ELECTROMAGNETIC RADIATION PRESSURE ON FREE ELECTRONS

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The pressure of electromagnetic radiation on free electrons in a spectral field of radiation is calculated. It is shown that the expression for the force acting on an electron in a spectral field differs essentially from that for the force in the field of a plane electromagnetic wave.

IN recent years considerable interest has been aroused by effects associated with induced Compton scattering on free electrons.[1-8] Induced scattering has recently been achieved experimentally.[6] Ya. B. Zel'dovich and the present author have previously[4,8,9] considered the statistical properties of a system that consists of a free electron gas and a photon gas. We considered in particular the establishment of a stationary state and we determined the effective temperature of electrons in an arbitrary nonequilibrium radiation field. It was found that under certain conditions induced scattering effects play a decisive role and lead to the establishment of an electron temperature that exceeds considerably the mean energy of nonequilibrium radiation quanta. However, we did not consider in detail the mean systematic force exerted by an arbitrary radiation field on an electron. In other words, we did not investigate the dynamic behavior of a single electron in a radiation field.

It is well known from classical electrodynamics that the force acting on an electron at rest in the field of a plane monochromatic wave is given by $\mathbf{f}_0 = \sigma_T \mathbf{q}/c$, where σ_T is the total Thomson cross section and $\mathbf{q}$ is the energy flux in the wave. We emphasize that for an electron moving in a monochromatic wave a systematic force arises only when we take into account the reaction of the electron to its own secondary (scattered) radiation.

Since the relation between the force and the flux is independent of the frequency, it was tacitly assumed that the expression for the force can be retained in the case of an electron within a spectral radiation field. For a moving electron in the presence of waves having different directions the force exerted on the electron by the radiation field is given by the Lorentz transformation of the force calculated for an electron at rest in the radiation field. In particular, for an electron moving with velocity $v \ll c$ in an isotropic radiation field it was assumed that the electron is subject to a decelerating force that is proportional to its velocity and to the radiation energy density ϵ, i.e., $\mathbf{f}_0 = -\frac{4}{3}\sigma_T \epsilon \mathbf{v}/c$.[10] These well known results are presented here only because our subsequent analysis will show that they are far from complete and in some instances do not reflect, even qualitatively, the behavior of electrons in real electromagnetic fields even when the conditions required by the derivations ($h\nu \ll m_e c^2$) are fulfilled.

Indeed, in contrast with the case of a monochromatic field, an electron moving in a spectral field of radiation that has a spread of directions will be acted upon by a systematic force even if the reaction forces are neglected. This effect can be illustrated by the motion of an electron in the field of two crossed waves.

An elementary calculation shows that, in contrast with the periodic motion of an electron in the field of a single wave, an electron in the field of the two waves should exhibit, besides its oscillations, a systematic motion, i.e. a systematic accumulation of energy. This follows from the fact that an electron oscillating in the electric field of one of the waves is acted upon by the Lorentz force of the other wave.[9] Diffusive heating of an electron gas is associated with this nonlinear interaction between an electron and the radiation field. The motion of an electron will be purely periodic only in the case of unidirectional waves, where the total magnetic field coincides in phase with the total electric field.

It should be noted that the effect considered here is intimately linked to the retardation and acceleration of electrons by standing waves, i.e. by a combination of two determinate waves having opposite directions.[11]

In a field of radiation having random phases only the force obtained by averaging over all the phases has real meaning. Therefore the dynamical problem of electron motion in a radiation field requires statistical treatment.

We know that in systems possessing adiabatic invariants a quantum-statistical approach followed by a transition to the classical limit $h \to 0$ is simpler and clearer intuitively than a classical treatment.[12,5] We shall therefore consider the motion of an electron in a photon gas described by a distribution function $N(\nu, \mathbf{n})$ that represents the numbers of photons in individual radiation modes each having a frequency ν and a propagation direction given by a unit vector $\mathbf{n}$.

It will be convenient to use a reference system in which the electron is at rest. It is easily shown that the systematic force acting upon the electron is given by the expression

$$\mathbf{f} = \int \sigma c N(\nu, \mathbf{n})\,[N(\nu', \mathbf{n}') + 1]\,\Delta\mathbf{p}\,\frac{\nu^2\,d\nu}{c^3}\,d\mathbf{n}\,d\Omega. \qquad (1)$$

Here $\sigma d\Omega$ is the differential cross section for photon scattering; ν' and $\mathbf{n}'$ pertain to the scattered quantum. The electron momentum change $\Delta\mathbf{p}$ when scattering occurs is given by

$$\Delta\mathbf{p} = \Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 = \frac{h\nu}{c}(\mathbf{n} - \mathbf{n}') + \frac{h\nu^2}{m_e c^3}\mathbf{n}'(1 - \mathbf{n}\mathbf{n}'). \qquad (2)$$

In calculating an explicit expression for the force $\mathbf{f}$ acting upon the electron the small scattering-induced change of frequency was always previously neglected. Thus, $\Delta\mathbf{p} \approx \Delta\mathbf{p}_1$ was assumed and the scattering cross section was taken to be

$$\sigma \approx \sigma_0 = \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1+(\mathrm{nn}')^2}{2}.$$

In the integration the term proportional to $N(\nu', n')N(\nu, n)\Delta p_1$ vanished identically because of the antisymmetric integrand.

We shall show here that the neglect of frequency change in scattering leads to a qualitatively incorrect description of the interaction between an electron and spectral radiation. This neglect is an incorrect procedure even when we can confine ourselves to a classical approximation ($h\nu/m_e c^2 \ll 1$). When we transfer to a classical description, i.e., when $N(\nu, n)$ is replaced by the spectral function $F(\nu, n) = 8\pi h\nu^3 c^{-3} N(\nu, n)$, we find, for example, that $NN'\Delta p_2$ is independent of h and must be taken into account, together with $N\Delta p_1$, in the classical expression for the force. To obtain a complete expression for the force in the classical limit we must retain in (1) all terms that do not contain h. This calculation must in the first approximation include the change of frequency in the expression for Δp as well as in the spectral function and scattering cross section. We then obtain

$$\mathbf{f} = \mathbf{f}_{sp} + \mathbf{f}_{ind} = \left[\int \frac{\sigma_0}{8\pi} F(\nu, \mathbf{n})(\mathbf{n} - \mathbf{n}')\, d\nu\, d\mathbf{n}\, d\Omega\right]$$
$$+ \left\{\int \frac{\sigma_0 c}{64\pi^2} \frac{F(\nu, \mathbf{n})F(\nu, \mathbf{n}')}{m_e \nu^2} \mathbf{n}'(1 - \mathbf{nn}')\, d\nu\, d\mathbf{n}\, d\Omega\right.$$
$$\left. - \int \frac{\sigma_0 c F(\nu, \mathbf{n})}{64\pi^2 m_e} \left[\frac{\partial F(\nu, \mathbf{n}')\nu^{-3}}{\partial \nu}\right] (\mathbf{n} - \mathbf{n}')(1 - \mathbf{nn}')\nu^2\, d\nu\, d\mathbf{n}\, d\Omega\right\}. \quad (3)$$

We observe that the part of the force

$$\mathbf{f}_{sp} = \mathbf{f}_0 = \int \frac{\sigma_0}{8\pi} F(\nu, \mathbf{n})(\mathbf{n} - \mathbf{n}')\, d\nu\, d\mathbf{n}\, d\Omega = \frac{\sigma_T \mathbf{q}}{c} \quad (4)$$

determined by spontaneous scattering agrees with the usual expression for radiation pressure on free electrons. The second term in (3), as is evident from its structure, describes the force resulting from induced scattering. The induced pressure vanishes only in the special case where an electron is subjected to a single wave or a set of parallel waves.

On the other hand, it can be shown that in an arbitrary radiation field where at low frequencies $h\nu \ll m_e c^2$ we have a brightness temperature $kT_{br} > m_e c^2$ the induced force exceeds the spontaneous force f_{sp}. Then the total radiation density can be small, with the bolometric temperature $T_{bol} = (\epsilon/a)^{1/4} < m_e c^2$. On the other hand, a direct calculation shows that for an electron moving in a field of Planckian equilibrium radiation we have $f_{ind} \sim (kT_r/m_e c^2) f_{sp}$, where T_r is the radiation temperature. In this case, in order of magnitude f_{ind} does not exceed the relativistic corrections to f_{sp}.

The character of the force $\mathbf{f}_{ind}$ depends essentially on the specific form of the nonequilibrium spectral function $F(\nu, n)$. We shall therefore consider certain special cases.

1. For a spectral distribution in the factorized form

$$F(\nu, \mathbf{n}) = F_0(\nu)\varphi(\mathbf{n}), \quad (5)$$

where F_0 and φ are arbitrary functions, we have

$$\mathbf{f}_{ind} = \frac{c}{64\pi^2 m_e} \int \frac{F_0^2}{\nu^2} d\nu \int \sigma_0 \varphi(\mathbf{n})\varphi(\mathbf{n}')\mathbf{n}'(1 - \mathbf{nn}')\, d\mathbf{n}\, d\Omega. \quad (6)$$

The second term for f_{ind} in (3) vanishes in F as given by (5) because of the antisymmetric integrand. In astrophysical applications it is of considerable interest to study the radiation from the surface of a homogeneous disk of radius r at distances $R \gg r$, in which case we have

$$\varphi(\mathbf{n}) = \varphi(\alpha, \psi) = \Theta(\cos\alpha - \cos\alpha_0), \quad (7)$$

where α and ψ are the polar and azimuthal angles, with $\alpha_0 \approx r/R \ll 1$, and Θ is a step function. In the cases of real objects with intense nonequilibrium radiation the spectral distribution is a cut-off power function of frequency:

$$F_0(\nu) = \begin{cases} a\nu^{-\gamma}, & \nu \geqslant \nu_0, \\ 0, & \nu < \nu_0, \end{cases} \quad (8)$$

where $\gamma > 1$.

In this case (6) gives

$$\mathbf{f}_{ind} = \frac{3\sigma_T}{64\pi} \frac{\alpha_0^2}{\nu_0^3} \frac{(-\gamma+1)^2}{2(\gamma + 1/2)} \frac{|\mathbf{q}|\mathbf{q}}{c}. \quad (9)$$

It is clear that f_{ind} can greatly exceed f_{sp} in the case of intense low-frequency radiation. For certain actually observed astrophysical objects f_{ind} is overwhelmingly greater than the ordinary pressure. It should also be noticed that with increasing distance from the emitter f_{ind} decreases as $(r/R)^6$, whereas the decrease of f_{sp} is proportional to $(r/R)^2$. Thus a distance from the surface of the disk can always be found where f_{sp} exceeds f_{ind}.

It should also be noted that this very effect appears in the calculation of the force that acts upon an electron moving at ultrarelativistic speed. It is easily shown by means of a Lorentz transformation of (9) that electron velocities $v \to c$ at which f_{sp} again exceeds f_{ind} can always be found.

2. We now consider the slow motion of an electron at a velocity $v \ll c$ in an isotopic radiation field described by a spectral function $F_0(\nu)$. Transforming to the rest system of the electron, we have

$$F(\nu, \mathbf{n}) = F_0(\nu) + \frac{\nu^4(\mathbf{pn})}{m_e c}\left(\frac{\partial}{\partial \nu}\frac{F_0}{\nu^3}\right). \quad (10)$$

Substituting (10) in the general equation for the induced force on the electron and integrating over all angles, we obtain the electron-decelerating force in terms of $F_0(\nu)$:

$$\mathbf{f}_{ind} = +\frac{\sigma_T}{\pi m_e} \frac{\mathbf{p}}{m_e}\left[-\frac{7}{120}\int\left(\frac{\partial F_0 \nu^{-3}}{\partial \nu}\right)^2 \nu^6 d\nu + \frac{13}{32}\int \frac{F_0^2}{\nu^2} d\nu\right]. \quad (11)$$

In connection with this last equation we must consider the following circumstance. The force $\mathbf{f}_{sp}$, which results from spontaneous scattering, has by definition the direction of the radiation energy flux **q**; in the present case of an isotropic radiation field this is opposite to the velocity of motion **v**. The same cannot be stated in the general case for $\mathbf{f}_{ind}$. However, it can be shown in the case of an isotopic radiation field that for any attainable spectral radiation function, i.e. for finite radiation energy density, the square brackets in (11) enclose a negative expression. Thus, in the special case of an isotropic radiation field $\mathbf{f}_{ind}$, like $\mathbf{f}_{sp}$, has the direction of the radiation energy flux.

3. In the next example we consider an electron in the field of oppositely directed radiation beams along the z axis. One of the beams is assumed to be coherent, so that

$$N_1 = J_1 \delta(\nu - \nu_1).$$

The other beam has a broad spectrum with the distribution $N_2(\nu)$. Using (3) and performing some lengthy calculations, we obtain the force component $f_{z\,ind}$:

$$f_{z\,ind} = \frac{6\sigma_T h^2 J_1}{c^5} \nu^{3/2} \frac{\partial}{\partial \nu} (N_2 \nu^{3/2}) \Big|_{\nu=\nu_1}. \quad (12)$$

We observe that, depending on the sign of $\partial N_2 \nu^{3/2}/\partial \nu|_{\nu=\nu_1}$, the force can have either sign, which will be independent of the sign (direction) of the total flux $q = q_1 + q_2$.

The direction of the induced force can oppose the direction of the combined energy flux of the electromagnetic field. Since $f_{z\,ind}$ is then proportional to the product of the intensities, the total force can also oppose the energy flux.

4. We have thus far limited ourselves to the nonrelativistic motion of electrons. The relativistic generalization of the derived equations is very complicated. However, for the special case of the spectral distribution used in Example 1 a calculation for ultrarelativistic electrons in an isotropic field yields a compact expression for f_{ind} in the laboratory system (omitting a dimensionless factor depending on γ, for the sake of simplicity)[1)]

$$\mathbf{f}_{ind} \sim -\frac{\sigma_T \varepsilon^2}{\nu_0^3 m_e} \frac{m_e c^2}{E} \frac{\mathbf{v}}{c}, \quad (13)$$

where E is the energy of the electron. It is interesting to compare this expression with the corresponding expression for f_{sp} in the ultrarelativistic case:

$$\mathbf{f}_{sp} = -\frac{4}{3} \sigma_T \varepsilon \left(\frac{E}{m_e c^2}\right)^2 \frac{\mathbf{v}}{c}. \quad (14)$$

We observe that with increasing electron velocity f_{sp} grows rapidly, but that f_{ind} decreases. This last effect can be understood intuitively. Increasing velocity is accompanied by narrowing of the angular cone within which induced scattering occurs. As a result there is a reduction in the weight of induced scattering processes as compared with spontaneous scattering.

Energy changes, the stationary distribution, and the effective temperature of electrons in an isotropic radiation field with a relativistic brightness temperature have been considered by Ya. B. Zel'dovich, R. A. Syunyaev, and the present author. It was found that induced scattering always heats electrons, but that their mean energy remains below the value obtained from the nonrelativistic equation given in[3,8]

In conclusion we wish to thank Ya. B. Zel'dovich, A. S. Kompaneets, V. G. Levich, and R. A. Syunyaev for many valuable discussions, and A. F. Illarionov for assistance with the calculations.

[1)] We are considering small values of ν_0, so that in the rest system of the electron we have $h\nu_0' \ll m_e c^2$, i.e., $h\nu_0 \ll (m_e c^2)^2/E$.

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