THETA-PINCH IN SEMICONDUCTORS

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Submitted January 19, 1971

Zh. Eksp. Teor. Fiz. 60, 2344–2350 (June, 1971)

A θ-pinch in semiconductors with bipolar conductivity is considered under the assumption that the magnetic field build-up time τ_H is much larger than both the skin time and the electron and hole momentum relaxation times. Approximate solutions are found for the carrier density equations in nondegenerate and strongly degenerate semiconductors in the case of cylindrical and plane geometry when the surface recombination rate s is small, such that $sa/D \ll 1$ (a is the sample size and D the ambipolar diffusion coefficient). It is shown that strong compression of an electron-hole plasma occurs in sufficiently strong magnetic fields for which $b_e b_h H^2/c^2 \gg 1$ (b_e and b_h are the electron and hole mobilities, respectively), if $\tau_H D/a^2 \ll 1$ and $\tau \gg \tau_H$ (τ is the volume recombination time).

1. In semiconductors with bipolar conductivity, a θ-pinch is possible, i.e., compression of the electron-hole plasma under the influence of a magnetic field H(t) that increases with time. Unlike a gas plasma[1], where the small skin effect causes the compression to occur as a result of a large magnetic-pressure gradient (the magnetic field does not have time to penetrate into the region occupied by the plasma during the time of the pulse), in semiconductors the skin layer as a rule is much thicker than the dimensions of the samples, and the θ-pinch results from the drift of the carriers in the crossed magnetic and induced electric fields (the so-called "non-skin" θ pinch).

In the present paper we derive equations describing the compression of an electron-hole plasma in nondegenerate and strongly degenerate semiconductors for the case of planar and cylindrical geometry under the assumption that the duration τ_H of the magnetic-field pulse is much longer than the skin time and the relaxation time of the electron and hole momentum. Approximate analytic solutions are obtained for these equations when the rate s of the surface recombination is small. It is shown that strong compression of the electron-hole plasma takes place if sufficiently strong magnetic fields are reached during the course of the pulse, $b_e b_h H^2/c^2 \gg 1$, where b_e and b_h are the mobilities of the electrons and of the holes. The θ-pinch is most strongly pronounced if the magnetic-field pulse duration is much shorter than the characteristic diffusion time a^2/D (a—dimension of sample, D—coefficient of ambipolar diffusion) and the time τ of the volume recombination.

2. To obtain the initial equations describing the compression law, we use Maxwell's equation $\partial \mathbf{H}/\partial t = -c\nabla\times\mathbf{E}$, the equations of motion of the electrons and holes, and the equation of continuity of the quasineutral plasma $n_e = n_h = n$, assuming the carrier flux on the sample surface to be ambipolar. For a nondegenerate semiconductor (the pressure is proportional to the density, $P \sim n$), these equations take the form

$$\mathbf{v}_e = -D_e\frac{\nabla n}{n} - b_e\mathbf{E} - \frac{b_e}{c}[\mathbf{v}_e\mathbf{H}], \quad (1)^*$$

$$\mathbf{v}_h = -D_h\frac{\nabla n}{n} + b_h\mathbf{E} + \frac{b_h}{c}[\mathbf{v}_h\mathbf{H}],$$

$$\frac{dn}{dt} + \nabla(n\mathbf{v}_e) = -\frac{n-n_0}{\tau},$$

$$\frac{dn}{dt} + \nabla(n\mathbf{v}_h) = -\frac{n-n_0}{\tau},$$

$$E_\varphi = -\frac{r}{2c}\dot{H}, \quad E_y = -\frac{x}{c}\dot{H}.$$

The magnetic field is directed along the OZ axis, and all the quantities depend on the coordinate r in a cylindrical sample of radius a and on the coordinate x in a plate of width 2a. It is assumed that the plate dimensions in the direction of the OY and OZ axes is much larger than the dimension in the OX direction (the OZ axis passes through the center of the plate). We consider the case of linear volume recombination, when the lifetimes of the electrons and of the holes are equal. By virtue of the quasineutrality, the ambipolarity condition takes the form

$$v_{er} = v_{hr} = v_r, \quad v_{ex} = v_{hx} = v_x.$$

In Eq. (1), $v_{i=e,h}$ are the velocities of the electrons and holes, D_i are the diffusion coefficients, n is the carrier density, and n_0 is the density in the absence of the pinch effect (in the case of an intrinsic semiconductor $n_0 = n_p$, where n_p is the equilibrium density).

The boundary conditions correspond to equality of the ambipolar flux on the surface of the sample to the flux of the surface recombination (generation):

$$nv_x|_{x=\pm a} = s(n-n_p), \quad nv_r|_{r=a} = s(n-n_p). \quad (2)$$

The initial conditions are:

$$n(x,0) = \begin{cases} n_0 & |x| \leqslant a \\ 0 & |x| > a \end{cases}, \quad n(r,0) = \begin{cases} n_0, & 0 \leqslant r \leqslant a \\ 0, & r > a \end{cases}. \quad (3)$$

The expressions for the ambipolar velocities in the plate and in the cylinder take the form

$$v_x = -\frac{D}{1+\gamma H^2}\frac{1}{n}\frac{dn}{dx} - \frac{\gamma H\dot{H}}{1+\gamma H^2}x,$$

$$v_r = -\frac{D}{1+\gamma H^2}\frac{1}{n}\frac{dn}{dr} - \frac{\gamma H\dot{H}}{1+\gamma H^2}\frac{r}{2},$$

where $\gamma = b_e b_h/c^2$.

The equations for the density in dimensionless variables are obtained in the form:

a) in a plate

$$\frac{\partial N}{\partial T} - \frac{h\dot{h}}{1+h^2}\frac{\partial}{\partial \xi}(\xi N) - \frac{1}{1+h^2}\frac{\partial^2 N}{\partial \xi^2} = -\frac{N-1}{\vartheta}, \quad (4)$$

*$[\mathbf{v}_e\mathbf{H}] \equiv \mathbf{v}_e \times \mathbf{H}$.

$$N(\xi,0)=\begin{cases}1 & |\xi|\leqslant 1\\ 0 & |\xi|>1\end{cases};\qquad (5)$$

$$\mp\frac{h\dot{h}}{1+h^2}N-\frac{1}{1+h^2}\frac{\partial N}{\partial\xi}\Big|_{\xi=\pm1}=\sigma\left(N-\frac{n_p}{n_0}\right);$$

b) in a cylindrical sample

$$\frac{\partial N}{\partial T}-\frac{1}{2}\frac{h\dot{h}}{1+h^2}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2N)-\frac{1}{1+h^2}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial N}{\partial\rho}\right)=-\frac{N-1}{\vartheta},\qquad (6)$$

$$N(\rho,0)=\begin{cases}1 & 0\leqslant\rho\leqslant 1\\ 0 & \rho>1\end{cases};\quad -\frac{1}{2}\frac{h\dot{h}}{1+h^2}N-\frac{1}{1+h^2}\frac{\partial N}{\partial\rho}\Big|_{\rho=1}=\sigma\left(N-\frac{n_p}{n_0}\right).\qquad (7)$$

Here $N = n/n_0$, $\xi = x/a$, $\rho = r/a$, $T = tD/a^2$, $\vartheta = \tau D/a^2$, $\sigma = sa/D$, $h = \sqrt{\gamma}H$, $\dot{h} = \partial h/\partial T$.

In a strongly degenerate semiconductor (degeneracy of one plasma component, say the electrons, suffices) we have $P \sim n^{5/3}$, and the diffusion terms in the analogous equations turn out to be nonlinear. The equations take the following form:

a) in a plate

$$\frac{\partial N}{\partial T}-\frac{h\dot{h}}{1+h^2}\frac{\partial}{\partial\xi}(\xi N)-\frac{1}{1+h^2}\frac{\partial}{\partial\xi}\left(N^{2/3}\frac{\partial N}{\partial\xi}\right)=-\frac{N-1}{\vartheta},\qquad (4')$$

$$N(\xi,0)=\begin{cases}1 & |\xi|\leqslant 1\\ 0 & |\xi|>1\end{cases};\quad \mp\frac{h\dot{h}}{1+h^2}N-\frac{1}{1+h^2}N^{2/3}\frac{\partial N}{\partial\xi}\Big|_{\xi=\pm1}=\sigma\left(N-\frac{n_p}{n_0}\right);\qquad (5')$$

b) in a cylindrical sample

$$\frac{\partial N}{\partial T}-\frac{1}{2}\frac{h\dot{h}}{1+h^2}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2N)-\frac{1}{1+h^2}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho N^{2/3}\frac{\partial N}{\partial\rho}\right)=-\frac{N-1}{\vartheta},\qquad (6')$$

$$N(\rho,0)=\begin{cases}1 & 0\leqslant\rho\leqslant 1\\ 0 & \rho>1\end{cases};\quad -\frac{1}{2}\frac{h\dot{h}}{1+h^2}N-\frac{1}{1+h^2}N^{2/3}\frac{\partial N}{\partial\rho}\Big|_{\rho=1}=\sigma\left(N-\frac{n_p}{n_0}\right).\qquad (7')$$

The problem will henceforth be solved for semiconductors with a pure surface, when $\sigma = 0$. This corresponds to conservation of the number of particles in the sample, and the boundary conditions are best written in integral form (which is the same for a degenerate and strongly degenerate semiconductor)

$$\int_0^1 d\xi N(\xi,T)=1,\qquad (8)$$

$$\int_0^1 d\rho\rho N(\rho,T)=\frac{1}{2}.\qquad (9)$$

3. In the case of strong magnetic fields, $h \gg 1$, it is possible to neglect the diffusion term in Eqs. (4), (6), (4′), and (6′). The first-order differential equations obtained in this case, at a given initial distribution, h have exact identical solutions

$$N(T)=[1+h^2(T)]^{1/2}e^{-T/\vartheta}\left\{1+\int_0^{T/\vartheta}\frac{d\beta e^\beta}{[1+h^2(\beta)]^{1/2}}\right\},\qquad (10)$$

which do not depend on the coordinates ξ or ρ. Such a distribution takes the form of a shelf, the height of which increases during the course of the pulse, and the width decreases, since the total number of particles is conserved. The solution of Eq. (10) is valid in that part of the sample where the particles are concentrated (on the periphery, near the surface, the density is equal to zero). On the edges of the distribution (10), the solution becomes discontinuous, $\partial N/\partial\xi$, $\partial N/\partial\rho \to \infty$, since it corresponds to the limiting case $D \to 0$. Inside the plasma column, Eq. (10) gives the correct value of the density.

The following conclusions can be drawn from this solution: 1) to obtain strong compression it is necessary to have large values of the magnetic field, $h(T) \gg 1$; 2) if the magnetic-field pulse duration exceeds the recombination time $T_H = \tau_H D/a^2 > \vartheta$, then the compression is small. The succeeding investigation of Eqs. (4), (6), (4′), and (6′) will therefore be carried out for the case $T_H \ll \vartheta$, when the volume recombination can be neglected. Then Eq. (10) greatly simplifies and coincides with the equation previously obtained in[2]

$$N(T)=[1+h^2(T)]^{1/2}.\qquad (11)$$

4. Let us consider the compression of a nondegenerate electron-hole plasma in a plate. We can verify by direct substitution in (4) that the expression

$$N(\xi,T)=\mu_1(T)\exp[-\nu_1(T)\xi^2]\qquad (12)$$

satisfies Eq. (4). The boundary condition (8) gives the connection between the functions $\mu_1(T)$ and $\nu_1(T)$, which takes the form

$$\frac{\sqrt{\pi}}{2}\mu_1(T)\nu_1^{-1/2}(T)\Phi(\sqrt{\nu_1(T)})=1,\qquad (13)$$

where $\Phi(\sqrt{\nu_1(T)})$ is the probability integral.

Since the functions $\mu_1(T)$ and $\nu_1(T)$ should increase with increasing magnetic field, it follows that at $\nu_1(T) \gg 1$, when $\Phi(\sqrt{\nu_1(T)}) \approx 1$, we can assume, with exponential accuracy, that

$$\mu_1(T)=2\pi^{-1/2}\nu_1^{1/2}(T).\qquad (14)$$

Substituting (12) in (4), and also using (14), we can find the functions $\mu_1(T)$ and $\nu_1(T)$, and the expression for the density takes the form

$$N(\xi,T)=\left[\frac{1+h^2(T)}{1+\pi T}\right]^{1/2}\exp\left[-\frac{\pi}{4}\frac{1+h^2(T)}{1+\pi T}\xi^2\right].\qquad (15)$$

If $T_H \ll 1$ and $h \gg 1$, then the solution (15) can be represented in the form

$$N(\xi,T)=h\exp\left\{-\frac{\pi h^2\xi^2}{4}\right\},\qquad (16)$$

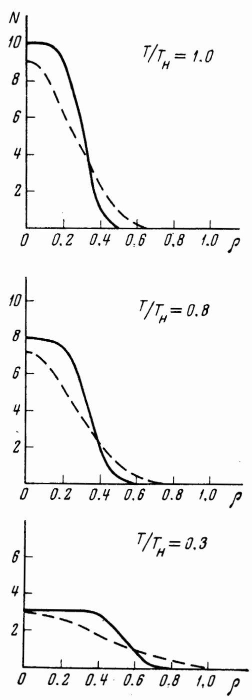
which coincides with (11) near the axis $\xi \to 0$. If $T_H \gg 1$ and $h \gg 1$, then the solution of (15) is

$$N(\xi,T)=(h/\sqrt{\pi T})\exp\left\{-\frac{h^2\xi^2}{4T}\right\}.\qquad (17)$$

As seen from a comparison of Eqs. (16) and (17), the θ-pinch is less noticeably pronounced when the magnetic-field pulse duration exceeds the characteristic diffusion time, and the magnitude of the compression is determined not only by the amplitude of the magnetic field but also by the rate of its growth.

Using similar reasoning for a cylindrical sample (Eq. (6)), we find that, with exponential accuracy, we have $\mu_2(T) = \nu_2(T)$, and the solution for the density turns out to be

$$N(\rho,T)=\frac{1+h^2(T)}{A}\exp\left\{-\rho^2\frac{[1+h^2(T)]^{1/2}}{A}\right\};\qquad A=1+4\int_0^T d\beta[1+h^2(\beta)]^{-1/2}.\qquad (18)$$



The figure shows the results of a numerical integration (solid curve) of Eq. (6) with boundary condition (7) in the case when the magnetic field increases linearly with time. The following parameters were specified: These parameters correspond to H_{max} = 450 kOe and $\tau_H = 3 \times 10^{-5}$ sec in a cylindrical sample of germanium with radius $a = 1.5 \times 10^{-1}$ cm. The dashed curve for the same parameters corresponds to formula (18). As seen from the figure, there is satisfactory agreement between the approximate solution (18) and the numerical integration of Eqs. (6) and (7).

5. In the case of a strongly degenerate electron-hole plasma, the process of compression has a somewhat different character. Let us consider Eqs. (4′) and (6′) without the right-hand side, $\vartheta \to \infty$, neglecting the volume recombination.

When the carrier density near the sample axis exceeds during the course of the compression the equilibrium value $N > 1$, then the particle density on the periphery decreases, since their total number is conserved. In the case of strong degeneracy the coefficient of ambipolar diffusion depends on the particle density $D \sim n^{2/3}$, and therefore in the near-surface layers of the sample the diffusion spreading decreases and the motion of the plasma periphery towards the center is more effective.

It can be shown that the solution

$$N(\xi, T) = \varphi_1(T)\,[1 - \xi^2 / \psi_1^2(T)]^{3/2}, \tag{19}$$

where $\varphi_1(T)$ and $\psi_1(T)$ are certain functions of the time, satisfies Eq. (4′). The function $\varphi_1(T)$ characterizes the law of variation of the density near the sample axis, and $\psi_1(T)$ determines the dimension of the region where the plasma is concentrated. The form of the solution (19) corresponds to pinching of the plasma[3], inasmuch as we have $N(\xi) = 0$ and $\partial N/\partial \xi = 0$ when $\xi = \psi_1(T)$. In such a situation, the boundary condition (8) takes the form

$$\int_0^{\psi_1(T)} d\xi N(\xi, T) = 1. \tag{8′}$$

Thus, the process of compression of a strongly degenerate electron-hole plasma can be represented in the following manner. The existing initial uniform distribution of the particles is transformed into $(1 - \xi^2)^{3/2}$ with increasing magnetic field, since $\psi_1(T)$ is still of the order of unity. Further increase of the magnetic field leads to a detachment of the plasma from the surface of the sample—pinching accompanied by a sharp increase of the density near the axis.

Using (19) and (8′), we obtain the connection between the functions $\varphi_1(T)$ and $\psi_1(T)$

$$\varphi_1(T)\psi_1(T) = 16/3\pi. \tag{20}$$

Further, substituting (19) in (4′) and also using (20), we obtain an ordinary differential equation for $\psi_1(T)$ (or for $\varphi_1(T)$). The final solution takes the form

$$\psi_1(T) = [1 + h^2(T)]^{-1/2}\left\{1 + 8\left(\frac{16}{3\pi}\right)^{2/3}\int_0^T d\beta\,[1 + h^2(\beta)]^{1/3}\right\}^{3/8},$$

$$\varphi_1(T) = \frac{16}{3\pi}[1 + h^2(T)]^{1/2}\left\{1 + 8\left(\frac{16}{3\pi}\right)^{2/3}\int_0^T d\beta\,[1 + h^2(\beta)]^{1/3}\right\}^{-3/8}. \tag{21}$$

From the obtained solution of Eqs. (19) and (21) we see that the initial distribution (5′) turns into $(1 - \xi)^{3/2}$ when the density on the sample axis turns out to equal $N(0, T) \approx 16/3\pi$.

Similar results were obtained for a cylindrical sample. In this case

$$N(\rho, T) = \varphi_2(T)\,[1 - \rho^2 / \psi_2^2(T)]^{3/2},$$

$$\psi_2(T) = [1 + h^2(T)]^{-1/4}\left\{1 + 10\left(\frac{5}{2}\right)^{2/3}\int_0^T d\beta\,[1 + h^2(\beta)]^{-1/6}\right\}^{3/10} \tag{22}$$

$$\varphi_2(T) = \frac{5}{2}[1 + h^2(T)]^{1/2}\left\{1 + 10\left(\frac{5}{2}\right)^{2/3}\int_0^T d\beta\,[1 + h^2(\beta)]^{-1/6}\right\}^{-3/5}.$$

6. The obtained solutions (15) and (18) for a nondegenerate plasma of a semiconductor are exact solutions of Eqs. (4) and (6) as $\vartheta \to \infty$ and satisfy with exponential accuracy the boundary conditions (5) and (7), starting with the instant of time when $\nu_{1,2}(T) \gg 1$. The distribution of the density takes the form of a Gaussian curve, the width of which decreases and the height of which increases with increasing magnetic field.

In the case of strong degeneracy, expressions (19), (21), and (22) are also exact solutions of Eqs. (4′) and (6′) as $\vartheta \to \infty$, and the boundary conditions corresponding to pinching of the plasma (8′) are satisfied exactly. The pinching of a degenerate plasma is the consequence of the fact that the pressure is $P \sim n^{5/3}$.

In conclusion, the authors thank V. F. Shanskiĭ for the numerical calculations, and A. M. Dykhne and V. N. Oraevskiĭ for valuable discussions.

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Translated by J. G. Adashko
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