MAGNETOELASTIC WAVES IN THE PRESENCE OF DOMAIN STRUCTURE

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Magnetoelastic waves are considered in a magnetic material with a periodic plane-parallel domain structure. The wavelengths are much larger than the thickness of a domain wall, but small in comparison with the dimensions of the specimen. The wave spectrum is of the band type; the width of the forbidden bands is proportional to the magnetoelastic coupling parameter far from resonance, and to the square root of this parameter in the neighborhood of the magnetoacoustic resonance. The effect of the domain structure on the resonance is investigated. It is predicted that there will be a "surface" magnetoacoustic resonance, which will arise when there is interaction of ultrasound with the surface magnetostatic oscillations that exist near the domain walls.

1. THE interaction of elastic and magnetostatic oscillations has been investigated repeatedly (see, for example, the review [1]). It is usually supposed that the crystal is placed in such a strong magnetic field that domain structure is absent and the specimen is magnetized to saturation. Comparatively recently there have appeared the first experimental observations of magnetoelastic waves in the presence of domain structure.[2,3] In [2], in particular, natural magnetoelastic resonance in an effective anisotropy field was detected. Theoretical researches relating to the propagation of magnetoelastic waves in crystals with a domain structure, as far as we know, are lacking.

Our communication will treat the propagation of ultrasound in a magnetic material with a plane-parallel periodic domain structure, in the absence of an external magnetic field. Spin (exchange) waves in such a domain configuration were investigated in [4], magnetostatic oscillations in [5]. We assume that the wavelength λ of the elastic and magnetostatic oscillations satisfies the condition $L \gg \lambda \gg \delta$ (L is a dimension of the specimen, δ is the thickness of a domain wall). Then, first, the dimensions and shape of the magnet are unimportant; and second, it is reasonable to neglect the exchange energy and to treat the domain wall as a geometric boundary. The relation between the alternating components of the magnetic moment and the effective field that acts on the magnetic moment within a domain is given by the magnetic susceptibility tensor $\kappa_{ik}(\omega)$. At the domain boundaries the magnetic potential φ, the normal components of the magnetic induction and the stress tensor, and the tangential components of the elastic displacements must be continuous. Besides the boundary conditions, the magnetic and elastic variables must satisfy the conditions of translational invariance.

According to [5], in a magnetic material with a periodic domain structure there are "volume" magnetostatic oscillations, whose spectrum consists of permitted and forbidden bands, and two branches of "surface" oscillations. The spectrum of one of these (ω_{II}) lies below the spectrum of the volume waves; the spectrum of the other (ω_I) overlaps the spectrum of the volume spin waves. The elastic waves interact both with the "volume" and with the "surface" oscillations. But as was shown in [6], in the frequency range in which short-wave spin waves ($\lambda_c \sim \delta$) can propagate, in the investigation of surface magnetostatic oscillations it may prove necessary to take account of the spatial dispersion of the tensor $\kappa_{ik}(\omega)$ (and therefore to take account of the structure of the domain wall). This nontrivial problem will be the subject of a special discussion; in the present communication, we shall restrict ourselves to an investigation of magnetoelastic waves with frequencies less than ω_I.

We shall now proceed to the solution of the problem.

2. We shall for definiteness suppose that the crystal has cubic symmetry and an anisotropy constant $K_1 > 0$.[1)] 180-degree boundaries of plane-parallel domains are oriented in the XZ plane; the Y axis is perpendicular to the boundaries, whose coordinates are $y_n = nd$, $n = 0, \pm 1, \pm 2, \ldots$. The static magnetization is $\mathbf{M}_0 = \{0, 0, M_Z^0\}$; in adjacent domains $M_Z^0 = \pm M_0$, where M_0 is the saturation magnetization. In the presence of a saturating magnetic field parallel to one of the principal axes of symmetry, it is possible, as is well known, to separate the two simplest types of magnetoelastic waves:[1] transverse waves propagated along the magnetic field, and transverse waves polarized along the field and propagated perpendicularly to it. It is easily seen that in the model under consideration, purely transverse waves of the first type cannot exist, since the components of the induction and of the stress tensor normal to the domain boundary differ.

For this reason, we shall consider only propagation of elastic waves in the XY plane. The elastic displacement vector in the wave is $\mathbf{u} = \{0, 0, u(x, y)\}$; all quantities are independent of z.

The oscillations of magnetic quantities inside the domains are described by the equations of magnetostatics $\mathbf{h} = -\nabla\varphi$, $\operatorname{div}\mathbf{b} = 0$, $\mathbf{b} = \mathbf{h} + 4\pi\mathbf{m}$; the relation connecting $\mathbf{m}$, φ, and u is given by

$$m_x = -\varkappa_1\frac{\partial\varphi'}{\partial x} - i\varkappa_2\frac{\partial\varphi'}{\partial y}, \quad m_y = -\varkappa_1\frac{\partial\varphi'}{\partial y} + i\varkappa_2\frac{\partial\varphi'}{\partial x},$$

$$m_z \equiv 0, \quad \varphi' = \varphi + \frac{B\sigma}{M_0}u, \tag{1}$$

1)The results will be valid also for a magnetically uniaxial crystal, provided its elastic properties may be considered isotropic in the plane perpendicular to the axis.

where $\sigma = \pm 1$ when $\mathbf{M}_0$ is directed parallel and antiparallel, respectively, to the Z axis, and B is the magnetoelastic interaction constant introduced by Kittel[7] ($B \equiv B_2$). The components of the tensor κ_{ik}, as is well known, are

$$\varkappa_{xx} = \varkappa_{yy} = \varkappa_1 = \frac{\gamma M_0 \omega_a}{\omega_a^2 - \omega^2}, \quad \varkappa_{xy} = -\varkappa_{yx} = i\varkappa_2 = -\frac{i\gamma M_0 \sigma \omega}{\omega_a^2 - \omega^2};$$
$$1 + 4\pi\varkappa_1 \equiv \mu, \quad 4\pi\varkappa_2 \equiv \mu'.$$

The remaining κ_{ik} vanish. We use the notation $\omega_a = \gamma H_a = 2\gamma K_1/M_0$, where γ (> 0) is the gyromagnetic ratio. The equation for the elastic displacement in our case has the form

$$\omega^2 u + s^2 \Delta u + \frac{B\sigma}{M_0 \rho} \operatorname{div} \mathbf{m} = 0;$$

$s = (c_{44}/\rho)^{1/2}$ is the speed of transverse elastic waves, ρ is the density of the body, and c_{44} is the appropriate elastic constant. The basic equations of the problem are conveniently presented in the following form:

$$\mu \Delta \varphi + \frac{B\sigma}{M_0} 4\pi\varkappa_1 \Delta u = 0,$$
$$\omega^2 u + s^2 (1 - \eta\varkappa_1) \Delta u - \frac{B\sigma\varkappa_1}{\rho M_0} \Delta \varphi = 0, \tag{2}$$

where $\eta = B^2/s^2\rho M_0$ ($\ll 1$) is a dimensionless parameter of interaction between elastic and magnetostatic oscillations. At the domain walls, φ, u, and the normal components of the induction b_y and of the stress tensor

$$\sigma_{yz} = \rho s^2 \frac{\partial u}{\partial y} + \frac{B\sigma}{M_0} m_y.$$

must be continuous.

We note first of all that in the absence of magnetoelastic interaction (B = 0), there are purely periodic magnetostatic oscillations of the "wave-guide" type,

$$\varphi \sim \text{const} \cdot \sin(q_{l0} y) e^{ipx},$$

satisfying the boundary conditions for $q_{l0} = l\pi/d$, $l = 1, 2, \ldots$; d is the domain dimension. The frequency of the oscillations is determined by the condition $\mu(\omega) = 0$ and is equal to the frequency of transverse volume magnetostatic waves in the effective anisotropy field,

$$\omega = \omega_3 = \sqrt{\omega_a(\omega_a + \omega_M)}, \quad \omega_M = 4\pi\gamma M_0.$$

The analogous magnetoelastic oscillations, satisfying equations (2) and the boundary conditions, have the form

$$u_l = A \sin(q_{l0} y) e^{ipx},$$
$$\varphi_l = -A \frac{4\pi\varkappa_1}{\mu} \frac{B\sigma}{M_0} \sin(q_{l0} y) e^{ipx}. \tag{3}$$

The frequencies of oscillation $\omega_l(p)$ are determined by the relation

$$\frac{\omega_l^2}{s^2} = \frac{1}{2}\left\{\frac{\omega_3^2}{s^2} + q_{l0}^2 + p^2 \pm \left[\left(\frac{\omega_3^2}{s^2} - q_{l0}^2 - p^2\right)^2 + 4\eta \frac{\omega_a \gamma M_0 (q_{l0}^2 + p^2)}{s^2}\right]^{1/2}\right\}. \tag{4}$$

The expression (4) agrees in form with the equation for the frequency of elastic waves in a saturating field H_a, but the values of the transverse wave number q are quantized. The character of the oscillations (3)–(4) depends on the relation between ω_3 and the "geometric" frequency $\omega_g = sq_{10} = s\pi/d$. If $\omega_3 < \omega_g$, then the upper sign in (4) corresponds to a predominantly elastic wave, the lower to a predominantly magnetostatic. If, however, $\omega_3 > \omega_g$, then the one or several oscillations with the lowest indices l ($l\omega_g < \omega_3$) have the character of magnetoelastic waves. Resonance of elastic and magnetostatic oscillations occurs near $p_l = (\omega_3^2/s^2 - q_{l0}^2)^{1/2}$; for $p < p_l$ the oscillation with frequency ω_+ has magnetostatic character, for $p > p_l$ elastic. For the oscillation with frequency ω_-, the properties are reversed.

Periodic solutions of the type (3) are a special type of magnetoelastic waves in the model of a magnetic material under consideration. In the general case, the solutions are not purely periodic in the transverse coordinate, but have the character of Bloch waves. In order to find these solutions, we proceed as follows. Inside the domain with index r we seek u and φ in the form

$$u_r = \{A_r e^{iqy} + B_r e^{-iqy}\} e^{ipx},$$
$$\varphi_r = -\frac{4\pi\varkappa_1}{\mu} \frac{B\sigma}{M_0} u_r + \varphi^r_{\text{surf}}$$
$$\varphi^r_{\text{surf}} = \{C_r \exp(-|p|y) + D_r \exp(|p|y)\} e^{ipx}. \tag{5}$$

Such a form of solution implies that in the propagation of ultrasound there are excited not only "volume" but also "surface" magnetostatic oscillations,[5] satisfying the equation $\Delta\varphi_{\text{surf}} = 0$. On substituting (5) in (2), we find the relation among ω, q, and p:

$$q^2 + p^2 = \frac{\mu\omega^2}{s^2} \frac{1}{\mu - \eta\varkappa_1} \equiv \frac{\omega^2}{s^2} \frac{\omega_3^2 - \omega^2}{\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0}. \tag{6}$$

The solution (5) must satisfy the condition of translational invariance

$$\varphi(y + 2d) = \varphi(y) e^{2i\varkappa d}, \quad u(y + 2d) = u(y) e^{2i\varkappa d};$$

2d is the period of the magnetic structure, and κ is the transverse wave number, analogous to the quasimomentum in quantum-mechanical problems with a periodic potential. On using this condition and the boundary conditions on the two domain walls, we get a system of homogeneous equations for the appropriate coefficients A, B, C, and D. Then the requirement that the determinant of the system must vanish gives the relation among ω, q, p, and κ:

$$\begin{gathered} q^2(\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0)^2 (\cos^2 qd - \cos^2 \varkappa d)\, \Pi(\omega, p, \varkappa) \\ + 2\eta\omega_a^2\omega_M\gamma M_0 pq \sin qd \operatorname{sh} pd (\omega_4^2 - \omega^2)(\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0) \\ \times (\cos^2 \varkappa d - \operatorname{ch} pd \cos qd) \\ + \eta^2 p^2 \gamma^2 M_0^2 \sin^2 qd \{(\omega_4^2 - \omega^2) \operatorname{sh}^2 pd [\omega^4 + \omega_a^2\omega_M^2 - \omega^2(\omega_a^2 + 2\omega_a\omega_M)] \\ - \omega^2 \sin^2 \varkappa d (\omega_a^2 + 2\omega_a\omega_M - \omega^2)^2\} = 0, \\ \Pi(\omega, p, \varkappa) = (\omega_3^2 - \omega^2)^2 \sin^2 \varkappa d - (\omega_4^2 - \omega^2)(\omega^2 - \omega_a^2) \operatorname{sh}^2 pd. \end{gathered} \tag{7}$$

In (7) the notation $\omega_4 = \omega_a + \omega_M$ is used. Since (7) depends on κ periodically with period π/d, κ may be considered to vary within the limits of the "elementary cell" $-\pi/2d \leq \kappa \leq \pi/2d$. Since (6) and (7) do not change with change of sign of q, p, and κ, we shall restrict ourselves to values q, p, $\kappa > 0$. Equations (6) and (7) determine implicitly the dispersion of magnetostatic waves, $\omega(p, \kappa)$.

3. First let $\eta \equiv 0$. Then (7) splits into several equations, describing different, noninteracting waves. The equation $\cos qd = \pm \cos \kappa d$ together with (6) determines the spectrum of elastic waves:

$$\omega_n = s(q_n^2 + p^2)^{1/2},\ \bar{\omega}_n = s(\bar{q}_n^2 + p^2)^{1/2},\ \omega_0 = s(\varkappa^2 + p^2)^{1/2};$$
$$q_n = n\pi/d + \varkappa,\ \bar{q}_n = n\pi/d - \varkappa,\ q_0 = \varkappa,\ n = 1, 2, \ldots$$

The infinite set of branches ω_n, $\bar{\omega}_n$ which merge when $\kappa = 0$ and when $\kappa = \pi/2d$, forms a continuous spectrum $\omega = s(q^2 + p^2)^{1/2}$. The equation $\omega = \omega_3$ gives the volume magnetostatic oscillations described above.

Finally, from the condition $\Pi(\omega, p, \kappa) = 0$ is found the spectrum of surface magnetostatic oscillations.[2)] In particular, the low-frequency branch ω_{II} is determined by the equation

$$(\omega_3^2 - \omega^2)(\mathrm{sh}^2\, pd + \sin^2 \varkappa d)^{1/2} - \omega\omega_M\, \mathrm{sh}\, pd = 0$$

and is contained within the limits $\omega_a \leq \omega \leq \omega_3$.

Allowance for magnetoelastic interaction leads to a change of the spectra of elastic and of magnetostatic oscillations. The interaction is manifested most strongly on the lines of intersection of the unperturbed spectra.

We now suppose that $\eta \neq 0$. It is not difficult to see that the spectrum of elastic waves in the presence of a domain structure has a band character, with discontinuities on the band edges; that is, at $\kappa = 0$ and at $\kappa = \pi/2d$. In fact, far from resonance and from the band edges we can set $q = q_n - \delta q_n$ in (7), where $|\delta q| \ll q$, and neglect terms of order η^2 in (6) and (7). Then to terms of order η we find

$$\delta q_0 = \frac{\eta G(\omega, p, \varkappa)}{\varkappa d},\quad \delta q_n = \frac{\eta G}{q_n d},\quad \delta \bar{q}_n = \frac{\eta G}{\bar{q}_n d},$$
$$G(\omega, p, \varkappa) = \frac{\omega_a^2 \omega_M \gamma M_0 (\omega_4^2 - \omega^2)\, p\, \mathrm{sh}\, pd\, [\mathrm{ch}\, pd \mp \cos \varkappa d]}{(\omega_3^2 - \omega^2)\, \Pi(\omega, p, \varkappa)},$$
$$\omega^{(n)} \cong \omega_n - \frac{s^2 q_n \delta q_n(\omega_n)}{\omega_n} - \eta \frac{\gamma M_0 \omega_a \omega_n}{2(\omega_3^2 - \omega_n^2)},$$
$$\bar{\omega}^{(n)} \cong \bar{\omega}_n - \frac{s^2 \bar{q}_n \delta \bar{q}_n(\bar{\omega}_n)}{\bar{\omega}_n} - \eta \frac{\gamma M_0 \omega_a \bar{\omega}_n}{2(\omega_3^2 - \bar{\omega}_n^2)}. \tag{8}$$

The signs $\mp$ in the expression for G correspond to even (upper sign) and odd (lower sign) n. The last term in the formulas for $\omega^{(n)}$ and $\bar{\omega}^{(n)}$ gives the shift of frequency caused by the magnetoelastic interaction within the domains. To a small change of the frequency of elastic waves inside the band there corresponds, as can be seen without difficulty, also a weak scattering of the elastic waves by the magnetic inhomogeneities: the ratio of the amplitudes of the scattered and incident waves is $|B_n/A_n| \sim \eta$.

The expansion (8) is valid as long as the distance to the zone extremum $\kappa = 0$ or to the zone edge $\kappa = \pi/2d$ is sufficiently large: $\delta\kappa \gg \eta/d$. We shall now consider the dispersion at the singular points. We shall first set $\kappa = 0$. Then (7) breaks up into two equations:

$$\sin qd = 0;$$
$$\sin qd \approx -\frac{2\eta\omega_a^2\omega_M\gamma M_0(\omega_4^2 - \omega^2)\, p\, \mathrm{sh}\, pd (\ldots \cos qd - 1)}{q(\omega_3^2 - \omega^2)\, \Pi(\omega, p, 0)} \tag{9}$$

The first of equations (9) determines the periodic volume magnetoelastic waves already investigated earlier, (3) and (4). Such solutions, as was to be expected from general considerations, correspond to singular points of the permitted bands. The second of equations (9) together with (6) gives the frequency at the bottom (for $\omega_3 > \omega > \omega_a$) or at the maximum ($\omega < \omega_a$, $\omega > \omega_3$) of the nearest neighbors of the band:

$$q^{(n)} \approx q_{n0} - 2\delta q_n(\omega, p, \varkappa = 0),\quad \delta q_n(\varkappa = 0) = \delta \bar{q}_n(\varkappa = 0),$$
$$\omega^{(n)} \approx \omega_{n0} - \frac{2s^2 q_{n0}\delta q_n(\varkappa = 0)}{\omega_{n0}} - \eta \frac{\omega_a \gamma M_0 \omega_{n0}}{2(\omega_3^2 - \omega_{n0}^2)},$$
$$q_{(0)}^2 \approx \frac{2\eta\omega_a^2\omega_M\gamma M_0 p\, \mathrm{sh}\, pd\, \mathrm{th}\, pd/2}{(\omega_3^2 - \omega^2)(\omega^2 - \omega_a^2)},$$
$$\omega_{(0)} \approx sp + \frac{q_{(0)}^2}{2p} - \eta \frac{\omega_a \omega_M \gamma M_0 sp}{2(\omega_3^2 - s^2p^2)}. \tag{10}$$

Thus the distance $\delta\omega$ between the bottom of the higher band and the top of the lower, far from resonance, is

$$\delta\omega = 2|\delta q_n(\varkappa = 0)|\, s^2 q_{n0}/\omega_{n0},\quad n \neq 0.$$

We note that the solution u, φ corresponding to the dispersion (10) for $n \neq 0$, like (3), is purely periodic in y but contains an admixture of surface magnetostatic oscillations and is neither even nor odd with respect to $y = 0$. The amplitude of the reflected wave in each domain is equal in modulus to the amplitude of the incident sound; that is, the solution can be chosen real. The value of $\omega_{(0)}$ determines the frequency of sound waves propagated parallel to the domain boundaries along the X axis and weakly modulated with respect to the transverse coordinate.

Now let $\kappa d = \pi/2$. Equation (7) again splits into the product of two independent equations:

$$q(\omega_3^2 - \omega^2 \pm \omega\omega_M\, \mathrm{th}\, pd)(\omega_3^2 - \omega^2 - \eta\gamma M_0\omega_a) \cos qd$$
$$= \eta\gamma M_0 p \sin qd \{\omega_M(\omega_3^2 - \omega^2)\, \mathrm{th}\, pd \pm \omega(\omega_4^2 - \omega^2 - \omega_M^2)\}. \tag{11}$$

On setting $q = \bar{\bar{q}}_n - \delta q_n^{\pm}$, $\bar{\bar{q}}_n = (2n + 1)\pi/2d$, $n = 0, 1, 2, \ldots$ and using (6), we find

$$\delta q_n^{\pm} \approx -\frac{\eta\gamma M_0 p[\omega_M(\omega_3^2 - \omega^2)\, \mathrm{th}\, pd \pm \omega(\omega_4^2 - \omega^2 - \omega_M^2)]}{\bar{\bar{q}}_n(\omega_3^2 - \omega^2)(\omega_3^2 - \omega^2 \pm \omega\omega_M\, \mathrm{th}\, pd)}$$
$$\omega_n^{\pm} \approx \bar{\bar{\omega}}_n - \frac{2s^2\bar{\bar{q}}_n\delta q_n^{\pm}}{\bar{\bar{\omega}}_n} - \eta \frac{\omega_a\bar{\bar{\omega}}_n\gamma M_0}{2(\omega_3^2 - \bar{\bar{\omega}}_n^2)}, \tag{12}$$

$\bar{\bar{\omega}}_n = \omega_n(\kappa = \pi/2d) = \bar{\bar{\omega}}_{n+1}(\kappa = \pi/2d)$.

Schematic graphs of $\omega_n(\kappa)$ and $\bar{\omega}_n(\kappa)$ are given in Fig. 1. The solutions u, φ at the edges of the bands are periodic in y; their period is equal to the doubled period of the magnetic structure, 4d.

4. We shall consider the peculiarities of magnetoacoustic resonance (MAR) in the presence of domain

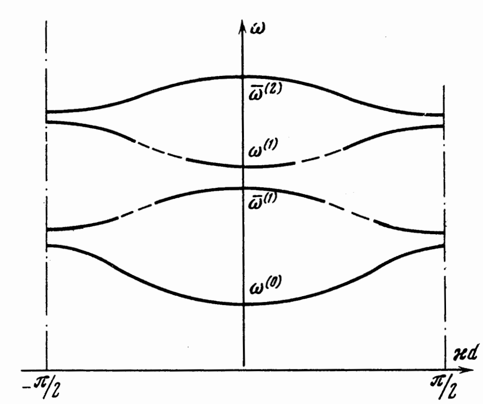

FIG. 1. Spectrum of elastic waves $\omega_n(\kappa)$ and $\omega_n(\kappa)$ for constant p. The neighborhoods of the frequencies of volume and surface magnetoacoustic resonance are shown dotted.

[2)] q = 0 corresponds to the trivial solution $\phi = u \equiv 0$.

structure. Near volume MAR we can set $\omega \approx \omega_3$ in all terms of (7) that do not contain the difference $\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0$. Then (6) breaks up into the product of two equations,

$$q(\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0)(\cos qd \pm \cos \varkappa d)\,\mathrm{sh}\,pd + \eta\omega_a\gamma M_0 p \sin qd\,(\mathrm{ch}\,pd \pm \cos \varkappa d) = 0. \quad (13)$$

The pair of equations (13) has solutions corresponding to even (−) and odd (+) band indices n. On setting

$$q = \begin{cases} q_n - \delta q_n \\ \bar{q}_n - \delta \bar{q}_n \end{cases}$$

we find

$$q_n\delta q_n = \bar{q}_n \delta\bar{q}_n \approx \frac{-\eta p\gamma M_0\omega_a(\mathrm{ch}\,pd \mp \cos\varkappa d)}{d(\omega_3^2 - \omega^2 - \eta\omega_a\gamma M_0)\,\mathrm{sh}\,pd}. \quad (14)$$

For investigating resonance far from the band edge, it is natural to introduce the wave number and the angle of propagation of the magnetoelastic wave by the relations

$$p = k_n \cos\theta_n, \quad q_n = k_n \sin\theta_n, \quad \bar{q}_n = k_n \sin\bar{\theta}_n. \quad (15)$$

The relations (15) actually signify a transition to the scheme of broadened bands usual for the case of weak coupling in quantum mechanics. From (6), (14), and (15) we obtain an equation that determines the frequency $\omega(k, \theta)$ near resonance:

$$k^2(\omega_3^2 - \omega^2 - \eta\gamma M_0\omega_a) + \frac{2\eta k\omega_a\gamma M_0\cos\theta[\mathrm{ch}(kd\cos\theta) - \cos(kd\sin\theta)]}{d\,\mathrm{sh}(kd\cos\theta)} = \frac{\omega^2}{s^2}(\omega_3^2 - \omega^2). \quad (16)$$

On setting $k = k_3 + \delta k$, where $|\delta k| \ll k_3 = \omega_3/s$, we find

$$\omega - \omega_3 \approx {}^1/_2\{s\delta k \pm [s^2\delta k^2 + \eta\omega_a\gamma M_0(1-\tau)]^{1/2}\},$$
$$\tau = \frac{2s\cos\theta}{\omega_3 d}\,\frac{\mathrm{ch}(\omega_3 d\cos\theta/s) - \cos(\omega_3 d\sin\theta/s)}{\mathrm{sh}(\omega_3 d\cos\theta/s)}. \quad (17)$$

The expression obtained for the dispersion of magnetoelastic waves near resonance is completely analogous to the well-known results for MAR in an external saturating field $H_{eff} = H_a$. The branches $\omega_\pm$ describe a transition from magnetostatic oscillations to elastic (ω_+) and from elastic to magnetostatic (ω_-) upon transition of δk from negative to positive values. The effect of the domain structure manifests itself formally in the substitution $1 \to 1 - \tau$ under the root in (17) and becomes vanishingly small when $\omega_3 ds^{-1}\cos\theta \gg 1$. The minimum distance between branches ($\delta k = 0$) is

$$\omega_+ - \omega_- = [\eta\omega_a\gamma M_0(1-\tau)]^{1/2}$$

and depends on the angle θ and the parameter $\omega_3 ds^{-1}$. As is seen from (17), the domain structure always decreases the distance between ω_+ and ω_- and, thereby, the width of the resonance at small attenuation.

Formula (14) and those following it are inapplicable in the vicinity of the extremes of the band, $\kappa = 0$. Near the resonance "Bragg" angles $\sin\theta_B = k_3 d/n\pi$, $|\theta - \theta_B| \lesssim \sqrt{\eta}$, the dependence $\omega(\theta)$ becomes more complicated. This fact is easy to understand if one takes into account that for $\theta \approx \theta_B$ the reflection of magnetoelastic waves from the domain boundaries rises sharply (the amplitude of the reflected wave inside a domain is $|B_r| \propto \eta|A_r|$ far from the Bragg angle, $|B_r| = |A_r|$ when $\theta = \theta_B$). Omitting the corresponding investigation, we present the result. At the band edge $\kappa = 0$ we have

$$\delta k \sin\theta_B + k_3\cos\theta_B\delta\theta = 0, \quad \delta\theta = \theta - \theta_B.$$

If we fix δk and change the angle $\delta\theta$, then the values of the resonance frequencies of each branch $\omega_\pm$ are different, depending on the sign of $\delta\theta - \delta\theta_{cr}$, $\delta\theta_{cr} = -k_3^{-1} \times \delta k\,\mathrm{tg}\,\theta_B$. If $\delta\theta - \delta\theta_{cr} \to +0$, then

$$\omega_- \to \omega_3 + (s\delta k - \delta\omega_1)/2, \quad \omega_+ \to \omega_3 + (s\delta k + \delta\omega_2)/2,$$
$$\delta\omega_1 = [s^2(\delta k)^2 + \eta\omega_a\gamma M_0(1-2\tau)]^{1/2}, \quad \delta\omega_2 = [s^2(\delta k)^2 + \eta\omega_a\gamma M_0]^{1/2}.$$

But if $\delta\theta - \delta\theta_{cr} \to -0$, then

$$\omega_- \to \omega_3 + {}^1/_2(s\delta k - \delta\omega_2), \quad \omega_+ \to \omega_3 + {}^1/_2(s\delta k + \delta\omega_1).$$

Formula (17) is valid only sufficiently far from the band edge, that is when

$$k_3 d\cos\theta_{cr}|\delta\theta - \delta\theta_{cr}|\,|\omega_3 - \omega| \gg \eta\omega_a\gamma M_0/\omega_3.$$

As is seen from (13), for $\kappa d = \pi/2$, in the lowest-order approximation with respect to the small parameter η, there are no gaps in the spectrum. In connection with this, the role of period of the structure, in the formula $\sin\theta_B = 2d/n\lambda$ for the "Bragg" angle, is played by the domain dimension, and not by the period 2d of the magnetic structure. From a physical point of view, this fact is not difficult to understand. Volume MAR is caused by magnetoelastic interaction within the domains, which is even with respect to the sign of the magnetization M_0. The presence of the structure leads to a change of the interaction but does not remove the degeneracy with respect to M_z^0. Far from the frequency of the volume MAR, the degeneracy is removed, and the period of the "lattice" is the period 2d of the magnetic structure.

We turn now to investigation of the surface MAR. We shall use the variables k and θ introduced in (15). The resonance frequencies are determined from the equation $\Pi(\omega, k, \theta) = 0$, if in it one sets $\omega = ks$. On going over to the dimensionless variables $x = kd\sin\theta$ and $\tilde{\omega}_i = s\omega_i d^{-1}\sin\theta$, we have

$$\sin^2 x = \frac{(\tilde{\omega}_4^2 - x^2)(x^2 - \tilde{\omega}_a^2)}{(\tilde{\omega}_3^2 - x^2)^2}\cdot \mathrm{sh}^2(x\,\mathrm{ctg}\,\theta) = f(x, \theta). \quad (18)$$

A graphic solution of this equation is shown in Fig. 2. The right member of (18) has the value $-\mathrm{ctg}^2\theta$ for $x = 0$, then increases monotonically together with its derivative, vanishing for $x = \bar{\omega}_a$ and becoming infinite for $x \to \bar{\omega}_3$. Thus for $\pi/2 - \theta \sim 1$, the equation has a single root, whose value depends on $\tilde{\omega}_a$, $\tilde{\omega}_3$, $\tilde{\omega}_4$, and θ (curve 1 in Fig. 2). For propagation angles θ close to $\pi/2$, that is for waves propagated almost perpendicularly to the domain boundaries, (18) can have three solutions, and there are three resonance frequencies. In principle, for $\Delta\theta = \pi/2 - \theta \ll 1$, $\tilde{\omega}_a \gg 1$, and $\tilde{\omega}_a\Delta\theta \lesssim 1$ the number of resonances may be even larger.

An interesting case is that of "double" resonance (curve 2 in Fig. 2), when at the point of intersection of the spectra of elastic and surface magnetostatic oscillations the group velocities also coincide:

$$s = d\omega_{II}/dk,$$
$$\omega_{II} = {}^1/_2\{[(\omega_4 + \omega_a)^2\,\mathrm{sh}^2(kd\cos\theta) + 4\omega_3^2\sin^2(kd\sin\theta)]^{1/2} - \omega_M\,\mathrm{sh}(kd\cos\theta)\}[\mathrm{sh}^2(kd\cos\theta) + \sin^2(kd\sin\theta)]^{-1/2}.$$

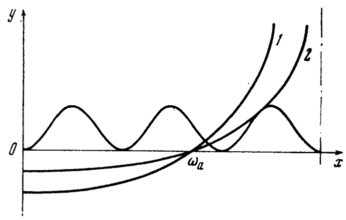

FIG. 2. Determination of the frequencies of surface MAR as points of intersection of the curves $y = \sin^2 x$ and $y = f(x, \theta)$. The graphs of $f(x, \theta)$ are given for two different values of the angle θ.

Then the dispersion curves near resonance have the form

$$\delta\omega = s\delta k - \tfrac{1}{4}\alpha_0(\delta k)^2 \pm \tfrac{1}{4}[\alpha_0^2(\delta k)^4 + 16\eta\beta\omega_a\gamma M_0]^{1/2},$$
$$\delta\omega = \omega - \omega_p,\quad \delta k = k - k_p,\quad \alpha_0 = d^2\omega_{\mathrm{II}}/dk^2, \tag{19}$$

where $\beta(>0)$ is a constant determined from (8).

For "double" resonance, in a comparatively broad range of wave numbers, $\delta k \sim (\eta\omega_a\gamma M_0/\alpha_0^2)^{1/4}$, there are two magnetoelastic branches with $v_{gr} \sim s$. It is not difficult to obtain a necessary condition for existence of "double" resonance. Since the derivative of the left member of (18) does not exceed unity in modulus, the same must be valid for $\delta_0 = f'(\omega_a)$. The dispersion of magnetoelastic waves with allowance for volume and surface resonances is given schematically in Fig. 3.

In conclusion, we note that in experiments[2] with yttrium ferrite-garnet, the condition $s\omega_a/d \gg 1$ was fulfilled. Under this condition, the frequency of surface MAR $\omega_{res} \sim \omega_a$, while the character of the volume resonance, as is seen from (17), is practically unaffected by the periodicity of the magnetic structure. The volume MAR for $s\omega_a/d \gg 1$ is determined solely by the magnetoelastic interaction within the individual domains. Precisely for this reason, explanation of the results[2] did not require information about the nature of the domain structure.

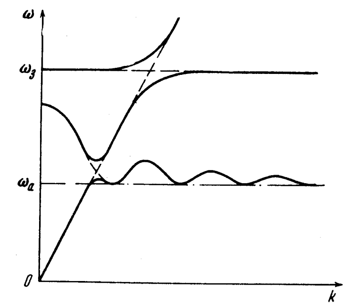

FIG. 3. Spectrum of magnetoelastic waves $\omega(k)$ for constant θ.

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[1] Fizicheskaya akustika (Physical Acoustics), collection of articles, **3B**, "Mir", 1968.
[2] V. V. Lemanov and A. V. Pavlenko, Zh. Eksp. Teor. Fiz. **57**, 1528 (1969) [Sov. Phys.-JETP **30**, 826 (1970)].
[3] A. W. Smith, J. Appl. Phys. **40**, 1979 (1969).
[4] M. M. Farztdinov, Fiz. Metallov i Metallovedenie **19**, 641 (1965) [Phys. Met. Metallog. **19**, No. 5, 1 (1965)]; M. M. Farztdinov and E. A. Turov, Fiz. Metallov i Metallovedenie **29**, 458 (1970) [Phys. Met. Metallog. **29**, No. 3, 11 (1970)].
[5] I. A. Gilinskiĭ and R. G. Mints, Zh. Eksp. Teor. Fiz. **59**, 1230 (1970) [Sov. Phys.-JETP **32**, 673 (1971)].
[6] L. N. Bulaevskiĭ, Fiz. Tverd. Tela **12**, 799 (1970) [Sov. Phys.-Solid State 12, 619 (1970)].
[7] C. Kittel, Rev. Mod. Phys. **21**, 541 (1949).

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