HIGH-FIELD DOMAINS IN GUNN DIODES WITH TWO KINDS OF CARRIERS

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The effect of holes on the parameters and the behavior of stable domains in the Gunn diode are considered. It is shown that holes accelerate domain motion and create the conditions required for the formation of domains moving in a direction opposite to that encountered in the case of a purely electronic semiconductor. For a sufficiently high hole concentration, the domains may move from cathode to anode and from anode to cathode with equal velocities, which exceed the electron drift velocity and are determined by the balance between the electron diffusion time and the hole Maxwellian time. In this case holes restrict the maximal field in the domain and lead to the possibility of existence of a trapezoidal domain.

1. INTRODUCTION

In Gunn diodes the electric field in a domain can become so strong that band-band breakdown takes place and as a consequence, electron-hole pairs are generated. The generation of electron-hole pairs leads to the occurrence of a current-controlled negative resistance (S-shaped characteristic), pinching of the current, and other phenomena, the mechanism of which is considered in[1]. The characteristic times of establishment of the S-shaped characteristic in "short" samples and of the occurrence of the phenomena associated with this characteristic are much longer than the domain transit time. However, if the holes have a sufficiently high mobility, they can also qualitatively change the physical picture of the "fast" process—formation and propagation of a strong-field domain. The present article is devoted to an investigation of the influence of holes on the behavior of domains in Gunn diodes.

In GaAs, which is the material most widely used for the preparation of Gunn diodes, the hole mobility $\mu_p \approx 300-400$ cm^2/V-sec is much smaller than the electron mobility in a weak field, $\mu_n \approx 5000-8000$ cm^2/V-sec and the maximum negative differential mobility of the electrons $(\mu_{nd})_{max} \approx 2000$ cm^2/V-sec. Nonetheless, at a high pair concentration p_0 the influence of the holes on the current-voltage characteristic of the sample is significant where μ_{nd} is small. This occurs, first, in the region of fields close to the threshold, and, second, in the region of strong fields, where the dependence of the drift velocity of the electrons on the field becomes saturated. In accordance with these field regions, the holes can influence the threshold field at which the instability sets in, as well as the behavior of the already formed strong-field domain. The question of the influence of holes on the occurrence of the instability was considered in[2] on the basis of the linear theory. In the present paper we investigate the parameters and behavior of the stable domain within the framework of the nonlinear theory.

To illustrate the influence of holes at large pair concentrations, Fig. 1 shows the current-voltage characteristic, normalized to the total number of electrons n_0, of a homogeneous sample of GaAs, for two limiting

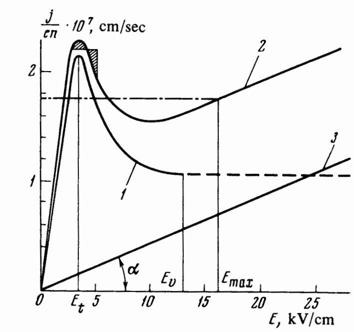

FIG. 1. Current-voltage characteristic of a homogeneous GaAs sample, normalized to the total number of electrons (j—current density, E—bias field): curve 1—$p_0 = 0$, $n_0 = n_D$; curve 2—$p_0 = n_0 \gg n_D$; curve 3—dependence of the drift velocity of the holes in an inhomogeneous sample on the field; $\tan\alpha = \mu_p$. The dashed curve represents an extrapolation of the experimental data obtained in [3]. The shaded areas illustrate the area rule for a domain propagating at high velocity u_0 in a sample with holes. The dash-dot line corresponds to the area rule for a trapezoidal domain.

cases: 1) $p_0 = 0$, $n_0 = n_D$, where n_D is the donor concentration (curve 1) and 2) $p_0 \approx n_0 \gg n_D$ (curve 2). Curve 1 is taken from[3], and curve 2 was obtained by adding curve 1 to curve 3 (for $\mu_p \approx 400$ cm^2/V-sec).

In addition to the situation mentioned above, when the holes are produced in the Gunn diode as a result of impact ionization in the strong-field domain, the problem of the influence of holes on the domain has a bearing on any case of drift instability in an intrinsic semiconductor or in a semiconductor in whose volume minority carriers or electron-hole pairs are produced, for example by external illumination or injection. For concreteness we assume, as before, that the dependence of the electron drift velocity v_n on the field E has a descending section, and that the differential conductivity of the holes is positive.

As will be shown below, at a low hole conductivity the motion of the domain accelerates, and the domain velocity increases with increasing hole concentration. Even at a relatively low hole concentration, there can exist, besides the domain propagating from the cathode to the anode (in analogy with the domain in a purely electronic semiconductor), also a domain of the same shape but propagating in the opposite direction (from the anode to the cathode) with the drift velocity of the holes, corresponding to the maximum field in the domain. (At low hole concentration, the occurrence of a domain propagating from the anode to the cathode is prevented by hole diffusion). At large hole concentration, the domain can propagate from the cathode to the anode and in the opposite direction with equal velocity, determined by the diffusion of the electrons and by the Maxwellian time of the holes ($D_n/u^2 \sim \tau_{mp}$, where D_n is the electron diffusion coefficient, u the domain velocity, and τ_{mp} the Maxwellian time of the holes). Positive conductivity of the holes leads also to the appearance of a rising section on the current-voltage characteristic in the region of strong fields, thus limiting the amplitude of the domain.

2. FUNDAMENTAL EQUATIONS OF THE PROBLEM

In the presence of two sorts of carriers, the fundamental equations of the phenomenological theory of the Gunn effect assume the form

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}\left(p\mu_p E-D_p\frac{\partial p}{\partial x}\right)=0, \quad (1)$$

$$\frac{\partial n}{\partial t}-\frac{\partial}{\partial x}\left[nv_n(E)+D_n\frac{\partial n}{\partial x}\right]=0, \quad (2)$$

$$\frac{\partial E}{\partial x}=\frac{4\pi e}{\varepsilon}(p-p_0-n+n_0), \quad (3)$$

where n and p are the concentrations, j_n and j_p the currents, and D_n and D_p the diffusion coefficients of the electrons and holes, respectively, μ_p is the hole mobility, ϵ is the dielectric constant of the material, and n_0 and p_0 are the electron and hole concentrations in the homogeneous sample. (In the case when the holes are produced by impact ionization, as for example in strongly-doped Gunn diodes, n_0 is the sum of the concentration n_D of the ionized donors and the concentration p_0 of the pairs: $n_0 = p_0 + n_D$.) For simplicity we assume that D_n and D_p do not depend on the field.

When $\mu_p \to 0$ and $p = p_0$, Eqs. (1)—(3) go over into the standard system of equations of the theory of the Gunn effect. At a finite value of μ_p, holes change the picture of the instability.

A linear theory of instability in a semiconductor with two types of carriers was considered in[2]. It is shown there, in particular, that the presence of carriers with positive differential conductivity makes it possible to produce in the sample, besides a growing wave propagating from the cathode to the anode, also a growing wave propagating from the anode to the cathode.

In the present paper we shall seek nonlinear solutions of Eqs. (1)—(3) that correspond to formed strong-field domains. All the quantities depend only on $z = x - ut$, where u is the velocity of the domain, and the system of equations (1)—(3) is transformed into a system of ordinary differential equations, in which (1) and (2) can be integrated. Substituting Eq. (3) in (1) and (2), we obtain

$$p(\mu_p E-u)-D_p\frac{dp}{dz}=p_0(\mu_p E_1-u), \quad (4)$$

$$n[v_n(E)+u]+D_n\frac{dn}{dz}=n_0[v_n(E_1)+u], \quad (5)$$

$$\frac{dE}{dz}=\frac{4\pi e}{\varepsilon}(p-p_0-n+n_0). \quad (6)$$

Here E_1 is the field outside the domain. Equations (4)—(6) can be rewritten in the form

$$\frac{4\pi eD_n}{\varepsilon}\frac{dn}{dE}=\frac{nv_n(E)-n_0v_n(E_1)+(n-n_0)u}{n-n_0-p+p_0}, \quad (7)$$

$$\frac{4\pi eD_p}{\varepsilon}\frac{dp}{dE}=\frac{\mu_p(pE-p_0E_1)-u(p-p_0)}{p-p_0-n+n_0}. \quad (8)$$

Let us consider first the case when it is possible to neglect the hole diffusion. The physical reason why the hole diffusion can be neglected is that the holes have a smaller mobility and a lower temperature than the electrons, since the electrons are heated. We shall obtain below criteria for the applicability of the approximation $D_p = 0$ and show that this approximation is good for a wide range of parameters, including the values corresponding to GaAs. (The case when hole diffusion is significant will be considered separately in Sec. 4.)

At $D_p = 0$ we obtain from (7) and (8) one first-order equation for the space-charge density $\rho = e(p - p_0 - n + n_0)$:

$$\frac{4\pi D_n}{\varepsilon}\rho\frac{d\rho}{dE}=e(n_D+p_0)[v_n(E)-v_n(E_1)]$$
$$-[u+v_n(E_1)]\left[\rho+\frac{ep_0\mu_p(E-E_1)}{\mu_pE-u}\right]+\frac{4\pi eD_np_0\mu_p}{\varepsilon(\mu_pE-u)^2}(u-\mu_pE_1)\rho. \quad (9)$$

We confine ourselves first to the case of large stationary electron concentration n_0. As will be shown below, the space-charge density in the domain is small in this case ($\rho/en_0 \ll 1$). A large stationary electron concentration n_0 can arise in the sample either as a result of a large concentration of electron-hole pairs produced in the sample by illumination or impact ionization[1)] ($n_0 = n_D + p_0$).

We seek the solution of (9) in the form of a series

$$\rho=\rho^{(0)}+\rho^{(1)}+\ldots$$

$\rho^{(0)}$ and $\rho^{(1)}$ are determined from the equations

$$\frac{4\pi D_n}{\varepsilon}\rho^{(0)}\frac{d\rho^{(0)}}{dE}=e(n_D+p_0)[v_n(E)-v_n(E_1)]$$
$$-[v_n(E_1)+u]\frac{ep_0\mu_p(E-E_1)}{\mu_pE-u}\equiv eF(E,u), \quad (10)$$

$$\frac{2\pi D_n}{\varepsilon}\frac{d}{dE}(\rho^{(0)}\rho^{(1)})=\frac{4\pi eD_np_0\mu_p}{\varepsilon(\mu_pE-u)^2}(u-\mu_pE_1)\rho^{(0)}-[u+v_n(E)]\rho^{(0)}. \quad (11)$$

Using (10), we can rewrite (11) in the form

$$\frac{d}{dE}(\rho^{(0)}\rho^{(1)})=-\frac{2}{3e(n_D+p_0)}\frac{d(\rho^{(0)})^3}{dE}-F_1(E,u)\rho^{(0)}, \quad (12)$$

$$F_1(E,u)=\frac{\varepsilon}{2\pi D_n}\Big\{u+v_n(E_1)-\frac{4\pi eD_np_0\mu_p(u-\mu_pE_1)}{\varepsilon(\mu_pE-u)^2}$$
$$+\frac{p_0\mu_p(E-E_1)}{(n_D+p_0)(\mu_pE-u)}[u+v_n(E)]\Big\}. \quad (12a)$$

From (10)—(12) we get

$$(\rho^{(0)})^2=\frac{\varepsilon e}{2\pi D_n}\int_{E_1}^{E}F(E',u)dE', \quad (13)$$

$$\rho^{(1)}=-\frac{2(\rho^{(0)})^2}{3e(n_D+p_0)}-\frac{1}{\rho^{(0)}(E)}\int_{E_1}^{E}dE'\rho^{(0)}(E')F_1(E',u). \quad (14)$$

As $\mu_p \to 0$ or $p_0 \to 0$, the solution (13) describes a

1) We note that impact ionization in GaAs Gunn diodes is usually observed in highly doped samples ($n_0 \gg 2.5 \times 10^{15}$ cm^{-3}), when, as shown in [4], the condition $(n-n_0)/n_0 \ll 1$ is satisfied.

stable strong-field domain in a strongly doped sample (see[4]).

We seek a solution describing the strong-field domain. From the Poisson equation it follows that at the maximum value of the field in the domain E_m the space-charge density is $\rho^{(0)}(E_m = 0)$. It follows from this condition that

$$\int_{E_1}^{E_m} F(E, u)\, dE = 0. \tag{15}$$

On the other hand, as is seen from (14), in order for $\rho^{(1)}$ to remain finite at the point E_m, it is necessary to have

$$\int_{E_1}^{E_m} dE\, \rho^{(0)}(E) F_1(E, u) = 0. \tag{16}$$

Simultaneous solution of the integral equations (15) and (16) determines the maximum value of the field in the domain E_m and the velocity of the domain u.

3. CASE OF LOW HOLE CONCENTRATION

In the case of sufficiently low hole concentrations (in which case the solutions (13) and (14) are valid at large n_D), $\rho^{(0)}$ has the same form as in the absence of holes. It follows from (15) that E_m is determined in this case (accurate to small corrections) with the aid of the usual "area" rule in the theory of strong-field domains[5,6]. Therefore in the zeroth approximation there is likewise no change in the dynamic characteristic of the sample with the domain (see[4]).

It follows from the form of the function $F_1(E, u)$ that Eq. (16) should have roots

$$u_1 = -v_n(E_1) + \Delta u_1, \tag{17}$$

$$u_2 = \mu_p E_m + \Delta u_2, \tag{18}$$

where Δu_1 and Δu_2 are small corrections proportional to the powers of p_0. The root u_1 is connected with the vanishing of the first (large) term in F_1 (see formula (12a)). The root u_2 is connected with the fact that as $u \to \mu_p E_m$ a small quantity Δu_2 arises in the denominators of the second and third terms and compensates for the smallness of the quantity in the numerator. We note that as $p_0 \to 0$ or $\mu_p \to 0$ the root u_1 goes over into the usual expression for the velocity of the strong-field domain ($u = -v_n(E_1)$ in the standard theory of the Gunn effect[5]).

From (16) and (12a) we get

$$\Delta u_1 = u_1 + v_n(E_1) = -\frac{4\pi e D_n}{\varepsilon} p_0 \mu_p [v_n(E_1) + \mu_p E_1] \times \int_{E_1}^{E_m} \frac{dE\,[n^{(0)}(E) - n_0]}{[v_n(E_1) + \mu_p E]^2} \left\{ \int_{E_1}^{E_m} dE' [n^{(0)}(E') - n_0] \right\}^{-1}. \tag{19}$$

Here

$$[n^{(0)}(E) - n_0]^2 = \frac{\varepsilon n_0}{2\pi D_n} \int_{E_1}^{E} dE' [v_n(E') - v_n(E_1)] \tag{20}$$

is the deviation of the electron concentration in the domain from the equilibrium value (compare with[4]). As seen from (19), the holes accelerate the motion of the domain. Estimates of the integrals in (19) and (20) are given in the Appendix. They were made for the most nontrivial case of mobile holes: $\mu_p E_V < v_n(E_1) < \mu_p E_m$ (E_V is the value of the field above which v_n becomes independent of E) and lead to the result

$$-\frac{\Delta u_1}{v_n(E_1)} \approx +\frac{6\pi e D_n p_0}{\varepsilon E_m v_n(E_1)} < 1. \tag{21}$$

In order for the results to be valid, it is necessary, besides satisfying inequality (21), that the second term in (10) be small compared with the first and the charge density $\rho^{(1)}$ be small compared with $\rho^{(0)}$ in the field region $E_V < E < E_m$, in which this requirement is most difficult to satisfy. The corresponding estimates, which are analogous to the estimates given in the Appendix, lead to the following inequalities:

$$n_0 \gg n_{cr} \equiv \frac{\varepsilon \mu_n E_v}{4\pi e D_n}(E_t - E_1) \tag{22a}$$

(this inequality corresponds to the condition of "strongly-doped" samples, obtained earlier in[4]) and

$$p_0 / n_D < (n_{cr} / 4 n_D)^{1/2}. \tag{22b}$$

(We note that in this case the second term in the right-hand side of (14) turns out to be larger than the second term in the right-hand side of (10)).

All the numerical estimates will be made for GaAs, with the following parameters: $\epsilon = 12.5$, $\mu_n = 6000$ cm²/V-sec, $\mu_p = 300$ cm²/V-sec, $E_1 = 1.5$ kV/cm, $E_t = 3$ kV/cm, $D_n = 400$ cm²/sec, $E_m \approx 90$ kV/cm, and $E_V \approx 15$ kV/cm. We then obtain from (21), (22a), and (22b) that $n_{cr} \approx 2.5 \times 10^{15}$ cm^{-3}, and from inequalities (21), (22a), and (22b) it follows that

$$p_0 < 10^{16}\ \text{cm}^{-3}, \quad n_D > 2.5 \cdot 10^{15}\ \text{cm}^{-3}, \quad p_0 / n_D \ll (6 \cdot 10^{14} / n_D)^{1/2}.$$

Thus, the results obtained above describe, for example, Gunn diodes at their relatively low level of impact ionization (see, for example,[7]).

We now consider the root u_2 (see formula (18)). Just as for u_1, the first two terms are of importance in the calculation of the integral in (16), the main contribution from the second term to the integral being made by the field region E_m. The calculation of the integral (16) yields

$$\frac{\Delta u_2}{\mu_p E_m} = \left\{ \frac{3\pi^2 D_n e p_0}{\varepsilon E_m [\mu_p E_m + v_n(E_1)]} \right\}^2 < 1. \tag{23}$$

Just as in the case of root u_1, in order for the employed approximations to be valid it is necessary, in addition to satisfaction of inequality (23), that the second term in (10) be small compared with the first and the charge density $\rho^{(1)}$ be small compared with $\rho^{(0)}$ in the field region $E_V < E < E_m$. The corresponding estimates lead to the following inequalities:

$$\frac{p_0}{n_D} > \frac{32}{9\pi^2}\left(\frac{n_{cr}}{n_D}\right)^2 A^3, \qquad A = \frac{(\mu_p E_m + \mu_n E_1) E_m}{\mu_n E_v (E_t - E_1)}; \tag{24a}$$

$$^{16}/_9 A^2 < n_D / n_{cr} \tag{24b}$$

As before, it is necessary also to satisfy the inequalities (22a).

Numerical estimates for GaAs, make for the indicated values of the parameters, yield in this case

$$p_\theta < 2{,}5 \cdot 10^{16}\ \text{cm}^{-3}, \quad p_0/n_D > (1.7 \cdot 10^{17}/n_D)^2, \quad n_{cr}/n_D < 10^{-3},$$

i.e., $n_D > 2.5 \times 10^{18}$ cm^{-3}.

The hole concentration in the domain is

$$p = p_0 \frac{\mu_p E_1 - u}{\mu_p E - u} + \frac{4\pi D_p \mu_p p_0 (u - \mu_p E_1)}{\varepsilon (\mu_p E - u)^3} \rho. \tag{25}$$

Expression (25) was obtained from (8) by formal expansion with respect to D_p. In the derivation of (9) we took into consideration only the zeroth term of the expansion (25). From (25), (18), and (23) it follows that the hole concentration in a domain moving with velocity u_2 increases with decreasing hole concentration p_0 outside the domain. Therefore the hole concentration gradient increases with decreasing p_0 and, consequently, the diffusion of the holes becomes all the more appreciable. At excessively small concentrations p_0, the hole diffusion makes it impossible to realize the domains corresponding to the root u_2. Therefore the solution obtained by us is valid down to concentrations satisfying the inequality

$$4\pi D_p\mu_p\rho / \varepsilon(\Delta u_2)^2 \ll 1,$$

as follows from a comparison of the second term with the first in (25). Substituting in this inequality the estimate given in the Appendix for the charge density as a function of the electric field, we obtain an upper bound on the hole density:

$$\frac{p_0}{n_D} > \frac{4}{3}A\left(\frac{4\pi eD_p n_{\rm cr}}{\varepsilon\mu_p E_m^2}\right)^{1/4}\left(\frac{n_{\rm cr}}{n_D}\right)^{7/8}. \qquad (26)$$

Substituting the numerical estimates given above for GaAs, we obtain at $eD_p/\mu_p = T = 300°K$:

$$p_0/n_D > 6(n_{\rm cr}/n_D)^{7/8}.$$

Thus, as shown above, in the presence of holes, unlike a semiconductor with one kind of carrier, there are two types of solutions corresponding to strong-field domains. The solution of the first type (root u_1) describes a domain propagating, just as in the absence of holes, from the cathode to the anode, with the holes accelerating the motion of the domain. Solutions of the second type, which are realized even at relatively low values of p_0, correspond to a domain propagating in the opposite direction, from the anode to the cathode. Which of these two solutions is realizable in the experiment depends on the boundary conditions at the contacts as determined by the technology of their preparation.

4. CASE OF SUFFICIENTLY HIGH HOLE DENSITY

As follows from the solutions obtained in Sec. 3, the absolute value of the domain velocity increases with increasing p_0. (This is valid for both roots.) One can therefore expect that at sufficiently large hole densities there will be realized a situation in which $u \gg \mu_p E_m + v_n(E_1)$. We then have from (10) and (15)

$$\int_{E_1}^{E_m}\{(n_D+p_0)[v_n(E)-v_n(E_1)]+p_0\mu_p(E-E_1)\}dE = 0. \qquad (27)$$

Expression (27) is a generalization of the area rule of the standard theory of strong-field domains[5,6]. This rule connects the density of the external current $j = ep_0\mu_p E_1 + en_0 v_n(E_1))$ with the maximum field in the domain E_m (see Fig. 1). From the area rule (27) it follows that the maximum field in the domain E_m cannot be larger than a certain critical value E_{max} that depends on the hole conductivity, even if the characteristic $v_n(E)$ does not impose any limitations on the amplitude of the domain in a purely electronic

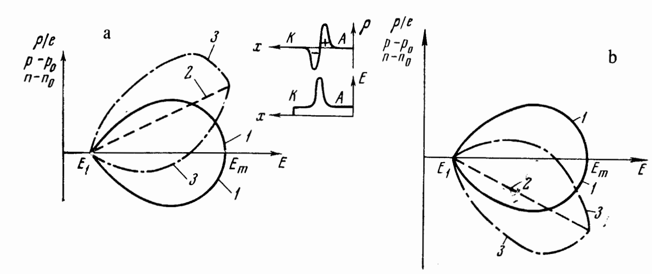

FIG. 2. Schematic field dependences of the space charge density ρ/e (curve 1), of the excess hole density $p-p_0$ (curve 2), and excess electron density $n-n_0$ (curve 3) in the domain walls: a—for a domain propagating from the anode to the cathode, b—for a domain propagating from the cathode to the anode. In the upper right corner of Fig. 2a are shown the distribution profiles of the space charge and of the field along the sample, corresponding to the strong-field domain. K—cathode, A—anode.

semiconductor (see Fig. 1). Thus, in a Gunn diode with two kinds of carriers there can exist a "trapezoidal" domain with a constant amplitude E_{max}, the width of which increases with increasing bias voltage.

Using the condition $u \gg \mu_p E_m$, $v_n(E_1)$, we obtain from (12a) and (16)

$$u = \pm u_0 = \pm\left[\frac{4\pi eD_n}{\varepsilon}\mu_p p_0\right]^{1/2} = \pm\sqrt{\frac{D_n}{\tau_{mp}}}, \qquad (28)$$

where τ_{mp} is the Maxwellian time of the holes.

Just as in the case of relatively low hole density, the domain can move not only from the cathode to the anode, but also in the opposite direction, and in this case the domain velocity is the same in both directions. The concentration of the electron-hole pairs in the domain varies with the direction of the main motion: in the case of motion from the cathode to the anode, the concentration of the electron-hole pairs in the domain is smaller than outside the domain, while in the case of motion in the opposite direction the pair concentration in the domain is higher. At the same time, the distribution profiles of the space charge and of the field in the domain do not depend on the direction of its motion (see Fig. 2). As a already noted above, the choice of the domain-motion direction may be determined by the conditions on the sample contacts.

Let us consider now the criteria for the applicability of the results obtained for the case of relatively high hole densities. From the conditions $u \gg \mu_p E_m$, $v_n(E_1)$ we obtain lower bounds for the hole density:

$$p_0 \gg \varepsilon v_n^2(E_1) / 4\pi eD_n\mu_p, \qquad (29a)$$

$$p_0 \gg \varepsilon\mu_p E_m^2 / 4\pi eD_n. \qquad (29b)$$

Inasmuch as in the case of high hole density the field amplitude in the domain is bounded (Fig. 1), we assume for numerical estimates $E_m \sim 30$ kV/cm. Then, using the numerical values given above for the remaining parameters, the criteria (29) yield $p_0 > 10^{16}$ cm^{-3}. Such hole-density values in Gunn diodes correspond to the experimental situation (see, for example,[5]).

The value of E_m used for the estimates pertains to the case $n_D \ll n_0 \sim p_0$ (see Fig. 1). If $n_D \gg p_0$, the holes can also strongly alter the velocity of the domain, even if E_m is approximately the same as in the domain propagating in a purely electronic sample. The hole density, which increases appreciably the velocity of the domain, will in this case be larger in absolute value and will increase in proportion to $\sqrt{n_D}$, since in a purely electronic sample the value of E_m is larger and increases in proportion to $n_D^{1/4}$ [4]. The hole charge in the domain

$$e(p-p_0)=\frac{e\mu_pE_m}{u}p_0=E_m\sqrt{\frac{\mu_pp_0}{4\pi eD_n}}$$

is in this case small compared with the electron charge $\rho_n \sim e\sqrt{n_0n_{cr}}$. The reason why the holes can alter the velocity of the domain strongly even in this case is as follows. In strongly-doped samples ($n_0 \gg n_{cr}$) the electron space charge $e(n^{(0)}-n_0)$ is small and, in first approximation, independent of the domain velocity (see[4]). The domain velocity is connected only with the small correction $en^{(1)}$ to the space charge of the electrons. Therefore in order to change the velocity of the domain, it suffices for the hole charge to be comparable with the small correction $en^{(1)}$.

From the inequalities (29) follows the condition

$$p_0 \gg \frac{\varepsilon v_n(E_1)E_m}{4\pi eD_n}\equiv n_{cr}\frac{E_1E_m}{E_v(E_t-E_1)}. \quad (29c)$$

(The inequality (29c) is obtained by multiplying the inequalities (29a) and (29b).) It follows from (29c) that the domain can propagate with velocity $\pm u_0$ only if $p_0 \gg n_{cr}$, since $E_1 \sim E_t - E_1$ and $E_m \gg E_v$. The electron and hole density oscillations in the domain, just as in a strongly-doped n-type sample (at $n_0 \gg n_{cr}$, see[4]), are small in this case in comparison with the stationary density. When $p_0 \gg n_D$ the charge density ρ in the domain is proportional in the case under consideration to $\sqrt{p_0}$, just as $n - n_0 \propto \sqrt{p_0}$ and $p - p_0 \propto \sqrt{p_0}$.

We shall show that in order for the results (27) and (28) to be valid it is necessary only to satisfy the criteria (29), and the validity of (27) and (28) is not connected, for example, with the approximations used in the derivation of (10)—(12). Indeed, at $u \gg \mu_pE_m$, $v_n(E_1)$ we have from (9)

$$\frac{2\pi D_n}{\varepsilon}\frac{d\rho^2}{dE}=e(n_D+p_0)[v_n(E)-v_n(E_1)]+ep_0\mu_p(E-E_1)-\rho u\left(1-\frac{u_0^2}{u^2}\right). \quad (30)$$

From (30) we get

$$\frac{2\pi D_n}{\varepsilon}\rho^2=e\int_{E_1}^{E}dE'\{(n_D+p_0)[v_n(E')-v_n(E_1)] + \mu_pp_0(E'-E_1)\}-u\left(1-\frac{u_0^2}{u^2}\right)\int_{E_1}^{E}\rho(E')dE'. \quad (31)$$

The strong-field domain consists of an increasing-field region, in which $\rho > 0$ everywhere, and a decreasing-field region, in which $\rho < 0$. Therefore the sign of the second term in the right-hand side of (31) at $u^2 \neq u_0^2$ will depend on the region (of increasing or decreasing field) to which the integration path corresponds. Therefore when $u^2 \neq u_0^2$ it is impossible to satisfy the condition $\rho(E_m) = 0$. Thus, it remains to put $u = \pm u_0$ (see formula (28)). Using the condition $\rho(E_m) = 0$, we then obtain from (31) the formula (27).

The picture of high-velocity domain motion is best considered for the limiting case of a trapezoidal domain, the motion of which constitutes synchronous motion of two charge layers separated by the width of the flat top of the domain. Neglecting diffusion, these layers are discontinuity planes moving through the semiconductor. When the discontinuity plane moves with velocity u, the laws for the continuity of electron and hole flow through this plane should be satisfied. Changing over to a coordinate system in which the discontinuity plane is at rest, we obtain, by equating the fluxes of electrons and holes on both sides of the discontinuity surface, the following relations:

$$(u-\mu_pE_m)p_m=(u-\mu_pE_1)p_0, \quad (32)$$

$$[u+v_n(E_m)]n_m=[u+v_n(E_1)]n_0. \quad (33)$$

Here n_m and p_m are the electron and hole densities in the domain at a field $E = E_m$. From (32) and (33) we find that in the case of a discontinuity surface moving with a velocity greatly exceeding the drift velocities of the electrons and the holes, the concentration jumps $n_1 = n_m - n_0$ and $p_1 = p_m - p_0$ are equal to

$$p_1=\frac{\mu_pE_m}{u}p_0,\quad n_1=-\frac{v_n(E_m)-v_n(E_1)}{u}n_0,$$

with $p_1 = -n_1$, since $\rho(E_m) = 0$. Thus, $\mu_pE_m = V_n(E_1) - v_n(E_m)$.

The electron diffusion smears the discontinuity surface out into a layer, the order of magnitude of whose width d can be estimated by starting from the fact that the diffusion flux in the layer is of the same order as the flux connected with the electron drift:

$$D_nn_1/d \sim n_1u.$$

Hence $d \sim D_n/u$. As follows from the Poisson equation, the field jump is connected with the space charge in the layer: $E_m - E_1 \sim (4\pi/\epsilon)\rho d$. Since inside the layer we have $p_1 \sim n_1 \sim (\mu_pE_m/u)p_0$, it follows that $E_m \sim (4\pi\mu_p/\epsilon u)E_mp_0d$, i.e., $d \sim \epsilon u/4\pi ep_0\mu_p$. Comparing the two relations $d \sim D_n/u$ and $d \sim \epsilon u/4\pi ep_0\mu_p$, we obtain an expression for the velocity with which the discontinuity surface can move through the electron-hole plasma of the semiconductor. The motion of such a discontinuity surface causes the uncompensated charge in the layer into which this surface becomes smeared out. In order for the system to remain neutral, it is necessary that there exist a second discontinuity surface, with an equal and opposite charge. This is possible only as a result of the presence of negative differential conductivity of the electrons. The charge ρd on the surface is uniquely connected with the velocity of the surface. Thus, the trapezoidal domain constitutes synchronous motion, with velocity $\pm u_0$, of two discontinuity surfaces that are smeared out by diffusion.

The picture presented above for the motion of the trapezoidal domain makes it possible to estimate the upper limit of the domain velocity, i.e., the upper value of the hole density, at which our theory is still valid. The smeared-out width of the domain wall cannot be smaller than the electron mean free path l, since the smearing of the discontinuity surface is connected with diffusion of the electrons:

$$d \sim D_n/u > l.$$

Substituting the estimate $D_n \sim lv_T$, where v_T is the thermal velocity of the electrons, we obtain $v_T > u$, i.e., the trapezoidal domain cannot move with a velocity exceeding the thermal velocity. The thermal velocity of the electrons in the domain can be estimated by using the results of[8]. Assuming for GaAs that $T \sim 10^4\,^\circ K$ and $m^* = 0.40m$ (for heavy electrons), we obtain $v_T \sim 7 \times 10^7$ cm/sec. Thus, $u_{max} \sim v_T \sim 7 \times 10^7$ cm/sec, i.e., $p_0 < \epsilon v_T^2/4\pi e D_n \mu_p \sim 2 \times 10^{17}$ cm^{-3}. At large values of p_0 it is necessary to take into account the temporal and spatial dispersions of the kinetic coefficients.

Another limitation on the region of applicability of our theory may be connected with the fact that we have assumed μ_p to be independent of the field. We note that if the drift velocity of the electrons becomes saturated in strong electric fields, then all the effects connected with the influence of the holes on the parameters and on the behavior of the strong-field domains will be less strongly pronounced. However, as shown by the experimental data[9], the dependence of μ_p on the field in GaAs becomes significant only in fields on the order of 60 kV/cm and above. Therefore, as seen from the foregoing estimates, this limitation is immaterial as applied to GaAs.

Let us see now how the hole diffusion changes the domain parameters in the case of large hole density. The hole density is determined in this case by formula (25), which is an expansion in terms of the reciprocal of the high velocity u. Substituting (25) in (7) and using the inequality $u \gg \mu_p E_m$, $v_n(E_1)$, we obtain

$$\frac{2\pi D_n}{\varepsilon e}\frac{d\rho^2}{dE}\left(1+\frac{D_p u_0^2}{D_n u^2}\right) = n_0[v_n(E) - v_n(E_1)] + p_0\mu_p(E - E_1) - u\left(1-\frac{u_0^2}{u^2}\right)\frac{\rho}{c} - \frac{12\pi D_p}{\varepsilon e u^3}\mu_p u_0^2 \rho^2. \tag{34}$$

Just as for (30), the term linear in ρ in the right-hand side of (34) has opposite signs in the increasing- and decreasing-field regions in the domain. We can therefore conclude that the hole diffusion does not change the velocity of the domain $u^2 = u_0^2$. By obtaining the domain velocity, we can get the $\rho(E)$ dependence from (34):

$$\rho^2(E) = \frac{\varepsilon e}{2\pi(D_n + D_p)}\int_{E_1}^{E} dE' \exp\left[\frac{6D_p\mu_p}{u(D_n + D_p)}(E' - E)\right] \times \{n_0[v_n(E') - v_n(E_1)] + p_0\mu_p(E' - E_1)\}. \tag{35}$$

In order for the hole diffusion to be significant, it is necessary to satisfy the condition

$$6D_p\mu_p E_m/u(D_n + D_p) > 1, \tag{36}$$

which follows from a comparison of (35) and (27). However, since $u > \mu_p E_m$ and $D_n + D_p > D_n$, condition (36) cannot be satisfied in the case of a large hole density. Thus, in this case the hole diffusion exerts practically no influence on the parameters and velocity of the domain.

APPENDIX

Let us estimate the integrals (19) and (20). Using the results of[4], we can obtain for the case of strongly-doped Gunn samples in the region $E_V < E < E_m$:

$$(n^{(0)} - n_0)^2 \sim \frac{\varepsilon n_0 \mu_n}{4\pi e D_n} E_v(E_t - E_1)\left(1 - \frac{E}{E_m}\right), \tag{A.1}$$

where E_t is the field corresponding to the maximum of the $v_n(E)$ curve (see Fig. 1). The maximum value of $n^{(0)} - n_0$ is reached in a field $E \sim E_V$ (for GaAs, $E_V/E_m \ll 1$ as $p_0 \to 0$). Using (A.1), we get

$$\int_{E_1}^{E_m} dE(n^{(0)} - n_0) \sim E_m\left[\frac{\varepsilon n_0 \mu_n E_v}{9\pi e D_n}(E_t - E_1)\right]^{1/2}. \tag{A.2}$$

In obtaining the estimate (A.2), the integration was carried out in the region from E_V to E_m. The region of values of E from E_1 to E_V does not make a noticeable contribution to the integral. (This can be verified by replacing $n^{(0)} - n_0$ in this region by the maximum value, which is reached at $E \sim E_V$.)

We estimate the second integral in (19) by assuming for simplicity $\mu_p E_V < v_n(E_1) < \mu_p E_m$ (which is satisfied in strongly doped GaAs samples). Then the main contribution to this integral is made also by the region $E \gtrsim E_V$, and the estimate of the integral takes the form

$$\int_{E_1}^{E_m} \frac{dE[n^{(0)}(E) - n_0]}{[v_n(E_1) + \mu_p E]^2} \sim \frac{1}{\mu_p v_n(E_1)}\left[\frac{\varepsilon n_0 \mu_n E_v(E_t - E_1)}{4\pi e D_n}\right]^{1/2}. \tag{A.3}$$

Substituting (A.2) and (A.3) in (19), we obtain the correction to the velocity:

$$-\frac{\Delta u_1}{v_n(E_1)} \approx +\frac{6\pi e D_n p_0}{\varepsilon E_m v_n(E_1)} < 1. \tag{A.4}$$

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248