

## INFLUENCE OF MAGNETIC QUANTIZATION ON THE NORMAL SKIN EFFECT IN SEMIMETALS

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Submitted February 11, 1971

Zh. Eksp. Teor. Fiz. 61, 753–761 (August, 1971)

A phenomenological quantum theory of the normal skin effect in semimetals in a magnetic field which is valid for an arbitrary relation between the wave frequency  $\omega$  and the electron-hole recombination time  $T$  is developed. A feature of the theory is that the appearance of nonequilibrium carriers in the wave, including quantum effects due to the dependence of carrier concentration in the semimetal on magnetic induction  $\mathcal{B}$  is consistently taken into account. It is assumed that  $\omega^{-1}$  is large compared to the intraband relaxation times and hence the kinetic analysis is combined with a thermodynamic description of the carriers in each of the bands. Consequently the impedance and wave attenuation factor can be related to the dependence of carrier concentration and magnetic susceptibility  $\mu$  on  $\mathcal{B}$  under thermodynamic equilibrium conditions. In the quasiclassical limit in a certain frequency range, the skin-effect pattern for a single oscillation of  $\mu(\mathcal{B})$  may vary from that characteristic of a monopolar metal to that characteristic of a compensated metal (with a greatly increased skin depth). Some features of wave propagation under purely quantum conditions are also investigated. It is shown, in particular, that for a certain relation between the parameters, a wave with a purely longitudinal electrical field and transverse magnetic field, but with vanishing magnetic induction, may arise.

### INTRODUCTION

IT is known that in the bipolar system, when the time of the electron-hole recombination  $T$  is large compared with the relaxation time  $\tau$ , the skin effect in a magnetic field has singularities connected with the appearance of nonequilibrium carriers. Under classical conditions<sup>[1]</sup>, it is due to the inhomogeneity of the alternating electric field in the volume and to the conditions for flux conservation on the surface. Under quantum conditions, the inhomogeneity of the magnetic induction in the wave is an additional cause for the occurrence of nonequilibrium carriers. Indeed, in a semimetal, owing to the Landau quantization, the equilibrium concentration of the electrons depends on the magnetic induction, and therefore the time-varying magnetic induction will generate nonequilibrium carriers.

The condition  $T \gg \tau$  makes it possible to construct a phenomenological theory that is valid at frequencies  $\omega \ll 1/\tau$ , but at arbitrary  $\omega T$ . Owing to the condition  $\omega\tau \ll 1$ , there is time for local equilibrium to become established between the carriers belonging to each of the bands, making it possible to introduce separate thermodynamic functions for the electrons and holes. In this approach, the skin-effect parameters for arbitrary  $\omega T$  turn out to be connected with several thermodynamic quantities pertaining to complete-equilibrium conditions, such as the magnetic permeability and the carrier density, which are functions of the magnetic field. Naturally, the results of the theory include, besides the direct consequence of the occurrence of nonequilibrium concentrations—the appearance of the additional wave—also other quantum effects, for example oscillations of the magnetic susceptibility, the role of which in the surface-impedance oscillations was elucidated in<sup>[2]</sup>.

We have analyzed in detail the simplest model of a semimetal with spherical electron and hole bands. A

generalization of the results to a more realistic model of a semimetal, and also to multivalley semiconductors, entailed no fundamental difficulties. The formulas obtained explain features of the skin effect in the quasiclassical and quantum regions.

### PHYSICAL MODEL. DISPERSION EQUATION

We shall assume that the semimetal has one electron valley and one hole valley and has isotropic dispersion, and that the times of the intravalley relaxation of the electrons and holes,  $\tau_n$  and  $\tau_p$ , are much shorter than the time of intervalley relaxation  $T$  and the reciprocal wave frequency  $1/\omega$ . Under these conditions, the distribution of the carriers within the limits of each band can be regarded as at equilibrium at any instant of time, and we can introduce Fermi quasilevels for the electrons and holes ( $\eta_n$  and  $\eta_p$ ). It is convenient to reckon them from the edges of the corresponding bands.

Under conditions of total thermodynamic equilibrium the quasilevels are connected by the relation

$$\eta_n + \eta_p = \Delta, \quad (1)$$

where  $\Delta$  is the band overlap in the semimetal. If the magnetic induction  $B$  is spatially inhomogeneous, but the characteristic length over which it varies is much larger than the Larmor radius  $R$  and of the screening length  $l_d$ , then the values of the electron and hole concentrations  $n$  and  $p$  are uniquely determined by the value of  $B$ <sup>[3]</sup> at the same point,  $n = p = n(B)$ , and the distribution of the electrostatic potential  $\varphi(\mathbf{r})$  can be obtained from the condition that the electrochemical potential be constant.

Therefore, when considering long-wave oscillations ( $\lambda \gg R, l_d$ ), it is natural to represent the recombination term in the continuity equation for the electrons in the form  $(n - n(B))/T$ , taking  $B$  to mean the instantaneous

value of the induction at the point under consideration. Then

$$\frac{\partial n}{\partial t} - \frac{1}{e} \operatorname{div} j_n + \frac{n - n(\mathcal{B})}{T} = 0. \quad (2)$$

The electron and hole currents are expressed by the formulas

$$j_n = \hat{\sigma}_n \left( \mathbf{E} + \frac{1}{e} \nabla \eta_n \right), \quad j_p = \hat{\sigma}_p \left( \mathbf{E} - \frac{1}{e} \nabla \eta_p \right), \quad (3)$$

where  $\hat{\sigma}_n$  and  $\hat{\sigma}_p$  are the tensors of the electron and hole conductivities. Since  $\eta_n$  and  $\eta_p$  depend on  $\mathcal{B}$ , there arise, in addition to the usual field and diffusion fluxes, also fluxes proportional to  $\operatorname{grad} \mathcal{B}$ ; these naturally, are purely quantum in nature.

We shall consider below the propagation of waves in the half-space  $\underline{z} > 0$  along the  $\underline{z}$  axis, when a strong constant field  $\mathcal{H}$  (with induction  $\mathcal{B}$ ) is applied along the  $y$  axis, and the electric and magnetic fields in the incident wave are oriented along  $x$  and  $y$ . Then, neglecting the displacement current ( $\omega \tau_{\text{Maxwell}} \ll 1$ ), the following components of the vectors of the high-frequency fields and of the current differ from zero:

$$\mathbf{E} = (E_x, 0, E_z), \quad \mathbf{H} = (0, H, 0), \quad \mathbf{B} = (0, B, 0), \quad \mathbf{j} = (j, 0, 0), \quad (4)$$

where for a wave  $\exp[i(kz - \omega t)]$  we have, as a result of Maxwell's equations,

$$E_x = \frac{\omega}{ck} B, \quad H = \frac{4\pi i}{ck} j. \quad (5)$$

In this notation, the last term in (2) takes the form  $[n - n(\mathcal{B} + \mathbf{B}(t))]/T$ .

The condition  $j_z = 0$  in conjunction with (3) enables us to determine  $E_z$ :

$$E_z = -\frac{\sigma_{xz}}{\sigma_{zz}} E_x + \frac{1}{e\sigma_{zz}} \left( \sigma_{xz}^p \frac{\partial \eta_p}{\partial z} - \sigma_{xz}^n \frac{\partial \eta_n}{\partial z} \right), \quad (6)$$

where

$$\hat{\sigma} = \hat{\sigma}_n + \hat{\sigma}_p. \quad (7)$$

Formula (6) allows us to eliminate  $E_z$  from the expressions for the currents:

$$j = j_x = \sigma E_x - \frac{a\sigma^*}{e} \frac{\partial}{\partial z} (\eta_n + \eta_p), \quad (8)$$

$$j_{nz} = \sigma^* \left[ -aE_x + \frac{1}{e} \frac{\partial}{\partial z} (\eta_n + \eta_p) \right]; \quad (9)$$

here

$$a = \frac{\sigma_{xz}^n}{\sigma_{zz}^n} - \frac{\sigma_{xz}^p}{\sigma_{zz}^p}, \quad \sigma = \sigma_{xx} - \frac{\sigma_{xz}\sigma_{zx}}{\sigma_{zz}}, \quad \sigma^* = \frac{\sigma_{zz}^n \sigma_{zz}^p}{\sigma_{zz}}, \quad (10)$$

and as a consequence of the spherical symmetry of the bands we have  $\sigma_{xx} = \sigma_{zz}$  and  $\sigma_{xz} = -\sigma_{zx}$ .

The material equation relating  $\mathbf{H}$  with  $\mathbf{B}$  and  $n$  is<sup>1)</sup>

$$H = \left( \frac{\partial \mathcal{H}}{\partial \mathcal{B}} \right)_n B + \left( \frac{\partial \mathcal{H}}{\partial n} \right)_{\mathcal{B}} (n - n(\mathcal{B})) \quad (11)$$

If we introduce

$$N = n - n(\mathcal{B} + B) \approx n - n(\mathcal{B}) - \frac{dn(\mathcal{B})}{d\mathcal{B}} B. \quad (12)$$

then (11) takes the form

$$H = \frac{B}{\mu} + \left( \frac{\partial \mathcal{H}}{\partial n} \right)_{\mathcal{B}} N, \quad \frac{1}{\mu} = \frac{d\mathcal{H}}{d\mathcal{B}} = \left( \frac{\partial \mathcal{H}}{\partial \mathcal{B}} \right)_n + \left( \frac{\partial \mathcal{H}}{\partial n} \right)_{\mathcal{B}} \frac{dn}{d\mathcal{B}}, \quad (13)$$

<sup>1)</sup>The influence of the electric field of the wave on the magnetic moment connected with the shift of the magnetic levels in the electric field can be neglected, since it is of the order of  $E^2$ .

i.e.,  $\mu^{-1}$  is the total derivative calculated under conditions of thermodynamic equilibrium. In perfect analogy we obtain for the nonequilibrium part of  $\eta_n + \eta_p$

$$\delta(\eta_n + \eta_p) = bN, \quad b = \left( \frac{\partial \eta_n}{\partial n} \right)_{\mathcal{B}} + \left( \frac{\partial \eta_p}{\partial p} \right)_{\mathcal{B}}, \quad (14)$$

where, as a consequence of (1),  $d(\eta_n + \eta_p)/d\mathcal{B} = 0$ .

Equations (5), (8), (13), and (14) enable us to connect  $\mathbf{B}$  with  $\mathbf{N}$ , after which we get from (2) and (9), as a result of simple but cumbersome calculations using (A.4), the following dispersion equation:

$$k^4 + k^2 \left( \frac{1}{L^2} - \frac{i}{\delta_0^2} \right) - \frac{i}{\delta^2 L^2} = 0. \quad (15)$$

Here

$$\frac{1}{L^2} = \frac{1 - i\omega T}{L_D^2}, \quad L_D^2 = DT, \quad D = \frac{\sigma^*}{e^2} b, \quad (16)$$

i.e.,  $L_D$  plays the role of the diffusion length and  $D$  is the coefficient of bipolar diffusion. The quantity

$$\delta^2 = c^2 / 4\pi\omega\sigma\mu \quad (17)$$

is the classical skin length and

$$\frac{1}{\delta_0^2} = \frac{4\pi\omega\mu}{c^2} \left\{ \sigma_n + \sigma_p - 2eac \frac{dn}{d\mathcal{B}} + e^2 c^2 \sigma^* \left( \frac{dn}{d\mathcal{B}} \right)^2 \right\}; \quad (18)$$

here  $\sigma_n$  and  $\sigma_p$  are made up of the components of the tensors  $\hat{\sigma}_n$  and  $\hat{\sigma}_p$  in analogy with  $\sigma$  (formula (10)), and we have used the identity  $\sigma_n + \sigma_p - \sigma = a^2 \sigma^*$ .

Equation (15) differs from the dispersion equation of the classical theory<sup>[1]</sup> in that account is taken of the magnetic permeability  $d\mathcal{B}/d\mathcal{H}$  and that characteristic corrections to  $\sigma_n + \sigma_p$  are introduced in the curly bracket of (18). It is convenient to rewrite (18) in the form

$$\frac{1}{\delta_0^2} = \frac{1}{\delta^2} \left\{ 1 + a^2 \frac{\sigma^*}{\sigma} (1 - gG) \right\}, \quad (19)$$

where

$$G = \frac{d \ln n}{d \ln \mathcal{B}}, \quad g = \frac{enc}{a\mathcal{B}\sigma^*}. \quad (20)$$

The amplitudes of the different physical quantities in the wave are connected by the relations

$$B = -\mu \frac{1 - (gG)^{-1}}{1 - i(\delta k)^{-2}} \left( \frac{\partial \mathcal{H}}{\partial n} \right)_{\mathcal{B}} N = \frac{ck}{\omega} E_x, \quad (21)$$

$$H = \frac{1 - igG(\delta k)^{-2}}{1 - gG} \frac{B}{\mu}, \quad (22)$$

$$j_{nz} = - \left\{ a\sigma^* + \frac{\sigma}{a} \frac{1 + i(\delta k)^2}{1 - gG} \right\} E_x. \quad (23)$$

The boundary conditions include, together with the ordinary electrodynamic conditions for the continuity of  $E_x$  and  $H_y$ , a condition connected with recombination of electron-hole pairs on the surface<sup>2)</sup>. We consider two limiting cases: 1) infinite rate of surface recombination ( $S \rightarrow \infty$ , when  $N(z=0) = 0$ , and 2) zero rate ( $S = 0$ ), when  $j_{nz}(z=0) = 0$ . Selecting the solutions of Eq. (15) with  $\operatorname{Im} k_{1,2} > 0$  and using the boundary conditions, we obtain, taking (21)–(23) into account, formulas for the surface impedance  $\zeta = E(0)/H(0)$  in the indicated limiting cases:

$$\zeta_\infty = -i \frac{\omega\mu}{c} \frac{i + \delta^2 k_1 k_2}{k_1 + k_2}, \quad (24)$$

<sup>2)</sup>The dependence of the rate of surface recombination in semimetals on the near-surface bending of the bands is considered in [4].

$$\zeta_0 = -i \frac{\omega\mu}{c} \delta^2 (k_1 + k_2) \left[ 1 + a^2 \frac{\sigma^*}{\sigma} + \frac{-\delta^2/L^2 + ia^2(\sigma^*/\sigma)(gG)^2}{\delta^2 k_1 k_2} \right]^{-1}$$

$$a^2 \sigma^* \approx \sigma_n + \sigma_p \approx \frac{1}{\sigma^*} \left( \frac{enc}{\mathcal{B}} \right)^2 \sim \sigma(\omega_c \tau)^2 \sim \frac{e^2 n \tau}{m} \gg \sigma; \quad (25)$$

The quantum effects are significant only if

$$\tau_n \omega_n, \tau_p \omega_p \gg 1, \quad (26)$$

where  $\omega_n$  and  $\omega_p$  are the cyclotron frequencies. Under these conditions, it follows from the well known formulas for the conductivity tensor that

$$\sigma_{xz}^n \approx -\sigma_{xz}^p \approx \frac{enc}{\mathcal{B}}, \quad a\sigma^* \approx \frac{enc}{\mathcal{B}}, \quad g \approx 1, \quad (27)$$

Here  $\omega_c \sim \omega_n, \omega_p$  and  $\tau \sim \tau_n, \tau_p$ . These relations, which we shall use constantly in what follows, enable us to simplify the final formulas. In particular, it follows from them that  $\delta^2 \gg \delta_0^2$ , provided only  $1 - gG \neq 0$ .

### ANALYSIS OF SOLUTIONS. LIMITING CASES

We must first emphasize one general circumstance. In spite of the fact that in this theory the carrier concentrations differ from their instantaneous-equilibrium values  $n(\mathcal{B} + B(t))$ , the dispersion equation (15) contains the equilibrium magnetic permeability (compare (17) and (18)); therefore at all  $\omega T$  the singularities of  $\mu$  should be reflected in the singularities of  $\zeta$ . An important role can be played also by singularities connected with  $dn/d\mathcal{B}$  and  $b$  (formula (16)).

It is convenient to consider separately two limiting cases.

#### 1. Quasiclassical Case

The oscillations of the various quantities can readily be determined by using the formulas of the Appendix and the usual Poisson expansion for the thermodynamic potential. If the first terms predominate in this series and the temperatures are low, then it follows from (A.2) and (A.5) that the relative magnitude of the oscillations  $b$  and the quantity  $G$  itself are of the order of the small quasiclassical parameter  $(\hbar\omega_c/\Delta)^{1/2}$ , i.e., of the order of the Shubnikov oscillations of the kinetic coefficients. At the same time, the oscillating part of  $\mu^{-1}$ , in accordance with (A.3), is of the order of

$$(\mu^{-1})_{\text{osc}} \sim q \left( \frac{\hbar\omega_c}{\Delta} \right)^{1/2}, \quad q = \frac{4\pi n \Delta}{\mathcal{B}^2}, \quad (28)$$

and can greatly exceed  $(\hbar\omega_c/\Delta)^{1/2}$ ; we shall assume that  $q \gg 1$ . Therefore, as was already discussed in<sup>[2]</sup>, oscillations of  $\mu$  can dominate in the surface impedance. In the bipolar situation, however, they have essential singularities.

At  $\delta_0 \ll \delta$ , the roots of the dispersion equation

$$k_1^2 \approx -\frac{1 - iL^2/\delta_0^2}{L^2}, \quad k_2^2 \approx \frac{i/\delta^2}{1 - iL^2/\delta_0^2} \quad (29)$$

depend on the ratio  $(L/\delta_0)^2$ , which can be reduced to the form

$$\left( \frac{L}{\delta_0} \right)^2 = \frac{\omega T}{1 - i\omega T} q \mu \frac{(1 - gG)^2}{^{3/2} - gG} \approx \frac{2}{3} \frac{\omega T}{1 - i\omega T} q \mu. \quad (30)$$

It follows from (28) that  $q\mu \gg 1$  always. If, in addition,  $\mu$  oscillates in order of magnitude, as is possible when  $q(\hbar\omega_c/\Delta)^{1/2} \sim 1$ , then there exist three frequency bands

with essentially different behaviors of the surface impedance.

**A. Upper band:**  $\omega T q \mu_{\text{min}} > 1$ . In this case  $(L/\delta_0)^2 > 1$ ,  $k_1 \approx \sqrt{i}/\delta_0$ ,  $k_2 \approx i\delta_0/\delta L$ , and in accordance with (24) and (25) we have

$$\zeta_\infty \approx \frac{\omega\mu}{c} \delta_0 \left( \frac{\delta}{L} + \frac{1}{\sqrt{i}} \right), \quad \zeta_0 \approx \frac{1}{\sqrt{i}} \frac{\omega\mu}{c} \delta_0. \quad (31)$$

The oscillations of  $\zeta_0$  are proportional here to  $\sqrt{\mu}$ , as in a monopolar metal<sup>[2]</sup>, i.e., they have a purely electrodynamic meaning, and the oscillations of  $\zeta_\infty$  are weakened. We note that (31) contains the length  $\delta_0$  corresponding to the conductivity  $\sigma_n + \sigma_p$ , and the latter is not altered by the magnetic field (just as in a monopolar metal).

**B. Lower band:**  $\omega T q \mu_{\text{max}} < 1$ . In this case  $L/\delta_0 < 1$ ,  $k_1 \approx i/L$ ,  $k_2 \approx \sqrt{i}/\delta$ ,

$$\zeta_\infty \approx \frac{1}{\sqrt{i}} \frac{\omega\mu}{c} \delta, \quad \zeta_0 \approx \frac{1}{\sqrt{i}} \frac{\omega\mu}{c} \delta \left( 1 + \frac{1}{\sqrt{i}} \frac{\delta L}{\delta_0^2} \right)^{-1}. \quad (32)$$

Here  $\zeta_\infty \sim \mu^{1/2}$ , and the oscillations of  $\zeta_0$  are weakened. Equation (32) contains the usual length  $\delta \sim \delta_0 \omega_c \tau$  for compensated metals.

**C. Intermediate frequency band.** In this case, on that section of the  $\mu(\mathcal{B})$  oscillations which corresponds to large  $\mu$  we have  $\omega T q \mu > 1$  and the values of  $k$  and  $\zeta$  are the same as in item A, whereas on the section with small  $\mu$ , when  $\omega T q \mu < 1$ , they are the same as in item B. Thus, in this frequency band the form of the oscillations of  $\zeta(\mathcal{B})$  is determined both by the direct dependence on  $\mu$  and by the transition to new characteristic lengths near the point  $\omega T q \mu \sim 1$ . These two factors, as follows from (31) and (32), operate in opposite directions, and their relative role is determined mainly by the ratio of the factors  $\mu_{\text{max}}/\mu_{\text{min}}$  and by  $\omega_c \tau$ .

We have omitted in all the foregoing the small terms of order of  $G$ . We note that formally the retention of the term with  $G^2$  in (25) ensures a finite limit for  $\zeta_0$  as  $\mu \rightarrow \infty$ , i.e., on approaching the instability region<sup>[5]</sup>. It follows, however, from the aggregate of the criteria of the present theory, that this can take place only if

$$\frac{\hbar}{\tau \Delta} \sqrt{\frac{T}{\tau}} \gg 1,$$

which corresponds to excessively large  $T$ .

#### 2. Quantum Case

We shall now assume that a small number of levels is filled in each band. These conditions are obtained in strong fields, when  $q \ll 1$ , and since  $\mu \sim 1$  except for the singular cases, we have in the quantum case in accordance with (30)  $L/\delta_0 \ll 1$  and the impedance is described by formula (32).

We confine ourselves below to absolute zero temperature. Then, starting from the usual expression for  $\Omega_n$  in the absence of scattering

$$\Omega_n = A_n \mathcal{B}^{3/2} \sum_{l=0}^{\infty} \sum_{s=\pm 1} \left( \frac{\eta_n}{\hbar\omega_n} - l - \frac{1}{2} - \frac{s}{2} \frac{m_n}{m_n'} \right)^{3/2}$$

$$A_n = -\frac{1}{3\pi^2 m_n} \left( \frac{2e^5}{\hbar c^3} \right)^{1/2} \quad (33)$$

where  $m_n$  and  $m_n'$  are the orbital and spin masses of the electrons, and from the analogous expression for  $\Omega_p$  we

can establish, using (A.1), (A.2), (A.3), and (A.5), the following exact relations:

$$G = \frac{1}{2} - \Delta / nb, \quad (34)$$

$$\mu^{-1} = 1 + q(\frac{1}{2} + G + \frac{1}{2}\Omega / n\Delta). \quad (35)$$

From the definition (14) and from the fact that the chemical potential depends monotonically on the concentration, it follows that the individual terms in  $b$  are positive. They are always bounded and vanish when the Fermi level passes through the edge of one of the Landau subbands. Since in the general case these passages in the electron and hole bands occur at different values of  $\mathcal{B}$ , it follows that  $b \neq 0$  and is of the order of  $\Delta/n$ . Therefore, as a consequence of (34),  $G \sim 1$ . Since furthermore  $\Omega \sim n\Delta$ , it follows from (35) that  $\mu \approx 1$ . In the quantum case, therefore, the dependence of the impedance on  $\mathcal{B}$  is determined by the changes of  $G$ ,  $b$ , and of the kinetic coefficients, whereas  $\mu$  remains practically constant.

A special situation arises when the Fermi level passes simultaneously through the edges of the electron and hole Landau subbands,<sup>3)</sup> Then  $b \rightarrow 0$  and, in accord with (34),  $G \rightarrow -\infty$ . Actually the growth of  $|G|$  is limited by the condition  $\mu > 0$  which, according to (35), is of the form  $1 + qG > 0$ ; on approaching this limit we have  $\mu \rightarrow \infty$ . In this situation, the impedance is equal to

$$\zeta_{\infty} \approx \frac{\omega\mu}{ci} \frac{L_p}{(1 - i\omega T\mu)^{1/2}}, \quad (36)$$

$$\zeta_0 \approx \sqrt{i} \frac{\delta}{cT} \frac{(1 - i\omega T)^{1/2} (1 - i\omega T\mu)^{1/2}}{qG^2}.$$

We see that the structures of the formulas for  $\zeta_0$  and  $\zeta_{\infty}$  are entirely different, and an unusual frequency dependence via the factor  $(1 - i\omega T\mu)$  is obtained.

A unique situation arises when  $gG = 1$ . Then, according to (19),  $\delta_0 = \delta$  and the roots of the dispersion equation (15) are equal to  $k_1 = \sqrt{i}/\delta$  and  $k_2 = i/L$ . The ratio of the amplitudes in the first wave can be determined from (21) and (22) with allowance for the fact that as  $gG \rightarrow 1$  the root is  $k_1 \approx \sqrt{i}\delta^{-1}[1 + O((1 - gG)^2)]$ . It turns out that  $N = 0$ , and all the remaining amplitudes differ from zero, with  $B = \mu H$ . For the second wave it follows directly from (21), (28), and (16) that  $E_x = B = 0$ , and the quantities different from zero are  $N$ , the magnetic field  $H = (\partial\mathcal{H}/\partial n)_{\mathcal{B}} N$ , and the longitudinal electric field  $E_z$ . Thus, in the first wave, which can be called electromagnetic in the literal sense, the concentration at each point has its own instantaneous-equilibrium value; at the same time, the second wave is quite extraordinary.

For the impedance, using (24) and (25), we obtain

$$\zeta_{\infty} = \frac{\omega\mu}{\sqrt{ic}} \delta, \quad \zeta_0 = \frac{\omega\mu}{\sqrt{ic}} \delta \left(1 + \frac{1}{\sqrt{i}} a^2 \frac{\sigma^* L}{\sigma \delta}\right)^{-1}. \quad (37)$$

The usual form of the formula for  $\zeta_{\infty}$  is connected with the fact that when  $S = \infty$  we have on the boundary  $N(0) = 0$ , and consequently the second wave is not excited at all. At the same time, a comparison of (37) with (32) shows that near  $gG = 1$  an essential singularity oc-

curs in  $\zeta_0(\mathcal{B})$ . Indeed, the denominator in (32), in a broad region  $\delta_0 \ll \delta$ , is equal to, in accord with (19),

$$1 + \frac{1}{\sqrt{i}} a^2 \frac{\sigma^* L}{\sigma \delta} (1 - gG)^2.$$

When this expression is extrapolated to the point  $gG = 1$ , the second term in it vanishes, whereas according to the exact formula (36) it takes on the value  $a^2 \sigma^* L / \sqrt{i} \sigma \delta$ . It follows from this that in a narrow interval of variation of  $\mathcal{B}$ , with width  $\sim \mathcal{B}(\omega_c \tau)^{-1}$ , in which  $\delta_0 \sim \delta$ , a sharp spike should appear in the impedance.

We take the opportunity to express our sincere gratitude to E. P. Vol'skiĭ for fruitful discussions that have stimulated the performance of the present work.

## APPENDIX

We present here a number of relations between the derivatives of the thermodynamic quantities; these relations are valid when  $n = p$ . We introduce the thermodynamic potentials of the electrons  $\Omega_n(\mathcal{B}, \eta_n)$  and holes  $\Omega_p(\mathcal{B}, \eta_p)$ , defining the total potential as  $\Omega = \Omega_n + \Omega_p$ . The concentration of the electrons and their magnetic moment are determined by the usual formulas

$$n = -\partial\Omega_n / \partial\eta_n, \quad \mathcal{M}_n = -\partial\Omega_n / \partial\mathcal{B} \quad (A.1)$$

and analogously for the holes.

Under conditions of total equilibrium, Eq. (1) is also satisfied, and  $n = p$  together with (A.1) leads to  $\partial\Omega / \partial\eta = 0$  (where  $\eta \equiv \eta_n$ ), from which we can find the equilibrium  $d\eta/d\mathcal{B}$ . Differentiating (A.1) and eliminating  $d\eta/d\mathcal{B}$ , we readily obtain

$$\frac{dn}{d\mathcal{B}} = - \left( \frac{\partial^2\Omega_n}{\partial\eta_n^2} \frac{\partial^2\Omega_p}{\partial\eta_p \partial\mathcal{B}} + \frac{\partial^2\Omega_p}{\partial\eta_p^2} \frac{\partial^2\Omega_n}{\partial\eta_n \partial\mathcal{B}} \right) / \frac{\partial^2\Omega}{\partial\eta^2}, \quad (A.2)$$

$$\frac{d\mathcal{M}}{d\mathcal{B}} = - \frac{\partial^2\Omega}{\partial\mathcal{B}^2} + \frac{(\partial^2\Omega/\partial\mathcal{B} \partial\eta)^2}{\partial^2\Omega/\partial\eta^2}. \quad (A.3)$$

Expression (A.3) enters directly into the material equation (13) for  $\mathcal{H} = \mathcal{B} - 4\pi\mathcal{M}$ ,  $\mathcal{M} = \mathcal{M}_n + \mathcal{M}_p$ .

Using the relation  $(\partial\mathcal{M}_n/\partial n)_{\mathcal{B}} = -(\partial\eta_n/\partial\mathcal{B})_{\mathcal{B}}$ , which follows from (A.1), as well as the connection between the partial derivatives of  $\eta_n + \eta_p$ , which follows from (1), we can obtain the following formula, which is needed for the derivation of the dispersion equation (15):

$$\left( \frac{\partial\mathcal{H}}{\partial n} \right)_{\mathcal{B}} = -4\pi \frac{dn}{d\mathcal{B}} b, \quad (A.4)$$

where  $b$  (cf. (14)) is equal to

$$b = - \left( \frac{\partial^2\Omega_n}{\partial\eta_n^2} \right)^{-1} - \left( \frac{\partial^2\Omega_p}{\partial\eta_p^2} \right)^{-1}. \quad (A.5)$$

<sup>1</sup>G. I. Babkin and V. Ya. Kravchenko, Zh. Eksp. Teor. Fiz. **60**, 695 (1971) [Sov. Phys.-JETP **33**, 378 (1971)].

<sup>2</sup>M. Ya. Azbel' and G. A. Begiashvili, ZhETF Pis. Red. **3**, 201 (1966) [JETP Lett. **3**, 128 (1966)]; R. G. Mints, ibid. **11**, 128 (1970) [11, 79 (1970)].

<sup>3</sup>D. Shoenberg, Phil. Trans. Roy. Soc. **A255**, 85, 1962.

<sup>4</sup>V. Ya. Kravchenko and É. I. Rashba, Zh. Eksp. Teor. Fiz. **56**, 1713 (1969) [Sov. Phys.-JETP **29**, 918 (1969)].

<sup>5</sup>J. H. Condon, Phys. Rev. **145**, 526, 1966.

<sup>3)</sup>This takes place always near the semimetal-dielectric transition point and in a few other cases, namely, in the symmetrical model when all the parameters of the electrons and holes coincide, under conditions when  $m'_n/m_n < 1$  and  $m'_p/m_p < 1$  and therefore several lower magnetic subbands correspond to one spin orientation, etc.