\infty}a_n^{(ph)}(\varkappa)$$

it suffices to retain the term with n = 0. Using in this case expression (10′) for $Q_0(\kappa)$, we obtain

$$\sum_n a_n^{(ph)}(\varkappa)\approx a_0^{(ph)}(\varkappa)\approx\frac{4\pi e^2}{(\varkappa^2+\varkappa_D^2)^{1/2}}\frac{1}{4\varepsilon_0\Omega}. \tag{11'}$$

B. Exciton branch of excitations at $\omega\gg\omega_i$ with

$$\omega^{(e)}(\varkappa)=\omega_1\left[\frac{1+\varepsilon_0\Omega\,\text{cth}\,\mathscr{L}/2}{1+\Omega\,\text{cth}\,\mathscr{L}/2}\right]^{1/2}.$$

This branch, of course, does not exist if $\epsilon(\omega)=1$, i.e., if the metallic film is situated in a vacuum and is not surrounded by a dielectric.

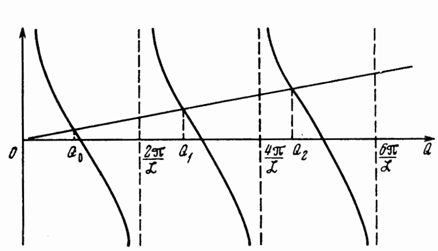

FIG. 2

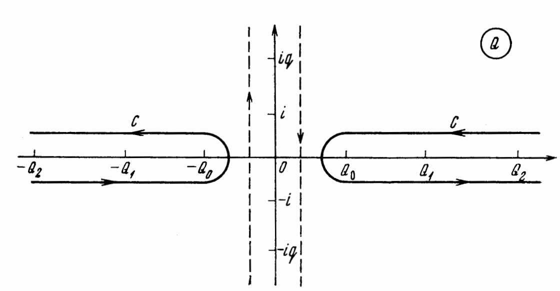

FIG. 3

The coupling constant of the exciton excitations is given by

$$a^{(e)}(\varkappa) = \lim_{\omega\to\omega^{(e)}(\varkappa)} [V_{\text{eff}}(\omega, \varkappa)(\omega - \omega^{(e)}(\varkappa))] \frac{1}{\omega^{(e)}(\varkappa)} \approx \frac{4\pi e^2}{\mathscr{L}^2(\varkappa^2+\varkappa_D^2)^{1/2}} \frac{(\varepsilon_0-1)\Omega}{[1+\varepsilon_0\Omega\,\mathrm{cth}(\mathscr{L}/2)][1+\Omega\,\mathrm{cth}(\mathscr{L}/2)]}. \quad (14)$$

6. To find the critical temperature in our system, we use the results of[4], where it is shown that if the electron interaction depends only on the transferred energy and momentum, then the transition temperature can be obtained in the weak-coupling approximation from the equation

$$\psi(\xi) = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\xi'}{\xi'}\,\mathrm{th}\frac{\xi'}{2T_c}W(\xi,\xi')\psi(\xi'), \quad (15)$$

where

$$W(\xi,\xi') = N(\xi)\left\{\overline{V_{\text{eff}}(\omega=\infty,\varkappa)} + \frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}\,\overline{V_{\text{eff}}(E,\varkappa)}\,dE}{E+|\xi|+|\xi'|}\right\}.$$

For two-dimensional systems, the density of states in the weak-coupling approximation is of the form $N(\xi) = m/4\pi^2$, and the superior bar denotes averaging over the angles, so that

$$\overline{f(\varkappa)} = \int_{(\xi-\xi')/v_F}^{2k_F} \frac{d\varkappa f(\varkappa)}{(4k_F^2-\varkappa^2)^{1/2}}.$$

It is convenient to change over from (15) to the equation

$$\psi(\xi) = -\frac{1}{2}\int_{-\bar{\omega}}^{\bar{\omega}}\frac{d\xi'}{\xi'}\,\mathrm{th}\frac{\xi'}{2T_c}V_{\text{sc}}(\xi,\xi')\psi(\xi'), \quad (16)$$

the kernel of which is given by

$$V_{\text{sc}}(\xi,\xi') = W(\xi,\xi') - \frac{N(\xi)N(0)\mathscr{V}|_{\xi'=0}\mathscr{V}|_{\xi=0}\ln(\bar{\xi}/\bar{\omega})}{1+\mathscr{V}|_{\xi=\xi'=0}\ln(\bar{\xi}/\bar{\omega})},$$

where $\mathscr{V} = \overline{V_{\text{eff}}(\omega=\infty,\kappa)}$, and the integration limits are determined by the formula

$$\ln\bar{\omega} = \sum_\nu \overline{a_\nu(\varkappa)\ln\omega_\nu(\varkappa)} \Big/ \sum_\nu \overline{a_\nu(\varkappa)}, \quad (17)$$

where $\omega_\nu(\kappa)$ are the frequencies of the excitations in the system, and $a_\nu(\kappa)$ are the corresponding coupling constants. The frequency $\bar{\xi}$ in the expression for $V_{\text{sc}}(\xi,\xi')$ is of the order of ϵ_F and depends only on $V_{\text{eff}}(\omega=\infty,\kappa)$.

Equation (16) determines the critical temperature, which is given, apart from a numerical factor on the order of unity, by the expression

$$T_c = \bar{\omega}\exp\{1/V_{\text{sc}}(0,0)\}. \quad (18)$$

Let us apply the foregoing theory of[4] to our problem. We note first that in our case

$$V_{\text{eff}}(\omega=\infty,\varkappa) = \frac{4\pi e^2}{\mathscr{L}^2(\varkappa^2+\varkappa_D^2)^{1/2}}\left(\mathscr{L} - \frac{2\Omega}{1+\Omega}\right)$$

and does not depend on the presence of the dielectric coating. From the spectral representation for $V_{\text{eff}}(\omega,\kappa)$ it follows that

$$W(0,0) = \left[\overline{V(\omega=\infty,\varkappa)}\Big|_{\xi=\xi'=0} + \frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}\,\overline{V(E,\varkappa)}\,dE}{E}\Big|_{\xi=\xi'=0}\right]N(0) = N(0)\overline{V_{\text{eff}}(\omega=0,\varkappa)}\Big|_{\xi=\xi'=0} = 0,$$

since $V_{\text{eff}}(\omega=0,\kappa) = 0$ in the "jellium" model. Therefore $V_{\text{sc}}(0,0)$ takes the form

$$V_{\text{sc}}(0,0) = -\frac{\mathscr{V}^2|_{\xi=\xi'=0}N^2(0)\ln(\bar{\xi}/\bar{\omega})}{1+\mathscr{V}|_{\xi=\xi'=0}\ln(\bar{\xi}/\bar{\omega})}. \quad (19)$$

As seen from (19), the dielectric affects $V_{\text{sc}}(0,0)$ only via $\ln\bar{\omega}$. This dependence is quite weak, so that we can assume with good accuracy that T_c depends on the presence of the dielectric coating exclusively via the pre-exponential factor in (18), and $V_{\text{sc}}(0,0)$ is practically the same for films with and without coating.

As shown in[4], when $\bar{\omega}$ is calculated for three-dimensional systems we can assume with logarithmic accuracy that

$$\sum_\nu \overline{a_\nu(\varkappa)\ln\omega_\nu(\varkappa)} \Big/ \sum_\nu \overline{a_\nu(\varkappa)} = \sum_\nu a_\nu(2k_F)\ln\omega_\nu(2k_F) \Big/ \sum_\nu a_\nu(2k_F). \quad (20)$$

Detailed calculations show that formula (20) also remains valid with the same degree of accuracy for two-dimensional systems. Then, using expressions (13) and (14) obtained above for $\sum_n a_n^{(ph)}(\kappa)$ and $a^{(e)}(\kappa)$, we obtain for $\mathscr{L}_0 \gg 1/\epsilon_0\Omega_0$:

$$\ln\bar{\omega} = \frac{a^{(e)}(2k_F)\ln\omega^{(e)}(2k_F) + \Sigma^{(ph)}\ln\bar{\omega}^{(ph)}}{a^{(e)}(2k_F)+\Sigma^{(ph)}} \approx \frac{2}{\mathscr{L}_0(1+\Omega_0)}\ln\omega^{(e)}(2k_F) + \left[1 - \frac{2}{\mathscr{L}_0(1+\Omega_0)}\right]\ln\bar{\omega}^{(ph)},$$

where

$$\Sigma^{(ph)} = \sum_n a_n^{(ph)}(2k_F), \quad \mathscr{L}_0 = L(4k_F^2+\varkappa_D^2)^{1/2}, \quad \Omega_0 = \frac{2k_F}{(4k_F^2+\varkappa_D^2)^{1/2}}$$

and we have introduced the average phonon frequency $\bar{\omega}^{(ph)} \approx \omega_i$.

For a film having no dielectric coating we have $\bar{\omega} = \bar{\omega}^{(ph)}$ with accuracy on the order of $\omega_i/\mathscr{L}_0$, as can readily be seen from Fig. 2. We obtain ultimately

$$\frac{(T_c)_{\text{coated}}}{(T_c)_{\text{uncoated}}} \approx \left[\frac{\omega^{(e)}(2k_F)}{\bar{\omega}^{(ph)}}\right]^{2/\mathscr{L}_0(1+\Omega_0)} \quad (21)$$

Let us obtain numerical estimates. At L = 3 Å and $k_F = 1\ \text{Å}^{-1}$, as is typical of metals, and $\omega^{(e)}/\bar{\omega}^{(ph)} = 30$ we have $\mathscr{L}_0(1+\Omega_0) \approx 15$ and $(T_c)_{\text{coated}}/(T_c)_{\text{uncoated}} \approx 1.5$. Thus we see that a dielectric coating changes the transition temperature of metallic films relatively little.

7. To what extent is the result (21) natural? It is shown in[4], with the generalized "jellium" model as an example, that the presence in a metal of exciton polarizability

$$\alpha^{(e)}(\omega) = \frac{1}{\pi}\frac{(\varepsilon_0-1)\omega_i^2}{\omega_i^2-\omega^2}$$

lead to coupling constants $a^{(e)} = 1 - 1/\epsilon_0$ and $a^{(ph)} = 1/\epsilon_0$. In our case, when the system consists of a metallic film coated with a dielectric, we can expect $a^{(e)} \sim 1$ and $a^{(ph)} \ll 1$ in a narrow layer near the metal-dielectric boundary, with a thickness on the order of the characteristic screening length R_{scr}. For the remaining mass of the metal we have $a^{(e)} \ll 1$ and $a^{(ph)} \sim 1$. On the whole, the system will have $a^{(ph)}/a^{(e)} \sim L/R_{scr}$ and

$$\frac{(T_c)_{\text{coated}}}{(T_c)_{\text{uncoated}}} \sim \left[\frac{\omega^{(e)}}{\omega_i}\right]^{\gamma R_{scr}/L}, \quad (22)$$

where γ is a numerical coefficient on the order of unity.

For simplicity, we have considered a very thin film with two-dimensional electron motion. In this case the screening acquires a unique character because of the two-dimensional electron motion, with $R_{scr} = (\kappa^2 + \kappa_D^2)^{-1/2}$ and $\kappa \sim 2k_F$. Exactly the same result

$$a^{(ph)} / a^{(e)} \sim \mathscr{L}_0 = (4k_F^2 + \varkappa_D^2)^{1/2} L = L / R_{scr}$$

was also obtained from the calculations.

Let us consider also the case when $\mathscr{L}_0 \ll 1/\epsilon_0$. In this case the exciton mechanism of superconductivity should play the principal role. Indeed, as can readily be seen from (10′), (11′), and (14), for such very thin films we have

$$\frac{(T_c)\,\text{coated}}{(T_c)\,\text{uncoated}} \approx \left(\frac{\omega^{(e)}}{\omega_i}\right)^{(1-1/\epsilon_0)} \approx \frac{\omega^{(e)}}{\omega_i},$$

which agrees with Ginzburg's result[2].

8. We have found above that the transition temperature in thin superconducting films with two-dimensional electron motion depends exponentially on the ratio R_{scr}/L.[2)] If this ratio is small, then the increase of T_c of the films as a result of the influence of the dielectric coating turns out to be insignificant. For a noticeable increase of T_c of the films it is therefore necessary not only to choose the dielectric correctly, but also to choose a material for the superconducting film such that R_{scr} is sufficiently large, as is the case, for example, in doped semiconductors at low carrier effective mass.

The foregoing numerical estimates show that to obtain an appreciable increase of T_c of a dielectric-coated film is not an easy task. It must be borne in mind, however, that these estimates were obtained on the basis of a concrete model in which the metal is described as a continuous medium up to scales on the order of atomic dimensions. When a more realistic electron interaction in the metal is employed, it is quite probable that the resultant numerical coefficients will be such that although the functional dependence will still retain the form (22), one can hope to obtain numerically an appreciable increase of the ratio $(T_c)_{coated}/(T_c)_{uncoated}$ at a large value of γ.

We have discussed above thin films with $L < \pi/k_F$. Of greatest interest for practical applications are films with larger thickness and three-dimensional electron motion. In this case the expressions obtained by us are not applicable directly, but we can expect formula (22) to remain in force, with $1/\kappa_D$ taking the place of R_{scr}.

It must be stated here that it was not the purpose of the present paper to obtain reliable quantitative results. We attempted to find an approach to the important and still unsolved problem of allowance for the influence of the boundaries on the superconducting properties of the films. In the future we hope to consider in greater detail some of the questions touched upon in this article.

In conclusion, we thank V. L. Ginzburg and the participants of the seminar under his direction for useful discussions.

[2)] A similar result was obtained in [13] for films with $L \gg \pi/k_F$ and in which the scale of variation of the electron interaction was much larger than the interatomic distances.

[1] V. L. Ginzburg, Usp. Fiz. Nauk 95, 91 (1968) [Sov. Phys.-Uspekhi 9, 355 (1968)].
[2] V. L. Ginzburg, ibid. 101, 185 (1970) [13, 335 (1970)].
[3] J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 108, 1175, 1957.
[4] D. A. Kirzhnits, E. G. Maksimov and D. I. Khomskiĭ, Preprint FIAN No. 108 (1970).
[5] P. C. Hohenberg, Phys. Rev. 158, 383, 1967.
[6] W. A. Little, Phys. Rev. 156, 396, 1967.
[7] V. L. Ginzburg and D. A. Kirzhnits, Zh. Eksp. Teor. Fiz. 46, 397 (1964) [Sov. Phys.-JETP 19, 269 (1964)].
[8] G. Rickayzen, Theory of Superconductivity, Ed. by R. E. Marshak, 14, New York, 1965.
[9] D. Pines, Elementary Excitations in Solids, Benjamin, 1963.
[10] P. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1965.
[11] J. R. Schrieffer, Theory of Superconductivity, Benjamin, 1964.
[12] A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 41, 569 (1961) [Sov. Phys.-JETP 14, 408 (1962)].
[13] D. A. Kirzhnits and E. G. Maksimov, Fiz. Met. Metallov. 22, 520 (1966).

Translated by J. G. Adashko
222