MAIN STATISTICAL CHARACTERISTICS OF THE MULTIQUANTUM PHOTOCURRENT PRODUCED BY MODULATED RADIATION

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The effect of modulation of the radiation incident on a multiquantum photocathode on the statistical characteristics of the multiquantum photocurrent is studied within the framework of the phenomenological theory of photoelectric detection of radiation. It is shown that the time distribution function for the number of emitted particles depends both on the statistical properties of the modulated radiation and on the type of modulation of the incident radiation. With increase of the order κ of the photoeffect the difference between the statistical characteristics of the photocurrent produced by the modulated radiation and the statistical characteristics of the photocurrent in absence of any modulation is found to increase.

IT is known that the statistical properties of optical fields are evaluated mainly from data on the distribution of the probability of photoelectric counts (see, for example,[1,2]). Recent theoretical and experimental investigations (see the review[3]) have shown that the elementary act of the external multiquantum photoeffect can, in principle, be a detector of the order and degree of coherence of radiation, since the amplitude values of the photocurrent depend on the correlation properties of the electromagnetic field producing this current.

In[4,5], using a number of concrete models of radiation fields, it was demonstrated that in two-quantum detection the temporal distribution function of the number of emitted particles not only differs significantly from the distribution function of the single-quantum photocurrent, but also carries information concerning the properties of radiation-field correlation functions of order higher than the second. The general results corresponding to the case of multiquantum detection of radiation with the aid of photodetectors of arbitrary order k, for which there is a characteristic relation $j_k = \alpha_k I^k$ between the photocurrent density j_k and the radiation intensity I (α_k is the quantum yield of the k-quantum photoeffect), were obtained in[6,7]. In particular, it is shown in[6] that if the condition $T \ll \tau_c$ is satisfied (T and τ_c are respectively the registration and coherence times of the incident radiation), then the distribution function of the number of emitted particles $p_k(n, T)$, which characterizes the probability of appearance of n photoelectrons during a time interval T over the surface of a detector of k-th order, is transformed with increasing order k of the photoeffect in such a way that, first, the distribution $p_k(n, T)$ becomes "flatter," and, second, its "center of gravity" shifts towards smaller values of n[1]). This fact was experimentally confirmed recently in[9] in a comparative investigation of the distribution functions of the photoelectron numbers $p_1(n, T)$ and $p_2(n, T)$ produced under the influence of radiation of the first and second harmonics of a helium-neon laser, since formally[10,11] the variation of the distribution function $p_k(n, T)$ with increasing order k of the photoeffect is analogous to the transformation of the distribution function $p(n, T)$ observed upon generation of higher harmonics of order k.

It must be stated, however, that in all the preceding investigations devoted to the statistics of the multiquantum photoeffect (or the photocurrent generated by emission of higher harmonics), they investigated the statistical characteristics of the photocurrent generated by a stationary random radiation field, i.e., radiation whose average intensity experienced only fluctuations induced by the statistical properties of the radiation during the time of registration of the distribution function $p_k(n, T)$. In practice, on the other hand, we deal as a rule with radiation that is intensity-modulated in a definite manner, and therefore the statistical characteristics of the photocurrent generated by modulated radiation should differ noticeably from the statistical characteristics of the photocurrent due to stationary (unmodulated) radiation.

The present paper is devoted to an elucidation of the influence of modulation of radiation with known statistical properties on the statistical characteristics of the multiquantum photocurrent (or the photocurrent generated by higher-harmonic radiation); the analysis will be carried out within the framework of the phenomenological (semiclassical) theory of radiation detection. Such a formulation of the problem is justified, incidentally, also by the fact that, as shown by a number of recent theoretical and experimental investigations[12-17] devoted to the statistical characteristics of photocurrent generated in single-quantum photodetectors, the modulation of the radiation leads to a noticeable change in the statistical characteristics of the photocurrent compared with the case when there is no modulation.

The statistical characteristics of the multiquantum photocurrent generated by intensity-modulated radiation having definite statistical properties are determined uniquely by the value of the instantaneous radia-

[1])There is an error in formula (4) of [6]. The operation of averaging in expression (3) should be carried out over the ensemble $P(W_k)$, and not $P(W)$, as is written in expression (4). Formally, however, all the expressions obtained in [6] remain valid if the operation of averaging over the field variables, denoted in the text by the brackets $\langle \ldots \rangle$, is taken to mean averaging over $P(W_k)$. Some of the results turn out to coincide in this case with the results of [8], where an analogous problem was considered, for the first time, for the case of single-quantum detection.

tion intensity $I(t)$, for which we can write the expression

$$I(t)=J(t)M(t), \tag{1}$$

where $J(t)$ is a function that varies rapidly with time and characterizes the intrinsic statistical properties of the radiation, while $M(t)$ is a determined function that varies slowly in time and describes the process of modulation of radiation whose intrinsic fluctuation characteristics are given by the function $J(t)$. As shown in[12-17], if the condition $T \lesssim \tau_c \ll T_1$ is satisfied (here T and T_1 are the registration time and the period of the modulation), the modulation of radiation with known statistical properties given by the function $J(t)$ can be described in terms of smooth variation of the average value of the radiation intensity, i.e., it can be assumed that the modulation process reduces to a time variation of the average value of the intensity $\langle I \rangle$, corresponding to a specified function $J(t)$, in accordance with some known law $M(t)$.

The distribution function of the number of photoelectrons emitted under the influence of modulated radiation can be obtained by substituting the expression for the instantaneous radiation intensity $I(t)$ in the form (1), into the following formula[6], which is valid for the case of multiquantum detection:

$$p_k^{\text{mod}}(n,T)=\int\limits_{(W_k)}\frac{(\alpha_k W_k)^n}{n!}\exp[-\alpha_k W_k]P(W_k)\,dW_k \tag{2}$$

and then averaging twice over the statistically independent processes $M(t)$ and $J(t)$. In expression (2) we have

$$W_k=\int\limits_0^T [I(t')]^k\,dt',$$

and $P(W_k)$ is the distribution function of the quantity $W_k(T)$. Simple calculations show that for the probability $p_k^{\text{mod}}(n, T)$ of counting n photoelectrons produced in a photodetector of k-th order during an interval of time $T \ll \tau_c$ under the influence of modulated radiation characterized by a distribution of the mean value of the intensity $\mathscr{P}(\langle I \rangle)d\langle I \rangle$, the following expression holds true:

$$p_k^{\text{mod}}(n,T)=\int\limits_{\langle I\rangle} p_k(n,T)\mathscr{P}(\langle I\rangle)\,d\langle I\rangle, \tag{3}$$

where $p_k(n, T)$ is the count distribution function corresponding to the case when there is no modulation, and for the distribution $\mathscr{P}(\langle I \rangle)d\langle I \rangle$ we have the expression

$$\mathscr{P}(\langle I\rangle)\,d\langle I\rangle = dt\,/\,T_1,$$

which reflects the fact that the start of the counting time of an observation extending over a time interval $T \ll T_1$ is equiprobably distributed on the time axis during the modulation period, and does not correlate in any way with the latter.

We shall estimate below how the character of the modulation, the order of the photoeffect k, and the intrinsic statistical properties of the modulated radiation influence the distribution function $p_k^{\text{mod}}(n, T)$ of the emitted particles, using as examples the particular cases of "sinusoidal" and "triangle" modulation of radiation fields with different statistical properties. As the models of the detected radiation fields we shall consider the emission of an ideal single-mode laser, when

$$P(I)=\delta(I-\langle I\rangle), \tag{4}$$

and the emission from a thermal source, when

$$P(I)=\frac{1}{\langle I\rangle}\exp\left(-\frac{I}{\langle I\rangle}\right). \tag{5}$$

For the case of triangular (or linear) modulation with a period T_1 we have

$$\frac{d\langle I\rangle}{dt}=\frac{\langle I\rangle_{max}-\langle I\rangle_{min}}{T_1}$$

and consequently

$$\mathscr{P}(\langle I\rangle)=\frac{1}{T_1}\frac{dt}{d\langle I\rangle}=\frac{1}{h[\langle I\rangle_{max}+\langle I\rangle_{min}]}. \tag{6}$$

Here

$$h=\frac{\langle I\rangle_{max}-\langle I\rangle_{min}}{\langle I\rangle_{max}+\langle I\rangle_{min}}=\frac{\beta-1}{\beta+1}$$

is the depth of modulation and $\beta = \langle I \rangle_{max}/\langle I \rangle_{min}$. For the case of sinusoidal modulation with frequency $\omega = 2\pi/T_1$, i.e., for the case when

$$\langle I\rangle=\langle I_0\rangle+\langle I_1\rangle\sin\omega t,\quad h=\frac{\langle I_1\rangle}{\langle I_0\rangle},\quad \beta=\frac{1+h}{1-h},$$

we have

$$\mathscr{P}(\langle I\rangle)=\pi^{-1}[\langle I_1\rangle^2-(\langle I\rangle-\langle I_0\rangle)^2]^{-1/2}, \tag{7}$$

Since $\langle I \rangle_{max} = \langle I_0 \rangle (1 + h)$ and $\langle I \rangle_{min} = \langle I_0 \rangle (1 - h)$.

Using the results of[6], which gives the values of the distribution function $p_k(n, T)$ which are characteristic of the distributions $P(I)$ chosen by us in the form (4) and (5) in the absence of modulation of the radiation, when $T \ll \tau_c$, and consequently $W_k \approx I^k T$, we can determine from formula (3), knowing the distributions $P(I)$, the distribution $p_k^{\text{mod}}(n, T)$ of the photoelectrons produced in a k-quantum detector upon detection of modulated radiation. The calculations show that

$$\begin{gathered}
p_k^{\text{mod}}(n,T)=\frac{[\alpha_k T\langle I\rangle_{min}^k]^n}{n!(\beta-1)}\sum_{m=0}^{\infty}A_{km},\\
A_{km}=\frac{(-1)^m}{m!}\frac{[\alpha_k T\langle I\rangle_{min}^k]^m}{[k(n+m)+1]}[\beta^{k(n+m)+1}-1]
\end{gathered} \tag{8}$$

for detection of linearly modulated radiation of an ideal single-mode laser (4), and

$$p_k^{\text{mod}}(n,T)=\frac{[\alpha_k T\langle I\rangle_{min}^k]^n}{n!(\beta-1)}\sum_{m=0}^{\infty}A_{km}[k(n+m)]! \tag{9}$$

for detection of linearly polarized ($\gamma = 1$) thermal radiation with intensity distribution given by (5).

The change produced in the form of the distribution functions by the modulation of the radiation can be seen clearly by writing down the connection between the distributions $p_k^{\text{mod}}(n, T)$ and $p_k(n, T)$. For expressions (8) and (9) we have respectively

$$\frac{p_k^{\text{mod}}}{p_k}=\frac{2^{kn}}{(\beta-1)(\beta+1)^{kn}}\exp\left[\alpha_k T\frac{\langle I\rangle_{min}^k}{2^k}(\beta+1)^k\right]\sum_{m=0}^{\infty}A_{km} \tag{10}$$

and

$$\frac{p_k^{\text{mod}}}{p_k}=\frac{2^{kn}}{\beta-1}\frac{\sum\limits_{m=0}^{\infty}A_{km}[k(n+m)]!}{\sum\limits_{m=0}^{\infty}\frac{(-1)^m}{m!}\frac{[\alpha_k T\langle I\rangle_{min}^k]^m}{2^{km}}[k(n+m)]!(\beta+1)^{k(n+m)}}. \tag{11}$$

We see from (10) and (11) that even for linear modulation of the light flux, the form of the distribution function of the emitted particles is appreciably altered, and the character of the variation depends both on the statistical properties of the modulated radiation and on the order k of the photoeffect. In particular, the distribution of the photoelectrons produced under the influence of linear modulated radiation of an ideal single-mode laser ceases to be of the Poisson type not only for multiquantum (nonlinear) detection, but also for single-quantum detection, and this deviation increases with increasing order k of the photoeffect. An analysis of expressions (8) and (9) shows that the distribution function $p_k^{\rm mod}(n, T)$ becomes flatter with increasing order of the photoeffect k and depth of modulation h. This causes, in particular, the interval of variation of the quantity $|\partial p_k^{\rm mod}(n, T)/\partial n|$, which characterizes the slope of the distribution (8) or (9), to decrease with increasing k[2].

For the mean values of the number of detected particles we have the expressions

$$\langle n_k\rangle^{\rm mod}=\sum_{n=0}^{\infty} n_k p_k^{\rm mod}(n, T)=\langle n_k\rangle\frac{2^k(\beta^{k+1}-1)}{(k+1)(\beta-1)(\beta+1)^k} \qquad (12)$$

for linear modulation of radiation of an ideal laser, and

$$\langle n_k\rangle^{\rm mod}=k!\,\frac{\alpha_k T\langle I\rangle_{min}^k(\beta^{k+1}-1)}{(k+1)(\beta-1)}=\langle n_k\rangle\frac{2^k(\beta^{k+1}-1)}{(k+1)(\beta-1)(\beta+1)^k} \qquad (13)$$

for detection of linearly modulated thermal radiation, since

$$\langle n_k\rangle^{\rm mod}=\int_{\langle I\rangle}\langle n_k\rangle\mathscr{P}(\langle I\rangle)\,d\langle I\rangle \qquad (14)$$

(in (12)–(14), $\langle n_k\rangle$ are the mean values of the number of particles detected under the influence of the statistically corresponding unmodulated radiation, for which $\langle I\rangle_{eq}=\langle I\rangle_{min}(\beta+1)/2$.

In the case when the modulation has a linear character, i.e., when $\mathscr{P}(\langle I\rangle)=\text{const}$, and the mean values of the particle numbers emitted upon detection of the unmodulated radiation can be represented in the form $\langle n_k\rangle=\text{const}\cdot\langle I\rangle^k$, the ratio

$$C_k=\langle n_k\rangle^{\rm mod}/\langle n_k\rangle,$$

which characterizes the relative increase of the number of emitted particles due to the modulation effect, is equal to unity at k = 1 (case of single-quantum detection), independently of the statistical properties of the modulated radiation. In the case of multiquantum photodetectors ($k\geq 2$) under the same conditions, we have

$$C_k=\frac{2^k(\beta^{k+1}-1)}{(k+1)(\beta-1)(\beta+1)^k}>1, \qquad (15)$$

from which we see that at $\beta=\text{const}$ the quantity $\langle n_k\rangle^{\rm mod}$ increases with increasing k. Thus, in detection of linearly modulated radiation by multiquantum photodetectors, the mean values of the number of emitted particles are larger than in detection, using a photodetector of the same order, of unmodulated radiation having the same statistical properties and the same equivalent mean value of the radiation intensity $\langle I\rangle_{eq}$.

For the case of linear modulation of the radiation, the quantity

$$A_k=\langle n_k\rangle_{P(I)}^{\rm mod}/\langle n_k\rangle_{\rm laser}^{\rm mod}$$

which characterizes the excess of the mean value $\langle n_k\rangle_{P(I)}^{\rm mod}$ of the number of particles detected under the influence of linearly-modulated radiation with intensity distribution P(I), compared with the mean number $\langle n_k\rangle_{\rm laser}^{\rm mod}$ of particles detected with modulation of the same character under the influence of radiation of an ideal laser, turns out to equal $A_k = k!$ for the distribution P(I) taken in the form (4), and consequently is the same as in the absence of modulation, i.e.,

$$\left(\frac{\langle n_k\rangle_{P(I)}}{\langle n_k\rangle_{\rm laser}}\right)_{\rm mod}=\left(\frac{\langle n_k\rangle_{P(I)}}{\langle n_k\rangle_{0\,\rm laser}}\right)_{\rm unmod}. \qquad (16)$$

Relation (16) is satisfied, as can readily be seen, for all values of k in the case when the statistical properties of the detected radiation are such that the initial moments of the distribution of the intensity of the radiation $\langle I^k\rangle$ can be written in the form

$$\langle I^k\rangle=\text{const}\cdot\langle I\rangle^k. \qquad (17)$$

This conclusion explains why in the experiment of Shiga and Imamura[18], where the amplitude values of two-quantum photocurrents ($j_2\sim\langle n_2\rangle$) produced by pulses of coherent and incoherent radiation with identical energy and spectral characteristics were compared, the ratio $(\langle n_2\rangle_{\rm random}/\langle n_2\rangle_{\rm laser})_{\rm mod}$ turned out to equal 2, i.e., the value characteristic of the case of two-quantum detection of unmodulated radiation. Indeed, both the laser and the "random" radiation obtained from it after passing through a system of diffraction gratings were modulated in[18] practically linearly, since the laser employed in the experiments operated in the giant-pulse regime and had, according to the authors' statement, an almost triangular radiation-pulse wave form. In this case $\mathscr{P}(\langle I\rangle)=\text{const}$ and consequently the ratios of the mean values of the numbers of emitted particles produced under the influence of the modulated and unmodulated light fluxes should be equal. At the same time, satisfaction of relation (16) means that the random radiation produced from the coherent laser radiation by passage through diffraction gratings is actually close in its statistical properties to radiation of thermal sources, for which condition (17) is satisfied.

The ratio $C_k=\langle n_k\rangle^{\rm mod}/\langle n_k\rangle$, which characterizes, as already mentioned, the relative growth in the number of particles emitted under the influence of modulated radiation compared with the average number of particles emitted from a photodetector of the same order k under the influence of unmodulated radiation with the same statistical properties and with an equivalent mean value of the radiation intensity does not depend on the statistical properties of the radiation field only in the case of the single-quantum photoeffect, when $C_1 = 1$. For example, in the case of detection of

[2] Recent experimental investigations performed for the case of single-quantum detection of a linearly-modulated distribution, corresponding to the case of a δ-like distribution of the source intensity, have confirmed the fact that the distribution function of the number of emitted particles becomes flatter when the radiation is modulated [17].

the radiation of an ideal laser modulated sinusoidally (see (7)), we have

$$C_k = \frac{1}{\pi \langle I_0 \rangle^k} \int\limits_{\langle I_0\rangle - \langle I_1\rangle}^{\langle I_0\rangle + \langle I_1\rangle} \langle I\rangle^k [\langle I_1\rangle^2 - (\langle I_0\rangle - \langle I\rangle)^2]^{-1/2} d\langle I\rangle.$$

In the particular case of the one-, two-, and three-quantum photoeffects we get for C_k

$$C_1 = 1, \quad C_2 = 1 + {}^1/_2 h, \quad C_3 = 1 + {}^3/_2 h^2,$$

i.e., at $k \geq 2$ we have $C_k > 1$. Thus, just as in the case of linear modulation, in the case of sinusoidal modulation it also turns out that the mean value of the number of particles detected with multiquantum photodetectors $(k \geq 2)$ is larger than the mean value corresponding to the case of detection of the same unmodulated radiation with an equivalent mean radiation intensity. This result, as can be readily understood, is the consequence of the nonlinearity of the process of detection, and is valid for all distribution functions of the modulated-radiation intensity $\mathscr{P}(\langle I \rangle)$.

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