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NONLINEAR THEORY OF PENETRATION OF *p*-POLARIZED WAVES INTO A CONDUCTOR

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Solutions are obtained for the internal and external electrodynamics problems that arise in the analysis of the reflection of incident p-polarized waves (i.e., waves with the electric vector in the plane of incidence) from a medium with a nonlinear dielectric permittivity. It is shown that, in the absence of an energy flux into a nondissipative medium, the reflection of p-polarized waves may occur not only under conditions of the existence of electromagnetic fields that vanish within the medium, but also under conditions of the existence of periodic fluxless field distributions. For almost normal wave incidence, the medium becomes stratified into regions in which the electromagnetic field is transverse and regions in which there is also a longitudinal field. The condition of nonlinear transparency with respect to longitudinal waves is satisfied in the latter regions, and spatial transformation of transverse and longitudinal electromagnetic waves occurs in them.

1. THE nonlinear theory of the penetration of a high-frequency field into a conductor was developed by one of the authors (V.P.S.) in [1] and the characteristic features of the distribution of transverse electromagnetic field in the medium were elucidated. The investigation was restricted to the case of normal incidence.

In the present work, we have investigated the case of oblique incidence of electromagnetic waves, whose electric vector lines in the plane of incidence (the case of p-polarized waves). The analysis given below showed that in oblique incidence of p-polarized waves and in the absence of energy flux within the medium, the reflection of waves from a half-space filled with a medium of nonlinear dielectric permittivity can correspond not only to the presence of electromagnetic fields that vanish far from the boundary, but also to the presence of spatially periodic distributions of the electromagnetic field. The latter are connected with the appearance of a spatially periodic transformation of the longitudinal and transverse degrees of freedom of the electromagnetic field in the nonlinear medium. The given phenomenon is most simply realized for almost normal incidence of p-polarized waves.

Following [1], we write the electric field in the form

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^+(\mathbf{r}) \cos \omega t + \mathbf{E}^-(\mathbf{r}) \sin \omega t. \tag{1.1}$$

Assuming the dielectric permittivity to be real and to depend on the energy density of the electric field averaged over the period of the high-frequency oscillations, we arrive at the following system of equations of electrodynamics:

$$-\Delta \mathbf{E}^\pm + \operatorname{grad} \operatorname{div} \mathbf{E}^\pm = k^2 \varepsilon \mathbf{E}^\pm, \quad \operatorname{div} \varepsilon \mathbf{E}^\pm = 0. \tag{1.2}$$

Here

$$\varepsilon = \varepsilon[\omega, (\mathbf{E}^+)^2 + (\mathbf{E}^-)^2], \quad k^2 \equiv (\omega / c)^2.$$

We assume that the medium fills the half-space $x \geq 0$, and that the z axis is directed along the boundary of separation. Let the (x, z) plane be the plane of incidence of the p wave with frequency ω and wave vector $\mathbf{k}(k_x, 0, k_z)$. We turn our attention to the internal problem, namely, we shall ascertain what types of p waves can exist in a medium with a nonlinear dielectric permittivity.

Assuming that all the quantities depend on the two spatial variables and that $E_y^\pm = 0$, we rewrite the system (1.2) in the form

$$-\Delta \mathscr{E}_z + \frac{\partial}{\partial x}\left(\frac{\partial \mathscr{E}_x}{\partial x} + \frac{\partial \mathscr{E}_z}{\partial z}\right) = k^2 \varepsilon \mathscr{E}_z,$$
$$-\Delta \mathscr{E}_x + \frac{\partial}{\partial z}\left(\frac{\partial \mathscr{E}_x}{\partial x} + \frac{\partial \mathscr{E}_z}{\partial z}\right) = k^2 \varepsilon \mathscr{E}_x,$$
$$\frac{\partial}{\partial x}(\varepsilon \mathscr{E}_x) + \frac{\partial}{\partial z}(\varepsilon \mathscr{E}_z) = 0. \tag{1.3}$$

Here

$$\mathscr{E}_x = E_x^+ + iE_x^-, \quad \mathscr{E}_z = E_z^+ + iE_z^-.$$

The system (1.3) allows a simple type of two-dimensional solution:

$$\mathscr{E}_x(x, z) = E_x(x) e^{ik_z z}, \quad \mathscr{E}_z(x, z) = E_z(x) e^{ik_z z},$$

which degenerates here into a system of ordinary differential equations with the parameter k_z:

$$-E_z'' + ik_z E_x' = k^2 \varepsilon E_z, \quad ik_z E_z' + k_z^2 E_x = k^2 \varepsilon E_x,$$
$$(\varepsilon E_x)' + ik_z \varepsilon E_z = 0. \tag{1.4}$$

2. Since the nonlinear medium is nondissipative ($\operatorname{Im} \epsilon = 0$), the system (1.4) should possess a set of first integrals (conservation laws), and the more general system (1.3) should have a set of divergent forms similar to those considered in [2]. The three first integrals of the system (1.4) have the form

$$|E_z'|^2 - k_z^2 |E_x|^2 + k^2 \int^{\tilde{h}} dq\, \varepsilon(\omega, q) = \text{const},$$
$$E_z' E_x^* + (E_z^*)' E_x = 0,$$
$$E_z (E_z^*)' - E_z^* E_z' + ik_z (E_z^* E_x + E_z E_x^*) = M, \tag{2.1}$$

where $\tilde{h} = |E_x|^2 + |E_z|^2$. In the case in which the energy flux is absent, M = 0. Then the system (1.4) has solutions of the form

$$E_x(x) \to E_x(x)e^{i\delta_x}, \quad E_z(x) \to E_z(x)e^{i\delta_z}.$$

Here the phases δ_Z and δ_X are constant, and their difference is $\delta_Z - \delta_X = \pi/2$. For real functions E_X and E_Z, the system (1.4) takes the form

$$-E_z'' + k_z E_x' = k^2\varepsilon E_z, \quad -k_z E_z' + k_z^2 E_x = k^2\varepsilon E_x, \qquad (\varepsilon E_x)' - k_z\varepsilon E_z = 0. \tag{2.2}$$

The last equation of the set (2.2) is the consequence of the first two and can be omitted in the subsequent analysis.

The properties of the distribution of the p waves in the medium are entirely determined by the integral curve in the phase space (E_Z', E_Z, E_X). The latter represents the intersection of the surface corresponding to the first of the integrals (2.1):

$$(E_z')^2 = k_z^2E_x^2 - k^2\int_{h_m}^{h} dq\,\varepsilon(\omega, q) + c_m, \qquad h = E_x^2 + E_z^2, \tag{2.3}$$

and the surface which represents the second of Eqs. (2.2):

$$-k_zE_z' = (k^2\varepsilon - k_z^2)E_x. \tag{2.4}$$

Here the constants of integration h_m and c_m are expressed at the point x_m in the following way:

$$h_m = (E_x^2)_m + (E_z^2)_m, \quad c_m = (E_z')^2_m - k_z^2(E_x^2)_m.$$

A single condition, which is a consequence of Eq. (2.4), is imposed on the three quantities $(E_Z')_m$, $(E_Z)_m$ and $(E_X)_m$. In particular, if $(E_Z')_m = (E_X)_m = 0$, then $c_m = 0$. If $(E_Z^2)_m$ satisfies here the relation

$$\int_0^{h_m} dq\,\varepsilon(\omega, q) = 0, \quad h_m = (E_z^2)_m,$$

then the first integral (2.7) takes the form

$$(E_z')^2 = k_z^2E_x^2 - k^2\int_0^h dq\,\varepsilon(\omega, q). \tag{2.5}$$

This means that among the solutions of the system (2.4) and (2.5) periodic, fluxless distributions can also exist in addition to the distributions of electromagnetic fields, which vanish at infinity (localized field distributions). The conditions for the existence and creation of these distributions will be made clear below.

For the elucidation of the topological features of the integral curve, we consider its projection on the phase plane (E_Z', E_Z), (E_X, E_Z) and (E_X, E_Z'). We make use of the following parametric representation of the integral curve namely, we take the quantity $h = E_X^2 + E_Z^2$ as a parameter. For example, excluding the normal component of the electric vector E_X from the system (2.4) and (2.5), and solving the resultant equations relative to the quantities E_Z' and E_Z, we arrive at the following parametric representation of the projection of the integral curve on the phase plane (E_X', E_Z):

$$(E_z')^2 = -\frac{(k^2\varepsilon - k_z^2)^2k^2}{(k^2\varepsilon - k_z^2)^2 - k_z^4}\int_0^h dq\,\varepsilon(\omega\ q). \tag{2.6}$$

$$E_z^2 = h + \frac{k_z^2k^2}{(k^2\varepsilon - k_z^2)^2 - k_z^4}\int_0^h dq\,\varepsilon(\omega, q). \tag{2.7}$$

The resultant relations, together with $h = E_X^2 + E_Z^2$, evidently also determine the projection of the integral curve on the planes (E_Z', E_X) and (E_X, E_Z). Taking this or some other model of the nonlinear medium, i.e., specifying an explicit form of the dielectric permittivity $\epsilon(\omega, h)$, there is no difficulty in calculating the projection of the integral curve on the phase plane by means of a high speed computer, of establishing the topology of the integral curve in the phase space (E_Z', E_Z, E_X), and of determining the dependence of the basic characteristics of the integral curve on the parameter k. The latter is connected with the angle of incidence ϑ of the wave by the simple relation $k_Z^2 = k^2\sin^2\vartheta$.

3. We consider the external problem of the reflection of transverse p-polarized waves from a nonlinear medium. First, we shall show that the electric and magnetic fields inside the medium are connected with the real amplitude functions $E_Z(x)$ and $E_X(x)$ by the relations (1.1) and

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}^+(\mathbf{r})\cos\omega t + \mathbf{H}^-(\mathbf{r})\sin\omega t. \tag{3.1}$$

Here, and also in (1.1),

$$\begin{gathered} E_z^+ = E_z\cos(k_zz + \delta_z), \quad E_z^- = E_z\sin(k_zz + \delta_z), \\ E_x^+ = E_x\sin(k_zz + \delta_z), \quad E_x^- = -E_x\cos(k_zz + \delta_z), \\ kH_y^+ = (k_zE_x - E_z')\sin(k_zz + \delta_z), \\ kH_y^- = (-k_zE_x + E_z')\cos(k_zz + \delta_z). \end{gathered} \tag{3.2}$$

For $x < 0$, we have incident E^i and reflected E^r waves of the electromagnetic field. Setting

$$E^i = E_i\cos(k_xx + k_zz - \omega t), \qquad E^r = RE_i\cos(-k_xx + k_zz - \omega t) \tag{3.3}$$

and taking into account the continuity of the tangential component of the electric field on the boundary, we get the following relations:

$$\begin{gathered} E_i\cos\vartheta(1 + R\cos\Psi) = E_z(0)\cos\delta_z, \\ RE_i\cos\vartheta\sin\Psi = E_z(0)\sin\delta_z. \end{gathered} \tag{3.4}$$

Here E_i is the amplitude of the incident wave, R the coefficient of reflection, Ψ the phase shift upon reflection. As a consequence of (3.4), we get the relations

$$\begin{gathered} E_z^2(0) = E_i^2\cos^2\vartheta(1 + R^2 + 2R\cos\Psi), \\ \operatorname{tg}\delta_z = R\sin\Psi\,/\,(1 + R\cos\Psi). \end{gathered} \tag{3.5}$$

The conditions of continuity of the tangential components of the magnetic field on the boundary lead to the relations

$$\begin{gathered} kE_i(1 - R\cos\Psi) = [-k_zE_x(0) + E_z'(0)]\sin\delta_z, \\ kE_iR\sin\Psi = [-k_zE_x(0) + E_z'(0)]\cos\delta_z. \end{gathered} \tag{3.6}$$

The relations (3.4) and (3.6) show that the reflection coefficient R = 1; here we have for the phase shift upon reflection

$$\sin\Psi = \frac{[E_z'(0) - k_zE_x(0)]E_z(0)}{2E_i^2k\cos\vartheta}. \tag{3.7}$$

Further, we find that

$$\begin{gathered} E_z(0) = 2E_i\cos\vartheta\cos(\Psi/2), \\ k\sin\vartheta E_x(0) - E_z'(0) = 2kE_i\sin(\Psi/2). \end{gathered} \tag{3.8}$$

Eliminating the phase shift of the reflected wave from (3.8), we get

$$\frac{E_z^2(0)}{4E_i^2\cos^2\vartheta} + \frac{[k_zE_x(0) - E_z'(0)]^2}{4k^2E_i^2} = 1. \tag{3.9}$$

For given values of the amplitude of the incident wave E_i and angle of incidence θ, the latter determines the surface of boundary conditions in the three-dimensional phase space (E_z', E_z, E_x). The intersections of the surface of boundary conditions with the integral curve determine those values $E_z'(0)$, $E_z(0)$, $E_x(0)$ which are realized on the boundary of separation for the given values of E_i, ϑ, and the relation (3.7)—the phase shift of the reflected wave.

4. To make clear the features of the internal problem and the type of integral curve, we use the following explicit form of the dielectric permittivity:

$$\varepsilon = \varepsilon_0(\omega) + \Delta(\omega) h. \quad (4.1)$$

In the case of a plasma medium, located in a strong high-frequency field, (4.1) corresponds to the weakly nonlinear approximation,[1] and

$$\varepsilon_0 = 1 - (\omega_0/\omega)^2, \quad \Delta = 1 - \varepsilon_0. \quad (4.2)$$

Let $\epsilon_0 < 0$. Then the relations (2.10), (2.11) take the form

$$\dot{e}_z^2 = \frac{(1+\alpha-s)^2(2-s)s}{(1-s)(1+2\alpha-s)}, \quad e_z^2 = \frac{(1-s)^2+\alpha(1-{}^3/_2 s)}{(1-s)(1+2\alpha-s)} s. \quad (4.3)$$

Here we use the following notation:

$$h = -\frac{\varepsilon_0}{\Delta} s, \quad e_z^2 = -\frac{\Delta}{\varepsilon_0} E_z^2, \quad \dot{e}_z^2 = \frac{2\Delta (E_z')^2}{(k\varepsilon_0)^2}, \quad \alpha = -\frac{\sin^2\vartheta}{\varepsilon_0}. \quad (4.4)$$

A. Let us consider the case of small angles of incidence:

$$\alpha < 1, \quad \sin^2\vartheta < -\varepsilon_0. \quad (4.5)$$

The projection of the integral curve on the phase plane ($\dot{e}_z$, e_z) which is shown in Fig. 1a, represents the set of two that form a curve of the "figure eight" type, and a closed curve of the "oval" type. The curves intersect the $\dot{e}_z$ axis at the points

$$2\dot{e}_\pm^2 = 1 + {}^3/_2\alpha + {}^3/_8\alpha^2 \pm [(1 + {}^3/_2\alpha + {}^3/_8\alpha^2)^2 - (1+\alpha)^3]^{1/2}. \quad (4.6)$$

We note that for loops which form curves of the "figure eight" type, the parameter s, which corresponds to the energy density of the electric field, changes within the range

$$0 \leqslant s \leqslant 1 + {}^3/_4\alpha - \sqrt{{}^9/_{16}\alpha^2 + {}^1/_2\alpha}, \quad (4.7)$$

while for the curve of the "oval" type,

$$1 + {}^3/_4\alpha + \sqrt{{}^9/_{16}\alpha^2 + {}^1/_2\alpha} \leqslant s \leqslant 2. \quad (4.8)$$

The projection of the integral curve on the phase plane (e_z, e_x), shown in Fig. 1b), represents a curve of the "figure eight" type and a curve of the "oval" type with protruberances along the e_z axis.

Finally, the integral curve is projected on the phase plane ($\dot{e}_z$, e_x) in the form of a pair of curvilinear segments which intersect at the solitary point $\dot{e}_z = e_x = 0$ and are symmetric relative to the transformation $\dot{e}_z \to -\dot{e}_z$, $e_x \to -e_x$ (Fig. 1b). Evidently, the integral curve in the space ($\dot{e}_z$, e_z, e_x) is represented by a pair of non-connecting curves, one of which—the "figure eight" type—corresponds to a localized distribution of the electromagnetic field in the medium ($\lim e_z = \lim \dot{e}_z = \lim e_x = 0$ as $x \to \pm\infty$), and the other—the "oval" type (more precisely, the type similar to a pair of silhouettes of tropical helmets)—to the excitation in the medium of a fluxless periodic field distribution.

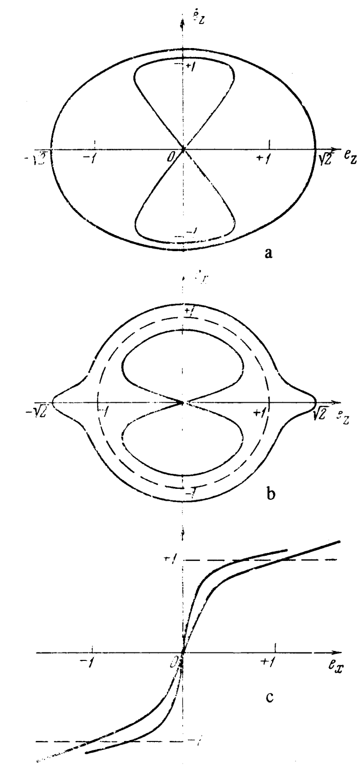

FIG. 1

The simplest and physically clearest picture is obtained in the analysis of the degenerate case—the "almost" normal incidence of p-polarized waves. For the dielectric permittivity (4.1) and $\theta = 0$, the set of equations (2.4) and (2.5) takes the form

$$\dot{e}_z^2 = (e_x^2 + e_z^2)(2 - e_x^2 - e_z^2), \quad (4.9)$$

$$(1 - e_x^2 - e_z^2) e_x = 0. \quad (4.10)$$

Let $e_x = 0$; then, on the basis of (4.9), we find that the projection of the integral curve on the phase plane ($\dot{e}_z$, e_z) is a curve of the "figure eight" type, located along the e_z axis. However, if $e_x \neq 0$, then (4.10) leads to the conclusion that

$$e_x^2 + e_z^2 = 1, \quad (4.11)$$

since, as a consequence of (4.9), it is established that

$$\dot{e}_z = \pm 1, \quad -1 \leqslant e_z \leqslant 1. \quad (4.12)$$

Comparison of the projection of the integral curve for the degenerate case (normal incidence) with the projections of the integral curve on the planes ($\dot{e}_z$, e_z), (e_x, e_z) and $\dot{e}_z$, e_x), shown in Fig. 1 for the case $\sin^2\vartheta < -\epsilon_0$, allows us to establish the fact that for "almost" normal incidence ($\sin^2\vartheta \ll -\epsilon_0$) two types of fluxfree distributions of the electromagnetic field can be excited in the medium with integral curves close to those shown in Fig. 2.

The first type of integral curve corresponds to a localized field distribution in the medium and is a continuous curve

$$\dot{e}_z = \sqrt{e_z^2(2 - e_z^2)}, \quad 1 \leqslant e_z \leqslant +1,$$

located in the ($\dot{e}_z$, e_z) plane, and has a break in the tan-

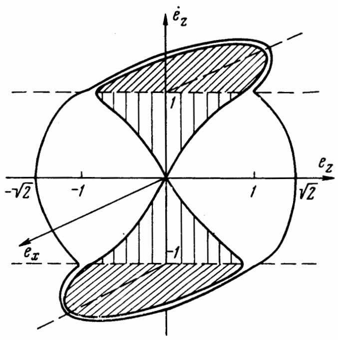

FIG. 2

gents and is closed by the arc of the semicircle

$$e_x^2 + e_z^2 = 1, \quad -1 \leqslant e_x \leqslant +1,$$

located in the plane $\dot e_z = +1$. A similar curve is located in the lower half-space $\dot e_z < 0$.

A second type of integral curve corresponds to a periodic fluxless field distribution and is the set of two arcs of semicircles $e_x^2 + e_z^2 = 1$, located in the planes $\dot e_z = \pm 1$ and joined to arcs of the curve

$$\dot e_z^2 = e_z^2(2 - e_z^2),$$

located in the plane $e_x = 0$.

For a localized field distribution, the dependence of the tangential and normal components of the electric field on the spatial variable has the form

$$e_z = \xi, \quad e_x = \sqrt{1-\xi^2}, \quad |\xi| \leqslant 1;$$

$$e_z = \frac{\sqrt{2}\,\mathrm{sign}\,\xi}{\mathrm{ch}[\sqrt{2}(1-\xi) - \ln(1+\sqrt{2})]} \quad e_x = 0, \quad |\xi| \geqslant 1 \qquad (4.13)$$

and is shown in Fig. 3. Here $\xi = \sqrt{-\epsilon_0}\,kx/\sqrt{2}$. Thus, for $|\xi| \le 1$, a layer develops in which the condition for nonlinearity of transmission relative to longitudinal excitation ($\epsilon = 0$), while the electric field vector, keeping its value, ex cutes a rotation by an angle equal to π in the limits of the range of transmission. For $|\xi| \ge 1$, the electric field is strictly transverse and tends to zero as $\xi \to \pm\infty$.

For periodic fluxless distribution of the field, the dependence of the tangential and normal components on the spatial variable is shown in Fig. 4 and corresponds

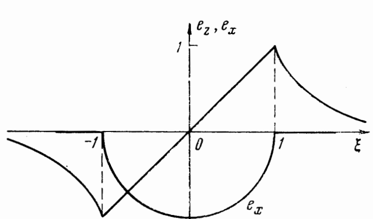

FIG. 3

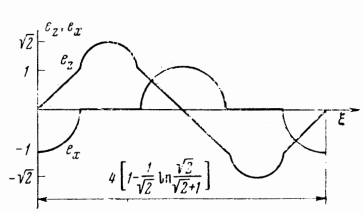

FIG. 4

to stratification of the medium with a period

$$l = 4[\sqrt{2} + \ln(\sqrt{2}+1)]/\sqrt{-\varepsilon_0}k \qquad (4.14)$$

into alternate layers, in one of which $\epsilon = 0$ and the electric field vector, preserving its value, undergoes rotation through the angle π, while in the other, the electric field is transverse and changes within the limits $-1 \le e_z \le 1$. In other words, a spatially periodic transformation of the transverse and longitudinal degrees of freedom of the electromagnetic field takes place in the nonlinear medium.

B. We proceed to the case of large angles of incidence, when

$$\alpha > 1, \quad \sin^2\vartheta > -\varepsilon_0.$$

The projection of the integral curve on the ($\dot e_z$, e_z) plane, shown in Fig. 5a, represents a curve of the "figure eight" type, located inside a large "oval." Moreover, there are two small "ovals," located symmetrically with reference to the $\dot e_z$ and touching the large "oval" at the points

$$\dot e_z = 0, \quad e_z = \pm(\alpha+1)/\sqrt{2\alpha}.$$

The projection of the integral curve on the (e_x, e_z) plane is shown in Fig. 5b. Finally, the integral curve is projected on the ($\dot e_z$, e_x) plane in the form of a pair of curvilinear segments which intersect at the origin and are symmetric relative to the transformation $\dot e_z \to$ e $\to -\dot e_z$, $e_x \to -e_x$. Consequently, the integral curve for $\alpha > 1$ represents a set of two unconnected curves, one of which, namely the curve of the type of a deformed "figure eight," is connected with the localized distributions of the field in the medium, and the other—a type of "oval" with leaves—corresponds to periodic fluxless field distributions.

5. Comparing the given analyses of the internal and external problems, one can make clear under what conditions (more precisely, what values of the amplitude of the incident wave and the angle of incidence) the reflection of p-polarized waves from the boundary of separation is connected with excitation in the medium of localized or periodic distributions of the electromagnetic field. As a first example, we consider the case in which the electric field on the boundary of the medium has only a longitudinal component $e_z(0) = 0$. Evidently, $s(0) = e_x^2(0)$. The given situation is realized for

$$s_\pm(0) = 1 + {}^3/_4\alpha \mp \sqrt{{}^9/_{16}\alpha^2 + {}^1/_2\alpha}.$$

Using the equation of the surface of boundary conditions (3.9) and the relation (4.3), we arrive at the following expressions:

$$\left.\begin{matrix}[1+s_+(0)]^2 s_+(0)\\ [1+s_-(0)]^2 s_-(0)\end{matrix}\right\} = \frac{4\Delta E_i^2\alpha}{\varepsilon_c^2} \qquad (5.1)$$

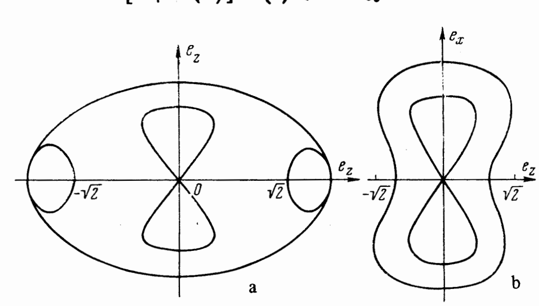

FIG. 5

If the amplitude of the incident wave E_i and the angle of incidence ϑ satisfy the first of the condition (5.1), then reflection leads to the excitation in the medium of a localized field distribution which vanishes at large distance from the boundary of separation. However, for E_i and ϑ satisfying the second of the conditions (5.1), periodic field distributions are excited in the medium. In both cases, the phase shift of the reflected wave is $\Psi = \pi$.

We turn our attention to the fact that in the first case the energy density of the electric field on the boundary of separation reaches the maximum value for which fluxless localized field distributions can be excited in the medium. In the second case there is realized on the separation boundary that minimum value of the energy density of the electric field, for which fluxless periodic distributions of field can be excited in the medium.

For $-\xi_0 \ll 1$ and $\Delta \sim 1$, the Eqs. (5.1) lead to the following asymptotic expressions in the range of small and large angles of incidence:

$$(4E_i^2)_\pm \sim {}^1/_2\varepsilon_0^2 \mp (-{}^1/_2\varepsilon_0)^{3/2}\sin\vartheta, \quad \sin^2\vartheta \ll -\varepsilon_0;$$

$$\left.\begin{matrix}(4E_i^2)_+ \sim -2\varepsilon_0^3/27\sin^2\vartheta \\ (4E_i^2)_- \sim {}^{27}/_8\sin^4\vartheta\end{matrix}\right\}, \quad \sin^2\vartheta \gg -\varepsilon_0. \tag{5.2}$$

The behavior of the corresponding curves on the plane $(rE_i^2, \sin\vartheta)$ is shown in Fig. 6.

As a second example, we consider the case in which the energy density of the electric field on the separation boundary reaches its maximum value compatible with excitation of fluxless periodic field distributions in the medium. In particular, $s(0) = 2$, $e_x(0) = \dot{e}_z(0) = 0$. Using the equation for the surface of boundary conditions, we find that

$$4E_i^2 = -2\varepsilon_0/\Delta\cos^2\vartheta. \tag{5.3}$$

The corresponding curve for the case $-\epsilon_0 \ll 1$ and $\Delta \sim 1$ is also shown in Fig. 6.

We now turn to the degenerate case of normal incidence. The surface of boundary conditions (3.9) for $k_z \to 0$ degenerates into an elliptical cylinder located along the e_x axis:

$$\frac{\varepsilon_0^2}{8\Delta E_i^2}\dot{e}_z^2 + \frac{-\varepsilon_0}{4\Delta E_i^2}e_z^2 = 1. \tag{5.4}$$

Considering the intersection of the integral curves

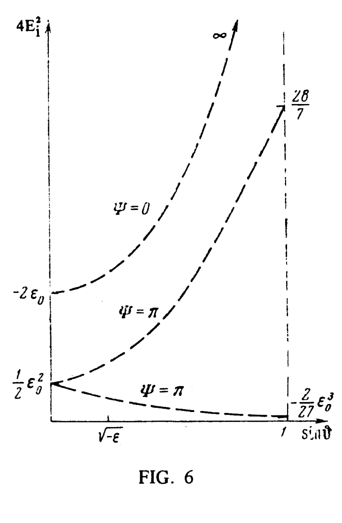

FIG. 6

(Fig. 2) with the surface of boundary conditions (5.4), we find that, for the amplitude of the incident wave satisfying the inequalities

$$4E_i^2 < \varepsilon_0^2/2\Delta, \quad 4E_i^2 < -\varepsilon_0/\Delta, \tag{5.5}$$

there is a unique solution connected with the excitation of a localized field distribution in the medium. The latter field falls monotonically to zero on going to infinity. The longitudinal field in the medium is not excited in the given case ($e_x \equiv 0$). If the amplitude of the incident wave satisfies the inequalities

$$\varepsilon_0^2/2\Delta < 4E_i^2 < -\varepsilon_0/\Delta, \tag{5.6}$$

then there are three solutions. The first corresponds to a monotonically decreasing localized distribution of the transverse field, the second to localized distribution of the field associated with the formation of a layer near the separation boundary in which a longitudinal electric field is excited. In passage through a layer with no linear longitudinal transmission, the vector of the electric field, without changing its magnitude, undergoes rotation through the angle necessary to restore the longitudinal field to zero. A phase of monotonic decrease in the transverse field should follow. Finally, the third solution corresponds to excitation in the medium of a periodic fluxless field distribution, i.e., excitation in space of a periodic transformation of the longitudinal and transverse degrees of freedom of the electromagnetic field. Thus the solution of the boundary problem of reflection of p-polarized waves for almost normal incidence for amplitudes of the incident wave satisfying the inequalities (5.6) is not unique.

In conclusion, we note that for the case of a real nonlinear dielectric permittivity, both the internal problem of electrodynamics connected with the finding of the proper distributions of the electromagnetic field in the medium, and the external problem associated with the penetration of the p-polarized waves into the nondissipative medium allow a complete study in three dimensional phase space (E_z', E_z, E_x). Here, both the transverse and longitudinal degrees of freedom of the electromagnetic field are taken into account. The analysis carried out above showed that account of the excitation of longitudinal components of the electromagnetic field leads to qualitatively new phenomena (for example, the spatial transformation of transverse and longitudinal degrees of freedom of the field in a nonlinear medium). We emphasize that the problem of reflection of s-polarized waves (the electric vector is normal to the plane of incidence) is essentially a very simple problem, since it turns out not to be connected with the excitation of longitudinal degrees of freedom of the electromagnetic field.

In spite of the fact that the case which was considered in detail above was the one for which the medium is opaque in the linear approximation ($\epsilon_0 < 0$), the relations obtained also make it possible to study the case of a transparent medium in the linear approximation ($\epsilon_0 > 0$). In particular, for the dielectric permittivity (4.1) with $\epsilon_0 > 0$ and $\Delta > 0$, Eq. (2.5) and the second of Eqs. (2.2) lead only to localized fluxless distributions of the electromagnetic field in the medium.

Finally, we show that uncomplicated numerical calculations of the integral curves allow us to investigate the distribution of the electromagnetic field in the medium even for an arbitrary dependence of the nonlinear dielectric permittivity on the energy density of the electric field. The latter circumstance is associated with the fact that the relations (2.6) and (2.7) determine the integral equation in parametric form for the case of an arbitrary dependence of the dielectric permittivity on the energy density of the electric field.

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