SOME FEATURES OF HELICON PROPAGATION IN METALS FOR LARGE DIFFERENTIAL SUSCEPTIBILITY AMPLITUDES IN THE DE HAAS-VAN ALPHEN EFFECT

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The influence of the de Haas–Van Alphen effect on the helicon spectrum in metals is considered for the region near the critical values of the differential magnetic susceptibility, which determines the region of appearance of domain structure. A number of new effects are predicted. Numerical calculations are carried out for aluminum.

1. THE de Haas–van Alphen effect changes the dispersion law of helicons in metals, generating quantum oscillations of their phase velocity as a function of the magnetic field.[1–3] It is significant that the amplitude of these oscillations is determined by the derivative of the magnetization of the metal with respect to the magnetic field and is therefore enhanced as a consequence of the rapidly oscillating dependence of the magnetic moment on the field.

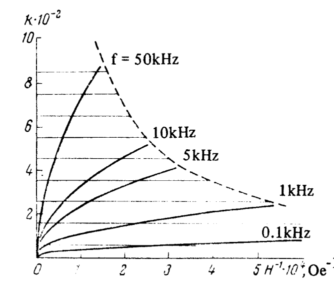
By lowering the temperature and improving the quality of the samples, one can in principle obtain such an amplitude of the de Haas–van Alphen effect for any metal, that the condition of thermodynamic stability ($4\pi\partial M/\partial B < 1$ in the isotropic case) ceases to be satisfied over a definite part of each period of oscillation. In this case, depending on the geometry of the experiment, either jumps of the field B in the metal or the division of it into domains are observed.[4]

As the amplitude of the de Haas–van Alphen oscillations approaches the critical value, the oscillations of the phase velocity of the helicons increase so much[5] that a number of new effects appear, the consideration of which is the subject of this paper.

The calculations used to plot the graphs in the paper were completed for aluminum, for which we had available all the necessary data: the values of the Hall constant and the amplitude of the oscillations of $\partial M/\partial B$ from [3] and the location of the Doppler-shifted cyclotron resonance.[6]

2. We start out from a simple experiment on the observation of helicon resonances in a plate of metal as a function of the magnetic field for fixed frequency of the wave. Figure 1 shows the dependence of the real part of the wave vector of the helicons on the magnetic field without account of the oscillations. The straight lines parallel to the abscissa determine the values of k for which an odd number of half-wavelengths are included in the thickness. For symmetric excitation, resonances appear each time when, following a change of the magnetic field, the curve k(B) intersects one of these straight lines. By the method of crossed coils,[1] we could observe a succession of resonance peaks in this case. The de Haas–van Alphen effect leads to the result that oscillations are superimposed on the monotonic k(B) dependence. While their amplitude is small in comparison with the width of the resonances, the same resonance peaks are observed, but are masked by the oscillations.[1] However, new effects appear upon increase in the amplitude of the oscillations.

FIG. 1. Dependence of the real part of the wave vector of helicons in aluminum on the magnetic field (without oscillations) for various frequencies. The boundary of the Doppler-shifted cyclotron resonance is shown (dashed curve) for the field direction [100], $\theta = 45°$, k in cm^{-1}



3. In order to determine the character of the effects that arise upon increase in the amplitude of the oscillations, we estimate the maximum possible range of change of k within the limits of a single period of an oscillation of the de Haas–van Alphen type. For this purpose, we use the expression for the dispersion law of helicons:[3]

$$k^2 = 4\pi\omega / c^2 RB\sqrt{q}\cos\theta, \tag{1}$$

$$q = 1 - 4\pi(M_{xx} + M_{yy}) + 16\pi^2(M_{xx}M_{yy} - M_{xy}^2). \tag{2}$$

Expressions (1) and (2) are written down in a system of coordinates whose z axis coincides with the direction of the wave vector and the normal to the plane surface of the metal. The magnetic field B lies in the xz plane, rotated an angle θ with respect to the z axis. Here R is the Hall constant, and $M_{ik} = \partial M_i/\partial B_k$ are the derivatives of the magnetic moment with respect to the magnetic field. The damping of the wave is assumed to be weak, and the corresponding term is omitted in (1). The field B is the field inside the metal, acting on the electrons.

It is seen from (1) that a strong dependence of k on the field will occur in the region of small values of q. The minimum value of q is determined by the condition of thermodynamic stability. In order to obtain it, it is necessary to reduce the tensor M_{ik} to diagonal form and to require that each diagonal component satisfy the condition $4\pi M_{aa'} < 1$. Then, expressing the components M_{ik} entering into (2) in terms of $M_{aa'}$, and setting $4\pi M_{aa'} = 1$, we get the minimum value of q within a single period of oscillation.

For a spherical Fermi surface or for a part of it whose symmetry axis is identical with the B direction, the magnetic moment is directed along the field, and it is necessary to require that the quantity $4\pi\partial M/\partial B$ not exceed unity. The expression (2) for q is transformed in this case to the form[3]

$$q = 1 - 4\pi \frac{\partial M}{\partial B} \sin^2\theta, \tag{3}$$

and the minimum value of q is equal to $1 - \sin^2\theta$, which is always greater than zero since θ must always be less than 90° for observation of helicons. Nevertheless, for $\theta = 80°$, the quantity k changes by a factor of three on going from minimum to maximum value over the period of oscillation.

If the principal contribution to q is made by the cylindrical part of the Fermi surface, which is parallel to the x axis in our set of coordinates, then

$$M = M_x, \qquad q = 1 - 4\pi\partial M / \partial B_x. \tag{4}$$

In this case, the quantity $4\pi\partial M/\partial B_x$ should be smaller than unity and the minimum value of q is thus equal to zero.

The latter case can exist in aluminum or in indium if we take a specimen with the normal along the [110] axis and direct a magnetic field at the angle θ to the normal in the (001) plane. One of the tubes in the third zone of the Fermi surface of these metals will be parallel to the normal, and will not affect the propagation of the helicons; the other will be parallel to the surface of the metal. All the graphs in the paper are constructed for an aluminum sample oriented in such fashion, in a magnetic field parallel to [100].

The dependence of k on the field in the limits of a single period of oscillation is shown in Fig. 2. This dependence is computed from Eq. (1) with $q = 1 - \sin\psi$. The abscissa here is not the quantity ψ, which is proportional to B inside the metal, but the quantity $\varphi = \psi + \cos\psi$, which is proportional to the field H, since the external field gives a component H_x parallel to the surface of the sample, and not B_x.

In Fig. 2, just as in Fig. 1, we show the location of the resonances of the standing waves in a plate of aluminum of thickness $d \approx 0.6$ mm. It is seen that for a change in the field within the limits of a single period of oscillations, the wave number runs through a whole succession of resonance values in the direct and reverse directions. And all this takes place in a range of magnetic field equal to several Gauss!

4. We can now show how the picture of helicon resonances in a plate, observed as a function of the magnetic field for fixed frequency by the method of crossed coils or in some other way, changes with temperature. At sufficiently high temperatures, when the amplitude of oscillations k(B) is less than the width of the helicon resonances in the plate, a succession of resonance peaks is observed, masked by the oscillations.[1] Near the vertices of the resonances, in a field interval of the order of the amplitude of the oscillations (the oscillations of the wave vector can be converted into the oscillations of the resonance field) a second harmonic of the oscillation will appear.[1)] Upon subsequent increase in the amplitude of the oscillations, there will first be observed two deep peaks in each period, corresponding to the passage of the resonance in the direct and reverse directions. Then, when the amplitude of the oscillations k surpasses the value $2\pi/d$, the previous resonance peaks and the space between them are replaced by an extraordinarily fine structure of sharp peaks, the sequence of which in the limits of each period of oscillation is determined by graphs of the type shown in Fig. 2.

5. If we fix the magnetic field within a single oscillation close to the maximum of $\partial M/\partial B$ and lower the temperature, then the fixed value $k(B_0, \omega_0)$ will successively pass through the resonance values k_n with increasing amplitude of the oscillations.

In helicon observation by the method of crossed coils, a series of resonance peaks as a function of temperature should appear. This interesting effect is illustrated by the graph of Fig. 3. The ordinates of this graph are proportional to the signal in the secondary coil and are computed from the equation[7]

$$\operatorname{Re}\mu = \frac{4\omega_c\tau}{\pi^2} \sum_n n^{-2} \left[1 + Q^2\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)^2\right]^{-1} \tag{5}$$

Here n, ω_n, and Q are the number, frequency, and quality factor of the helicon in the metal plate, n = 1, 3, 5, ...,

$$\omega_n = \left(\frac{n}{d}\right)^2 \frac{\pi c^2 RB\cos\theta}{4} \left(1 - 4\pi\frac{\partial M}{\partial B}\right)^{1/2}. \tag{6}$$

The amplitude value of $\partial M/\partial B$ is substituted in (6) from the formula[8]

$$\frac{\partial M}{\partial B} = A \frac{T\exp(-2\pi^2 kT^*/\hbar\omega_c)}{B^{5/2}\,\mathrm{sh}(2\pi^2 kT/\hbar\omega_c)}, \tag{7}$$

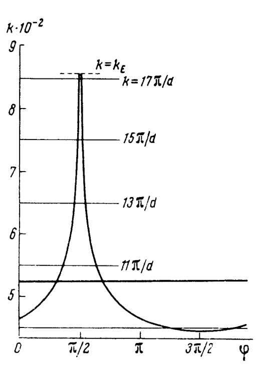


FIG. 2. Dependence of the real part of the wave vector of helicons on the magnetic field in the limits of a single period of oscillation for the amplitude $4\pi M_{xx} = 1$ (see text). The heavy straight line on the drawing indicates the k(B) dependence without oscillations; $\omega/2\pi = 16$ kHz

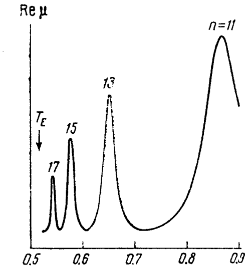


FIG. 3. Resonances as a function of temperature, $T^* = 0.06°$K

1)It seems to us that the appearance of the second harmonic in the experiments of Krylov on indium can be explained in this fashion. [2]

T^* is the Dingle temperature, the constant A is determined by the geometry of the Fermi surface and is computed from the data of [3], where the absolute magnitude of $\partial M/\partial B$ for the tubes of the aluminum Fermi surface in the third zone and the Dingle temperature were measured on the same sample. After the quantity A was determined, one could compute the amplitudes of $\partial M/\partial B$ for any values of T, T^*, and B.

6. The range of change of k within a single oscillation period could be so great that, as is seen from Fig. 2, one can go from the local region $kl \ll 1$ to the nonlocal $kl > 1$ and even pass through the threshold of Doppler-shifted cyclotron resonance. Strictly speaking, our analysis is not valid in the neighborhood of Doppler-shifted cyclotron resonance, since we are using a dispersion law that is obtained in the local limit. Nonetheless, it can be confirmed qualitatively that in this range of fields, where the k(B) curve in Fig. 2 rises above the boundary of the Doppler-shifted resonance (this boundary is indicated by the dashed line in Fig. 1), the damping of the wave increases sharply. The intervals of damping will arise periodically, with the period of the oscillations in some range of fields below the boundary curve in Fig. 1. The dimensions of this region depend on the amplitude of the oscillations k. It is possible that transparent intervals are formed beyond the threshold, due to those half-periods of the oscillations in which k diminishes. However, this problem requires special consideration.

It can also be established that a limiting temperature T_E exists for resonances as a function of temperature (Fig. 3) for which $k(B_0, \omega_0)$ reaches the threshold of the Doppler-shifted resonance.

7. In conclusion, it is useful to represent the real conditions, namely the temperature and field, in which the effects described above could be observed. For this purpose, we computed the upper envelope curve k(B), i.e., the line passing through the vertex of the oscillations of k. The calculations were made from Eq. (1) with q from (4) for various values of T and T^*. In place of $\partial M/\partial B$ the amplitude of the de Haas–van Alphen effect is substituted in q, taken from Eq. (7). The results of the calculations are shown in Fig. 4. The amplitude of the oscillations of $\partial M/\partial B$ and, consequently, of k, have a maximum with respect to the field, the height and location of which depend on T and T^*. Of course, the effects considered above are best observed in the region of this maximum. The calculations for aluminum plates of thickness $d \approx 0.6$ mm show that the change of k in the limits of a single half-period of oscillation is larger than $2\pi/d$ in a field of about 7kG for $T = 0.5$°K; $T^* = 0.07$–0.06°K, if the frequency of the wave is equal to 16 kHz. It is seen from Fig. 4 that the envelop of the oscillations for $T^* = 0.07$–0.06°K and $T = 0.5$°K goes beyond the threshold of Doppler-shifted resonance in the range of magnetic fields in which the helicons propagate without oscillations and with weak damping.

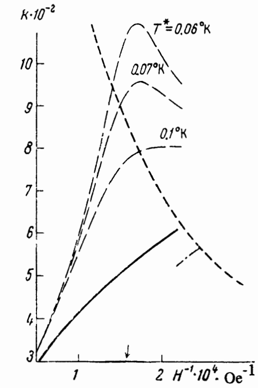


FIG. 4. Envelope (over the maxima) of the oscillation dependence of the real part of the wave vector of the helicons in aluminum on the field, in a magnetic field parallel to [100], $\theta = 45°$. The continuous curve represents the k(B) dependence without oscillations. The dot-dash curve indicates the part of the envelop over the maxima of k(B) for $T^* = 0.06$°K. The arrow shows the field for which the curve on Fig. 2 was computed. The boundary of Doppler-shifted cyclotron resonance is also shown in the figure. The temperature $T = 0.5°$ for all curves.

The temperature 0.5°K is entirely achievable at the present time if one works with He^3. Dingle temperatures less than 0.1°K can also be achieved by preparation of specimens with low density of dislocations.[9]

It should also be noted that the observation of the effects described in the present research requires a very high resolving power in terms of the magnetic field and in the temperature, i.e., high homogeneity and stability of the magnetic field and temperatures.

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