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DISCRETE LEVELS IN THE FORBIDDEN BAND OF A DISORDERED SEMICONDUCTOR

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The problem of the probability of appearance of discrete one- and two-electron levels in random force fields is considered. The conditions for which this probability is finite are indicated. It is concluded that there exists a sharp energy boundary between "current" and "non-conducting" electron states in random fields of a rather broad class. The total number of discrete levels in the forbidden band of a disordered semiconductor is estimated.

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

The problem of the energy spectrum of a disordered semiconductor has been examined repeatedly and from different points of view. Of special interest here is the region of energies corresponding to the forbidden band. Experimentally, it has clearly been established (cf., e.g., the survey[1]) that within the forbidden band of an amorphous substance there is a set of discrete levels, corresponding to localized states of the charge carriers. This is in agreement with the theoretical conclusions[2-4] that sufficiently deep levels of this type necessarily arise in practically any (semiconducting) disordered system. For shallower levels, however, the situation becomes less clear. It is also not clear to what extent the various model assumptions used in the different papers are essential. Finally, there is no theoretical estimate of the total concentration of discrete levels in various disordered systems.

In the present paper, we make an attempt to treat the problem of localized states in a disordered system from a somewhat different point of view. Namely, we shall seek the probability that discrete levels appear in a given random field[1]. If this probability turns out to be finite, then in a large system such levels will necessarily arise and their concentration will be proportional to the above probability. As will be seen from the following, this formulation of the problem evidently enables us to distinguish clearly the assumptions on the model character (these turn out to be associated with the statistical properties of the random field), and also to understand to what extent these assumptions are important.

We note that, generally speaking, these levels, if they exist, cannot be ascribed to any individual atoms or to any sharply defined aggregate of a small number of atoms: the levels can arise because of the finite probability of formation of the corresponding fluctuation-potential wells, i.e., as a result of collective effects. Furthermore, the existence of such levels is, generally speaking, in no way due to the absence of overlap between the wave functions of the electrons localized in the different wells, but is due to the random character of the levels. Under these conditions, overlap of the wave functions does not necessarily lead to the transformation of states belonging to L_2 into states of the continuous spectrum (here and below, the term "continuous spectrum" is to be understood in its purely mathematical sense, and not simply as a set of very closely spaced levels): because of the random spacing and random characteristics of the wells, the levels are not obliged to "resonate."

We shall consider the behavior of charge carriers in a macroscopically homogeneous and isotropic system. The possible periodic field is excluded by the standard effective-mass method,[2] in which for the beginning the effective mass will be assumed to be isotropic. In fact, this assumption is not very important in the present case (cf. Sec. 3).

2. STATISTICAL CHARACTERISTICS OF THE RANDOM FIELD

In a macroscopically homogeneous system, the average value of the potential energy V of a charge carrier in a random field is a constant. Consequently, it is sufficient to confine ourselves entirely to treating the fluctuations of a field with zero average value, putting

$$V(\mathbf{x})=\langle V\rangle+U(\mathbf{x}),\quad \langle U\rangle=0, \tag{1}$$

where the angular brackets denote averaging over the random field.

Denoting by $\mathscr{P}[U]$ the functional defining the probability of appearance of a given function $U(\mathbf{x})$, we obtain the obvious relation

$$\int\delta U\,\mathscr{P}[U]=1. \tag{2}$$

Physical considerations concerning regularity can lead to supplementary conditions to be imposed on the class of functions $U(\mathbf{x})$; however, we shall not need them.

The determination of the explicit form of $\mathscr{P}[U]$ is a problem in its own right; to solve it, it is necessary to introduce some or other assumptions of a model character. For the following, however, a phenomenological approach is convenient. Namely, all the average values of

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[1] It is obvious that such a formulation of the question has meaning only in the three-dimensional case.

[2] Expressions such as "effective mass," "dispersion law," etc., need not lead to misunderstandings: as always in such cases, we have in mind the corresponding characteristics of the auxiliary problem with a purely periodic field. As is well known [5], in the case of a macroscopically homogeneous system, such a problem can be uniquely defined.

interest to us can be calculated if we know the characteristic function

$$A(z) = \left\langle \exp\left\{-iz \int d\mathbf{k}\, U(\mathbf{k}) J(\mathbf{k})\right\}\right\rangle. \tag{3}$$

Here

$$U(\mathbf{k}) = \frac{1}{(2\pi)^3} \int d\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}}\, U(\mathbf{x}) \tag{4}$$

is the Fourier transform of the random function U(**x**), and J(**k**) = J*(−**k**) is a regular function.

In the case of a Gaussian field,

$$\mathscr{P}[U] = N \exp\left\{-\frac{1}{2} \int d\mathbf{k}\, U(\mathbf{k}) \Psi^{-1}(\mathbf{k}) U(-\mathbf{k})\right\}, \tag{5}$$

where N is a normalization factor, and Ψ(**k**) is the Fourier transform of the correlation function

$$\langle U(\mathbf{x}') U(\mathbf{x}'')\rangle \equiv \Psi(\mathbf{x}' - \mathbf{x}'') = \int d\mathbf{k}\, \Psi(\mathbf{k}) e^{i(\mathbf{k},\, \mathbf{x}' - \mathbf{x}'')}. \tag{6}$$

In this case,

$$A(z) = \exp\left\{-\frac{z^2}{2} \int d\mathbf{k} |J(\mathbf{k})|^2 \Psi(\mathbf{k})\right\}. \tag{7}$$

In a more general case, we put

$$A(z) = \exp\left\{-\frac{z^2}{2} \int d\mathbf{k} |J(\mathbf{k})|^2 \Phi(\mathbf{k})\right\} \sum_{n \geqslant 0} c_n H_n[F(z, J)], \tag{8}$$

where Φ is a function of the same class as Ψ, H_n are the Hermite polynomials, and c_n are certain coefficients, satisfying the conditions

$$\sum_{n \geqslant 0} c_n H_n(0) = 1, \quad c_{2m} \in \mathrm{Re}, \quad c_{2m+1} \in \mathrm{Im}, \tag{9}$$

$$F(z, J) = \sum_{l \geqslant 2} \frac{z^l}{l!} \int \delta(\mathbf{k}_1 + \ldots + \mathbf{k}_l) \Phi_l(\mathbf{k}_1, \ldots, \mathbf{k}_l) \prod_{i=1}^{l} J(\mathbf{k}_i) d\mathbf{k}_i, \tag{10}$$

and Φ_l are kernels subject only to the obvious regularity conditions. In particular, we shall assume that

$$\Phi_l(0, \ldots, 0) < \infty, \tag{11}$$

$$\Phi_l(0, \ldots, 0) \Phi^{-l/2}(0) l^{-3} \equiv \zeta_l < \infty \tag{12}$$

and for all finite values of z

$$\sum_{l \geqslant 2} \frac{z^l}{l!} \zeta_l \equiv S < \infty. \tag{13}$$

It is possible to represent functions of a fairly broad class in the form of the expansion (8). From the following, it will be clear that for the proof of the theorems of interest to us, only the properties (9)—(13) are important, and not the explicit form of the coefficients c_n.

We note certain obvious properties of the function Ψ(**k**). The condition of macroscopic isotropy and the normalization (2) give

$$\Psi(\mathbf{k}) = \Psi(-\mathbf{k}) \in \mathrm{Re}, \quad \Psi(\mathbf{k}) > 0. \tag{14}$$

Furthermore, we shall suppose that the correlation function (6) goes to zero only for $|\mathbf{x}' - \mathbf{x}''| \to \infty$. Then $\mathbf{k}^2\Psi(\mathbf{k}) < \infty$ for k → 0. We shall take a somewhat stronger condition:

$$\Psi(\mathbf{k}) \to \mathrm{const} \quad \text{as} \quad \mathbf{k} \to 0. \tag{15}$$

Finally, by definition,

$$\int \Psi(\mathbf{k}) d\mathbf{k} \equiv \psi_1 = \langle U^2\rangle < \infty. \tag{16}$$

In the framework of our formulation of the problem, the function Ψ(**k**) must be assumed known. Thus, in the case of a field created by randomly distributed atoms of a charged impurity,

$$\Psi(\mathbf{k}) = \frac{2Z^2 e^4 n_t}{\pi\varepsilon^2 (\mathbf{k}^2 + r_0^{-2})^2} \tag{17}$$

where Z, n_t, ϵ and r_0 are respectively the charge of an individual impurity atom in units of the electron charge, the impurity concentration, the dielectric constant of the substance and the screening radius.

The Fourier transforms of the functions Ψ, Φ and Φ_l are easily related. According to (8),

$$\Psi(\mathbf{k}) = \Phi(\mathbf{k}) - \Phi_2(\mathbf{k}, -\mathbf{k}).$$

Correspondingly,

$$\psi_1 = \varphi_1 - \int \Phi_2(\mathbf{k}, -\mathbf{k}) d\mathbf{k}, \tag{18}$$

where $\varphi_1 = \int d\mathbf{k}\Phi(\mathbf{k})$; by convention, $\varphi_1 < \infty$.

Condition (9) enables us to prove that the overwhelming majority of potentials U(**x**) are bounded. Indeed, we shall calculate the probability Q that U(**x**) is greater than a certain given value U_0 (of course, in a macroscopically homogeneous system, Q will not depend on **x**). Obviously,

$$Q = \langle \theta[U(\mathbf{x}) - U_0]\rangle, \tag{19}$$

where θ is the well-known step function. Using the standard integral representation for it and bearing (8) and (9) in mind, we easily find (for $U_0 \to \infty$)

$$Q \to \exp\{-U_0^2 / 2\varphi_1\}. \tag{20}$$

In an analogous way, we can also find the probability Q_1 that U(**x**) is less than ϵ, where $\epsilon \to +0$:

$$Q_1 = \frac{1}{2} + \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{ds}{s} \exp\left\{-\frac{s^2}{2}\varphi_1\right\} \sum_{n \geqslant 0} c_n H_n[F]. \tag{21}$$

In a Gaussian field, $Q_1 = 1/2$.

3. ONE- AND TWO-ELECTRON LEVELS

In this and the following paragraphs we shall consider the case of a Gaussian field.

In the formulation of the problem of interest to us, many-electron effects can be important only when several charge carriers fall into the same fluctuation well. In fact, in systems of the type under consideration, the concentration of free charge carriers is usually not great, so that effects such as the dependence of the effective mass on the concentration do not play a role. On the other hand, it makes sense (and is necessary) to take into account the screening of the potential only in the calculation of the correlation function Ψ(**k**) (cf. (17)).

Thus, being interested in the one-electron levels, we arrive at the standard problem of determining the possible energy values W of an electron in a random field V(**x**):

$$T\psi + U\psi = \lambda\psi. \tag{22}$$

Here

$$\lambda = W - \langle V\rangle,$$

and T is the kinetic energy operator of the charge carrier, the effective mass of which is m; the energy zero coincides with the edge of the corresponding band.

The localized states correspond to discrete energy levels, if they exist. Obviously, there is degeneracy with respect to the coordinates of the center of localization. For definiteness, we shall henceforth place this at the coordinate origin; expressions "at infinity" thereby also acquire meaning.

We denote by α^{-1} the characteristic dimension of the region of localization (this is independent of the volume of the system, provided that the latter is sufficiently great). Then the condition for the existence of discrete levels can be written in the form

$$\lambda < \nu_0 + O(1/\alpha r). \tag{23}$$

Here ν_0 is the smallest eigenvalue of the matrix

$$U_{lm;\,l'm'} = \oint d\Omega(\theta, \varphi)\, \bar{Y}_{l'}{}^{m'} Y_l{}^m U(r, \theta, \varphi), \tag{24}$$

where Y_l^m is a normalized spherical harmonic, and r, θ and φ are polar coordinates, with the value of r fixed and chosen in accordance with the condition $\alpha r \gg 1$.

It is obvious that condition (23) will certainly be fulfilled if we replace the quantity ν_0 by its minimum value ν_m on a sphere of radius r. Since it has been stipulated that the radius of the sphere is large only in comparison with α^{-1}, but is not macroscopically large, ν_m is by no means certain to coincide with the absolute minimum of the potential in the volume. Furthermore, by virtue of (20), $|\nu_m| < \infty$ with overwhelming probability. Below, we shall shift r to infinity.3)

The probability Q_b that a discrete level with energy λ appears is the probability that the inequality (23) is realized:

$$Q_b = \langle \theta[\nu_0 - \lambda] \rangle. \tag{25}$$

Since $\lambda = \lambda[U]$, the calculation of this average value in the general case is difficult. We note, however, that

$$\lambda \leqslant (\psi_{\alpha_1}(T + U)\psi_\alpha).$$

Here ψ_α is an arbitrary function of the class L_2 (also normalized to unity), which depends on a parameter or on an aggregate of parameters α. Replacing ν_0 by ν_m also, we only lower the required probability. Thus,

$$Q_b \geqslant \lim_{x \to \infty} \langle \theta[\nu_m - (\psi_{\alpha'}(T + U)\psi_\alpha)] \rangle \equiv \bar{Q}_b. \tag{26}$$

In contrast to (25), the form of the function ψ_α here is determined a priori and, consequently, the random quantity U appears only explicitly in (26). This makes it possible to calculate $\bar{Q}_b$ easily; it is convenient to make use of the momentum representation (we put here $\hbar = 2m = 1$). Proceeding as in the calculation of (19), we obtain

$$\bar{Q}_b = {}^1/{}_2\{1 - \mathrm{Erf}\, \eta_1\}, \tag{27}$$

where $\mathrm{Erf}\, \eta_1$ is the error function

$$\eta_1 = (a + |\nu_m|)/2b, \tag{28}$$

$$b^2 = \frac{1}{2} \int d\mathbf{k}\, \Psi(\mathbf{k}) B(\mathbf{k}) B(-\mathbf{k}), \tag{29}$$

$$a = \int |\psi_\alpha|^2 T(\mathbf{k})\, d\mathbf{k}, \quad B(\mathbf{k}) = \int d\mathbf{k}'\, \psi_\alpha^*(\mathbf{k}')\, \psi_\alpha(\mathbf{k}' - \mathbf{k}). \tag{30}$$

For obvious reasons, it is natural to take as the trial function $\psi_\alpha(\mathbf{k})$ the expression4)

$$\psi_\alpha(\mathbf{k}) = (8\alpha^5)^{1/2} / \pi(\alpha^2 + k^2)^2. \tag{31}$$

In this case, obviously,

$$\lambda - \nu_0 = -\alpha^2, \quad a = \alpha^2, \quad B = (1 + k^2/4\alpha^2)^{-2}. \tag{32}$$

Correspondingly, formula (28) gives

$$\eta_1 = (\alpha^{1/2} + |\nu_m|\alpha^{-3/2})/4 \left[\int d\mathbf{q}\, \Psi(2\alpha\mathbf{q})\, (1 + q^2)^{-4} \right]^{1/2}. \tag{33}$$

By virtue of (15), the denominator of (33) at sufficiently small (but finite) α is proportional to $\Psi(\mathbf{k})^{1/2}|_{\mathbf{k}=0}$. We see that the quantity η_1 is finite.

Thus, in the system under consideration, discrete levels are certainly formed in the forbidden band.5) It is not difficult to see that this result is not connected with the very simple form taken above for the iso-energetic surfaces close to the extremum.

As can be seen from (23), the eigenvalues of Eq. (22) are reckoned from the "random zero" ν_0. Naturally, in all phenomena in which only the differences between electron levels in the crystal are important, this circumstance plays no role, provided that the random field acting on the electrons and on the holes is one and the same.6) Accordingly, it makes sense to talk of sharp boundaries between the states of the discrete and continuous spectra.

On the other hand, in the macroscopic system that we are considering, the inequality $\alpha r \gg 1$ may refer to only a part of the sample. In this case, in the other parts the same energy value will correspond also to the continuous spectrum (in the thermodynamic limit, all these parts of the sample will be infinite). An example of such a force-field configuration is shown in the Figure.

It is clear that in this sense we can speak of the coexistence of a continuous and a discrete spectrum, corresponding (in different parts of the sample) to the same charge-carrier energy. We note, however, that in this case also, the density of states is not continuous in the range of energies considered. Accordingly, these levels are non-conducting: if at absolute zero temperature the Fermi level falls in the given region, then the static electrical conductivity of the substance will be equal to zero[6,7].

Our conclusion on the existence of a sharp energy boundary between the conducting and non-conducting states is in agreement with the assumptions of a number of authors (cf., e.g.,[8-10]), which were, however,

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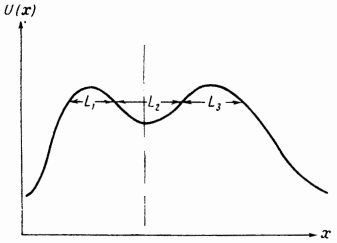
3) Taken literally, the limit of U(x) as x tends to some well-defined value may not exist: at any point in space the random potential may have an essential singularity (while remaining finite, by virtue of (20), with overwhelming probability). From the following, however, it will be clear that the expression of interest to us for the probability (26) that a discrete level exists always has meaning. We remark also that we are performing, in essence, two limiting processes: the first is expressible by the condition $\alpha r \gg 1$, and the second is the thermodynamic passage to the limit.

4) The function (31) may not be orthogonal to the eigenfunctions of the lower-lying discrete levels. To prove the existence theorem, this is of course, unimportant; however, it is not possible to calculate the density of states and the total number of states by this method. Below (Sec. 4), we shall estimate the total number of discrete levels by another method; this latter, however, has meaning only if we already know that the states under consideration exist.

5) The origin of the factor ½ in (27) is perfectly clear: in a Gaussian field, fluctuations with decrease and increase of potential energy are equally probable.

6) This condition, generally speaking, is not fulfilled when the field under consideration is caused by a deformation potential.



One of the possible configurations of a random field in which there is coexistence of a discrete and a continuous spectrum (obtained by rotation of the graph about the vertical axis indicated by the dashed line). The lengths L_1, L_2 and L_3 must be made to tend to infinity.

obtained by a different route. The approach taken above seems to be less bound up with hypotheses that are difficult to verify.

We turn now to the question of the probability of localization of two electrons in one fluctuation well. We shall confine ourselves to the Hartree approximation, bearing in mind that the role of inter-electronic repulsion is somewhat overestimated in this approximation; consequently, our conclusion on the possibility of formation of the corresponding levels will also be valid in a more exact solution of the problem of the interaction of the electrons. In place of Eq. (22), we now have the Schrödinger equation with the Hamiltonian

$$\mathscr{H}=\sum_{i=1,2}\{T_i+U(\mathbf{x}_i)\}+V_C(\mathbf{x}_1-\mathbf{x}_2). \tag{34}$$

Here the index i labels the electrons, and the potential energy of the Coulomb interaction between them is denoted by V_C. Since the dimensions of the region of localization can be rather large, it is better to write the expression for V_C with allowance for the screening:

$$V_C(\mathbf{x}_1-\mathbf{x}_2)=\frac{e^2}{\varepsilon|\mathbf{x}_1-\mathbf{x}_2|}\exp\left\{-\frac{|\mathbf{x}_1-\mathbf{x}_2|}{r_0}\right\},$$

where r_0 is the screening radius. The explicit expression for r_0 depends on the mechanism and conditions of the screening and is not important for the following.

We must now take the trial function ψ_α in the form of a product of the functions (31). Then for the probability that a bound two-electron state arises, we again obtain formula (27), in which, however, we must replace η_1 by

$$\eta_2=\frac{1}{4}\left(\alpha^{1/2}+\frac{5e^2}{16\varepsilon}\alpha^{-1/2}f(\alpha r_0)+|\nu_m|\alpha^{-3/2}\right)\left[\int d\mathbf{q}\Psi(2\alpha\mathbf{q})(1+q^2)^{-4}\right]^{-1/2}. \tag{35}$$

Here

$$f(y)=1-\frac{1}{2(1+2y)}\left[1+\frac{4y}{1+2y}+\frac{32y^2}{5(1+2y)^2}+\frac{32y^3}{5(1+2y)^3}\right]. \tag{36}$$

For finite values of α and $|\nu_m|$, the quantity η_2 is finite. We arrive at the same conclusions as in the case of the one-electron levels. Allowance for the Coulomb interaction between the electrons weakens the effect of the random field, but does not eliminate it.

It would also be possible to treat the problem of the formation of many-electron clusters in an analogous way: the corresponding probability turns out to be finite.

4. THE CONCENTRATION OF DISCRETE ONE-ELECTRON LEVELS

The exact calculation of the density of states and of the total number of discrete levels clearly requires rather far-reaching model assumptions. It is possible, however, to evaluate a lower bound for the total number of levels, by noting that among them there are also quasi-classical levels. The number ν of the latter is given by the known expression[11] (here we use the usual units):

$$\nu=\frac{(2m^3)^{1/2}}{3\pi^2\hbar^3}\int d\mathbf{x}[-U(\mathbf{x})]^{3/2}\theta[-U(\mathbf{x})]. \tag{37}$$

We need only average expression (37). That is easily done by means of formula (7). As we should expect, $\langle\nu\rangle$ turns out to be proportional to the volume Ω of the system; for the average concentration of levels, we obtain

$$\langle\nu\rangle/\Omega=\Gamma(5/4)\,m^{3/2}\psi_1{}^{3/4}/3\pi^{5/2}\cdot 2^{1/4}\hbar^3. \tag{38}$$

It is convenient to rewrite the right-hand side of (38) in the form

$$N_C(T^*)=2(2\pi mkT^*)^{3/2}/(2\pi\hbar)^3, \tag{39}$$

where T* is the effective temperature, defined by this relation, and k is Boltzmann's constant. Comparing (38) and (39), we obtain

$$T^*=0.23\,\psi_1{}^{1/2}/k.$$

The value of ψ_1 is determined by the properties of the given specific system. In particular, for a strongly alloyed semiconductor, it follows from formula (17) that $\psi_1 = 2\pi n r_0 e^4 Z^2 \epsilon^{-2}$. For $\psi_1^{1/2} = 0.05$ eV, we obtain $T^* = 1.3 \times 10^2\,°K$ and

$$N_c(T^*)\equiv\frac{\langle\nu\rangle}{\Omega}=3.7\left(\frac{m}{m_0}\right)^{3/2}\cdot 10^{19}\ [\mathrm{cm}^{-3}],$$

where m_0 is the free-electron mass. The total concentration of discrete levels exceeds (38), but is probably of the same order. We see that in "sufficiently disordered" substances, this concentration can be extremely high. It is not excluded that precisely this circumstance fixes the position of the Fermi level in the sample, making it comparatively insensitive to further alloying.[7)]

5. ONE-ELECTRON LEVELS IN A NON-GAUSSIAN FIELD

In the case of a non-Gaussian field, we must start as before from formula (26). Instead of (27), however, formula (8) now gives

$$\bar{Q}_b=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{ds}{s-i\varepsilon}e^{-ias-c^2s^2}\sum_{n\geqslant 0}c_nH_n[F], \tag{40}$$

where

$$c^2=\frac{1}{2}\left[\varphi_1+\int d\mathbf{k}\,\Phi(\mathbf{k})B(\mathbf{k})B(-\mathbf{k})\right], \tag{41}$$

$$F=\sum_{l\geqslant 2}\frac{sl}{l!}\int d\mathbf{k}_1\ldots d\mathbf{k}_l\,\delta(\mathbf{k}_1+\ldots+\mathbf{k}_l)\Phi_l(\mathbf{k}_1,\ldots,\mathbf{k}_l)B(\mathbf{k}_1)\ldots B(\mathbf{k}_l). \tag{42}$$

[7)] This idea is not new. The estimate given above only substantiates the ideas that have been stated in qualitative form by various authors.

The quantities a and B are given as before by the formulas (30).

Choosing $\psi_\alpha(\mathbf{k})$ in the form (31) and proceeding as in Sec. 3, we can rewrite (40) in the form

$$\bar{Q}_b = \frac{1}{2} - \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{ds}{s} e^{-ias - c^2 s^2} \sum_{n \geqslant 0} c_n H_{2n}[F_1], \tag{43}$$

where

$$F_1 = \sum_{l \geqslant 2} \frac{s^l}{l!} (2\alpha)^{3l-3} 4\pi^{2l} \Phi_l(0, \ldots, 0) \frac{1}{l^3}. \tag{44}$$

We see that the probability of the formation of localized states in the forbidden band turns out, generally speaking, to be finite, provided that

$$I = -\frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{dy}{y} e^{-y^2} \sum_{k \geqslant 0} c_{2k+1} H_{2k+1} \cdot \left[\frac{1}{2\alpha^3} \sum_{l \geqslant 2} \frac{(2^{7/2} \pi \alpha^{3/2} y)^l}{l!} \zeta_l \right] < \infty. \tag{45}$$

In fact, in the conditions indicated, the second term in the right-hand side of (43) is transformed to the integral I. (In the case when it diverges, it would not be permissible to neglect the term as in the exponent in (43).) Generally speaking, $I \neq 1/2$, since the right-hand side of (45) contains the parameters ζ_l.[8]

In particular, the condition (45) is certainly fulfilled when the coefficients c_n and Φ_l are cut off at any finite values n and l, i.e., the series in formulas (8) and (10) become polynomials.

It is obvious, a priori, that in random fields with a diverging value of ψ_1, when the representation (8)—(12) is inapplicable, the probability of formation of discrete levels will also be non-zero. This, for example, is the situation in a random field of the "Lorentz" type.

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119

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[8] Of course, by definition of the coefficients c_n, the integral $I \leqslant 1/2$.