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COLLISION INTEGRAL FOR A CLASSICAL PLASMA

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The Bogolyubov-Born-Green-Yvon-Kirkwood (BBGYK) set of equations for a classical gas in non-equilibrium is reduced to a set of integral equations for the correlation functions in terms of an auxiliary "time." These equations presuppose that the problems of the motion of two, three, ... particles are solved. This form of the equations is convenient for the case of a long-range interaction potential, in particular, for the Coulomb potential: it enables us to take the "screening" of the interaction easily into account at large distances while describing it exactly at small distances where the potential may be arbitrarily strong. We consider the spatially uniform case. We obtain results for the pair correlations and use those to derive a kinetic equation for an electron plasma in the first approximation in the density. The collision integral has no divergences. Its Fokker-Planck part, which contains the "dynamic polarization" effects of the plasma, resembles the well-known Balescu-Lenard-Guernsey integral, but in contrast to it is finite.

1. INTRODUCTION

In the present paper we propose a general method for solving the BBGYK set of equations for the reduced distribution functions of a non-equilibrium classical gas with a long-range potential for the interactions between the particles which is strong at small distances. It is well known that these equations are integro-differential equations in which the direct interaction between the particles is described by the differential part and their interaction through the medium by the integral part. In the case of short-range or of weak long-range forces this form of the equations is very convenient to find approximate solutions,[1] since the integral part or the differential part containing the interaction can appropriately be treated using perturbation theory. If, however, the long-range potential is not weak at small distances, very serious difficulties arise when we try to solve the equations in this form (see the recent paper by Myerscough[2] which deals with the Coulomb case; one can find there a short survey of earlier literature). We reduce for that reason the BBGYK equations, written in the form of equations for the correlation functions, to a purely integral form in terms of the coordinates and momenta, by means of transformations which presuppose that the problems of the motion of 2, 3, ... particles are solved. The equations obtained enable us to split off small terms for the case of a rarefied gas; these terms correspond physically to the interference of the two above-mentioned interactions. It then turns out advisable to consider separately the case of "globally weak" potential, since such a potential can be described in a unified way without being split into a strong and a weak component (a particular case of this is the purely Coulomb potential). We consider the spatially uniform case (Sec. 3). We obtain in explicit form the kinetic equation for a stable, spatially uniform plasma (Sec. 4).

2. EQUATIONS FOR THE CORRELATION FUNCTIONS

The BBGYK equations for the reduced distribution functions F_s have, for the case of an ensemble of systems of identical particles in a volume V, where $V \to \infty$, the form

$$\frac{\partial F_s}{\partial t} + [H_s; F_s] = n \sum_{i=1}^{s} \int dx_{s+1} \frac{\partial \varphi_{i,s+1}}{\partial \mathbf{r}_i} \frac{\partial F_{s+1}}{\partial \mathbf{p}_i},$$

$$F_s(x_1 \ldots x_s, t) = V^{-1} \int d\mathbf{r}_{s+1} \int_V d\mathbf{p}_{s+1} F_{s+1}(x_1 \ldots x_{s+1}, t), \quad s \geqslant 2 \tag{1}$$

$$V^{-1} \int_V d\mathbf{r} \int d\mathbf{p}\, F_1(x, t) = 1, \quad x_i \equiv \{\mathbf{r}_i; \mathbf{p}_i\},$$

where $\varphi_{i,j} \equiv \varphi(|\mathbf{r}_i - \mathbf{r}_j|)$ is the interaction potential, $H_s(x_1, \ldots, x_s)$ the Hamiltonian of s particles, $[\ldots; \ldots]$ the Poisson brackets, and n the particle density. We introduce in the usual way the correlation functions $g_s(x_1 \ldots x_s, t)$:

$$\begin{gathered} g_1(x, t) \equiv F_1(x, t), \\ g_2(x_1, x_2, t) \equiv F_2(x_1, x_2, t) - g_1(x_1, t)\, g_1(x_2, t), \\ g_s(x_1 \ldots x_s, t) \equiv F_s(x_1 \ldots x_s, t) - \sum \prod g, \end{gathered} \tag{2}$$

where the second term indicates a sum of all possible products of g's with indices less than s and a sum of indices equal to s; each set x_i occurs in the product exactly once. In terms of the g_s Eqs. (1) take the form

$$\frac{\partial g_s}{\partial t} + [H_s; g_s) = -\sum_{i>j} \sum_{j=1}^{s-1} \sum_{\alpha=1}^{s-1} [\varphi_{i,j}; g_\alpha(\ldots x_i \ldots) g_{s-\alpha}(\ldots x_j \ldots)]$$

$$+ n \sum_{i=1}^{s} \int dx_{s+1} \frac{\partial \varphi_{i,s+1}}{\partial \mathbf{r}_i} \frac{\partial}{\partial \mathbf{p}_i}$$

$$\times \Big\{ \sum_{\alpha=1}^{s} g_\alpha(\ldots x_i \ldots) g_{s-\alpha+1}(\ldots x_{s+1}) + g_{s+1}(x_1 \ldots x_{s+1}) \Big\},$$

$$V^{-1} \int_V d\mathbf{r}_i \int d\mathbf{p}_i g_s(x_1 \ldots x_i \ldots x_s) \to \delta_{s,1}, \quad V \to \infty. \tag{3}$$

We transform Eqs. (3) with $s \geq 2$ as follows. On both sides of the equation we act with the operator $S_{t-t'}^{(s)}(x_1 \ldots x_s)$ which describes the evolution of the coordinates and momenta of s particles with a Hamiltonian $H_s(x_1 \ldots x_s)$ during the interval from t′ to t. Since

$$\frac{d}{dt} \{S_{t-t'}^{(s)} g_s(t)\} = S_{t-t'}^{(s)} \frac{\partial g_s(t)}{\partial t} + S_{t-t'}^{(s)} [H_s; g_s(t)],$$

we get after integrating over t from some t_0 to t′, after some obvious transformations the following set of equations:

$$g_s(t) = S^{(s)}_{t_0-t} g_s(t_0) + \int_{t_0-t}^{0} d\tau S^{(s)}_\tau \{\Phi_1^{(s)}(t+\tau) + \Phi_2^{(s)}(t+\tau)\}, \quad s \geqslant 2, \quad (4)$$

where $\Phi_1^{(S)}$ and $\Phi_2^{(S)}$ indicate the first and second terms on the right-hand side of (3).

Let us consider the second integral term on the right-hand side of (4). We start with the case of a purely Coulombic interaction. The following fact is important: in the case of a rarefied gas the Coulomb potential is "globally weak" in the sense that

$$n \int_{r \lesssim n^{-1/3}} dr\, \varphi(r) \ll H_{1\,\mathrm{av}}.$$

One sees easily that if this condition is satisfied the contribution from the "strong potential" region ($\varphi(r) \gtrsim H_{1\,\mathrm{av}}$) is, when we integrate over x_{s+1} and τ, small compared to g_s for each term in $\Phi_2^{(S)}$; in the first approximation we can thus neglect the term in g_{s+1} and the φ-dependent part of the operator $S_\tau^{(S)}$. In other words, a potential which is strong at small distances but is globally weak must be taken into account exactly in the first integral term in (4) which does not contain g_s and g_{s+1}, but can be considered to be weak in the second term. To construct consecutive approximations we must thus solve equations which are analogous to the case of weak interactions. If the potential at small distances is so large that even for a rarefied gas ($nr_0^3 \ll 1$, $\varphi(r_0) \sim H_{1\,\mathrm{av}}$) it is not globally weak (for instance, the left-hand side of the inequality may become infinite), we must first split off in $\Phi_2^{(S)}$ the contribution from the strong potential region, which is small compared to $\Phi_1^{(S)}$, as nr_0^3.

We shall in what follows consider ensembles in which the correlation functions satisfy the Bobolyubov condition:[1]

$$S^{(s)}_{-\infty} g_s(t) = 0, \qquad s \geqslant 2. \quad (5)$$

If we introduce some auxiliary "times" we can obtain for them equations which permit the above-mentioned simplifications but which are with respect to the time of a differential nature which in a number of cases turns out to be more convenient. We act upon both sides of (3) for $s \geq 2$ with the operator $S_\tau^{(S)}(x_1 \ldots x_s)$ and integrate over τ from $-\infty$ to 0. As

$$S^{(s)}_\tau [H_s, g_s(t)] \equiv [S^{(s)}_\tau H_s, S^{(s)}_\tau g_s(t)] = \frac{d}{d\tau}\{S^{(s)}_\tau g_s(t)\}, \quad (6)$$

we have by virtue of (5)

$$g_s(t) + \frac{\partial}{\partial t}\int_{-\infty}^{0} d\tau S^{(s)}_\tau g_s(t) = \int_{-\infty}^{0} d\tau S^{(s)}_\tau \{\Phi_1^{(s)}(t) + \Phi_2^{(s)}(t)\}, \quad s \geqslant 2. \quad (7)$$

If the average time of flight of the particles through the region where there are strong correlations is small compared to the characteristic time for changes in $g_s(t)$ the second term on the left-hand side of (7) can be considered to be a small perturbation. If the time of flight of the particles through the region of strong interactions is small in this sense, that part of this term which contains the φ-dependent component of the operator $S_\tau^{(S)}$ will be small.

If we assume that (5) is valid, Eqs. (7) have a meaning only when there are no bound states of the s particles (or if we neglect them). In that case condition (5) is an obvious consequence of the vanishing of the correlations when the particles are at infinite distances apart. We assume now that as the result of a sufficiently long evolution from a state where the correlations vanish when the particles are at infinite distances apart the functions $g_s(t)$ asymptotically approach some "stationary" form which is independent of the initial conditions and determined only by the behavior of $F_1(t)$ in the "immediate" past. One sees then easily from (4) that for such "stationary" correlations the following limit relation must be satisfied:

$$g_s(t) \to \int_{t'-t}^{0} d\tau S^{(s)}_\tau \{\Phi_1^{(s)}(t+\tau) + \Phi_2^{(s)}(t+\tau)\}, \quad t - t' \to \infty, \quad t' - t_0 \to \infty.$$

As then they must also satisfy (4) where t_0 is replaced by t', it follows that the "stationary" correlations must satisfy the condition (5) as was assumed by Bogolyubov[1] (the more restrictive character of the assumptions of [1] are justified only in particular cases).

3. CORRELATIONS IN A SPATIALLY UNIFORM GAS

We consider the "stationary" correlations for the case of relaxation of a spatially uniform gas. We shall start from the set of Eqs. (7). The characteristic time for changes in the $g_s(t)$ is in that case the same as the relaxation time of the system so that the second term on the left-hand side of (7) is small compared to the first one. We rewrite Eqs. (7) as follows:

$$\begin{aligned} & g_s(x_1 \ldots x_s, t) - n \sum_{i=1} \int_{-\infty}^{0} d\tau S^{0(s)}_\tau (x_1 \ldots x_s) \int dx_{s+1} \\ & \times \frac{\partial \varphi_{i,s+1}}{\partial \mathbf{r}_i} \frac{\partial f(\mathbf{p}_i, t)}{\partial \mathbf{p}_i} g_s(x_1 \ldots x_{i-1}, x_{i+1}, \ldots x_{s+1}, t) \\ & = \int_{-\infty}^{0} d\tau \{S^{(s)}_\tau \Phi_1^{(s)}(t) + S^{0(s)}_\tau \Phi_2^{(s,s-1)}(t)\} \\ & + \int_{-\infty}^{0} d\tau \{S^{int(s)}_\tau \Phi_2^{(s)}(t) + n S^{0(s)}_\tau (x_1 \ldots x_s) \cdot \\ & \times \sum_{i=1} \int dx_{s+1} \frac{\partial \varphi_{i,s+1}}{\partial \mathbf{r}_i} \frac{\partial g_{s+1}(t)}{\partial \mathbf{p}_i} - S^{(s)}_\tau \frac{\partial g_s(t)}{\partial t}\}, \\ & S^{0(s)}_\tau + S^{int(s)}_\tau \equiv S^{(s)}_\tau, \quad f(\mathbf{p}_i, t) \equiv F_1(x_i, t), \end{aligned} \quad (8)$$

where $S_\tau^{0(S)}$ is the operator $S_\tau^{(S)}$ when there are no interactions and $\Phi_2^{(S,S-1)}$ is the second term on the right-hand side of (3), $\Phi_2^{(S)}$ without g_s and g_{s+1}. The first integral on the right-hand side of (8) contains only $g_{s-1}, g_{s-2}, \ldots$, and the second is a small perturbation when the conditions for the rarefaction of the gas are satisfied (we consider here the case of a globally weak potential). To find the successive approximations for $g_2, g_3, \ldots$ we must find g_s as a functional of the right-hand side of (8). Formally the left-hand sides of Eq. (8) are analogous to those considered in the case of weak long-range interactions.[1,3-5] We solve them for an arbitrary form of $H_1(p)$ (which is important for the electron plasma in solids) by applying the usual method of the theory of singular equations.

We start with the lowest approximation for g_2. When $s = 2$ there is no term $\Phi^{(S,S-1)}$ and on the right-hand side of (8) there remains only

$$g_2^0(x_1, x_2, t) \equiv -\int_{-\infty} d\tau S_\tau\ (x_1, x_2)[\varphi_{1,2}; f(\mathbf{p}_1, t) f(\mathbf{p}_2, t)]$$
$$= \{S_{-\infty}^{(2)}(x_1, x_2) - 1\} f(\mathbf{p}_1, t) f(\mathbf{p}_2, t) \tag{9}$$

(by virtue of the spatial uniformity of the gas and of Eq. (6)). According to (8) g_2 is in this approximation determined by the function $h(x_i, \mathbf{r}_j)$,

$$h(x_i, \mathbf{r}_j) \equiv \int d\mathbf{p}_j g_2(x_i, x_j).$$

Integrating both sides of (8) over $\mathbf{p}_2$ for s = 2, we get an equation for h. We write its Fourier transform with respect to $\mathbf{r}_2 - \mathbf{r}_1$ after integrating over τ in the following form:

$$\varepsilon_{\mathbf{k}}(\mathbf{kv}) \tilde{h}_{\mathbf{k}}(\mathbf{p}) + n\tilde{\varphi}_{\mathbf{k}} \mathbf{k} \frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} \int_{-\infty}^{\infty} \frac{d\omega H_{\mathbf{k}}^*(\omega)}{\omega - \mathbf{kv} - i\delta} = \tilde{h}_{\mathbf{k}}^0(\mathbf{p}),$$
$$\mathbf{v} \equiv dH_1(\mathbf{p})/d\mathbf{p}, \quad \delta \to +0, \quad \tilde{\varphi}_{\mathbf{k}} \equiv \int d\mathbf{r}\, \varphi(r) e^{-i\mathbf{kr}}, \tag{10}$$

where

$$\tilde{h}_{\mathbf{k}}(\mathbf{p}) \equiv \int d\mathbf{r}' e^{-i\mathbf{k}(\mathbf{r}'-\mathbf{r})} h(\mathbf{p}, \mathbf{r}, \mathbf{r}'),$$
$$H_{\mathbf{k}}(\omega) \equiv \int d\mathbf{p}\, \tilde{h}_{\mathbf{k}}(\mathbf{p}) \delta(\mathbf{kv} - \omega),$$
$$\varepsilon_{\mathbf{k}}(\omega) \equiv 1 + n\tilde{\varphi}_{\mathbf{k}} \int \frac{d\mathbf{p}}{\omega - \mathbf{kv} + i\delta} \mathbf{k} \partial f(\mathbf{p})/\partial \mathbf{p}, \tag{11}$$

while the function $\tilde{h}_{\mathbf{k}}^0(\mathbf{p})$ is determined through g_2^0 in the same way as $\tilde{h}_{\mathbf{k}}(\mathbf{p})$. Multiplying both sides of (10) by $\delta(\mathbf{k} \cdot \mathbf{v} - \omega)$ and integrating over p we obtain an equation for $H_{\mathbf{k}}(\omega)$:

$$\varepsilon_{\mathbf{k}}(\omega) H_{\mathbf{k}}(\omega) - \frac{1}{\pi} \varepsilon_{\mathbf{k}}''(\omega) \int_{-\infty}^{\infty} \frac{d\omega' H_{\mathbf{k}}^*(\omega')}{\omega' - \omega - i\delta} = H_{\mathbf{k}}^0(\omega),$$
$$\varepsilon_{\mathbf{k}}''(\omega) \equiv \operatorname{Im} \varepsilon_{\mathbf{k}}(\omega) = -\pi n \varphi_{\mathbf{k}} \int d\mathbf{p} \delta(\mathbf{kv} - \omega) \mathbf{k}\, \partial f(\mathbf{p})/\partial \mathbf{p}. \tag{12}$$

The algebraic connection between $\tilde{h}_{\mathbf{k}}(\mathbf{p})$ and $H_{\mathbf{k}}(\mathbf{k} \cdot \mathbf{v})$ is clear from (10) and (12).

To solve Eq. (12) we first consider its imaginary part:

$$K \operatorname{Im} H_{\mathbf{k}}(\omega) = \operatorname{Im} H^{0\mathbf{k}}(\omega), \tag{13}$$

where the operator K is defined by the equation

$$K\psi(\omega) \equiv \varepsilon_{\mathbf{k}}'(\omega)\psi(\omega) + \frac{1}{\pi} \varepsilon_{\mathbf{k}}''(\omega) ⨍_{-\infty}^{\infty} \frac{d\omega'\psi(\omega')}{\omega' - \omega}, \tag{14}$$

$$\varepsilon_{\mathbf{k}}'(\omega) \equiv \operatorname{Re} \varepsilon_{\mathbf{k}}(\omega) = 1 + \frac{1}{\pi} ⨍_{-\infty}^{\infty} \frac{d\omega' \varepsilon_{\mathbf{k}}''(\omega')}{\omega' - \omega}. \tag{15}$$

($⨍$ indicates a principal value integral). Applying to (13) a regularization "from the left."[6] We introduce the operator K′:

$$K'\psi(\omega) \equiv a(\omega)\psi(\omega) + \frac{1}{\pi} b(\omega) ⨍_{-\infty} \frac{d\omega'\psi(\omega')}{\omega' - \omega} \tag{16}$$

and consider the expression $K'K\psi(\omega)$. Writing the double integral which occurs in the form

$$\frac{b}{\pi^2} ⨍_{-\infty} d\omega' \varepsilon_{\mathbf{k}}''(\omega') ⨍_{-\infty}^{\infty} \frac{d\omega''\psi(\omega'')}{\omega'' - \omega} \left(\frac{1}{\omega' - \omega} + \frac{1}{\omega'' - \omega'}\right)$$
$$= \frac{b}{\pi^2} \int_{-\infty}^{\infty} \frac{d\omega'\psi(\omega')}{\omega' - \omega} \{\varepsilon_{\mathbf{k}}'(\omega) - \varepsilon_{\mathbf{k}}'(\omega')\} - b\varepsilon_{\mathbf{k}}''(\omega)\psi(\omega)$$

(we used the Poincaré-Bertrand rule [6] to change the order of integration in the second term and also used Eq. (15)), we get

$$K'K\psi(\omega) = \{a\varepsilon_{\mathbf{k}}'(\omega) - b\varepsilon_{\mathbf{k}}''(\omega)\}\psi(\omega)$$
$$+ \frac{1}{\pi}\{a\varepsilon_{\mathbf{k}}''(\omega) + b\varepsilon_{\mathbf{k}}'(\omega)\} ⨍_{-\infty}^{\infty} \frac{d\omega'\psi(\omega')}{\omega' - \omega}.$$

We put

$$a \equiv \operatorname{Re} \varepsilon_{\mathbf{k}}^{-}(\omega), \quad b \equiv \operatorname{Im} \varepsilon_{\mathbf{k}}^{-}(\omega). \tag{16'}$$

Then

$$K'K = 1 \tag{17}$$

and acting on both sides of (13) with the operator K′ we see that if there exists a solution of Eq. (13) it has the form

$$\operatorname{Im} H_{\mathbf{k}}(\omega) = K' \operatorname{Im} H_{\mathbf{k}}^0(\omega). \tag{18}$$

The right-hand side of (18) is clearly a solution of Eq. (13) for any form of Im $H_{\mathbf{k}}(\omega)$, provided

$$KK' = 1. \tag{19}$$

It is well known that in order that a spatially uniform distribution function is stable, $\epsilon_{\mathbf{k}}(z)$ as function of the complex z must not have any zeroes for Im $z \geq 0$. The function $\epsilon_{\mathbf{k}}^{-1}(z) - 1$ has therefore no singularities in that region. As, moreover, it vanishes as $|z| \to \infty$, the following relation holds:

$$\varepsilon_{\mathbf{k}}^{-1}(\omega) - 1 = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\varepsilon_{\mathbf{k}}^{-1}(\omega') - 1}{\omega' - \omega - i\delta} d\omega'. \tag{20}$$

In the case of stability there is thus a connection similar to (15) between Im $\epsilon_{\mathbf{k}}^{-1}(\omega)$ and Re $\epsilon_{\mathbf{k}}^{-1}(\omega)$ which one easily sees to lead to (19). In the case of instability there is in (20) a contribution from the zeros of $\epsilon_{\mathbf{k}}(z)$ for Im $z > 0$, and Eq. (19) does not hold which in this case corresponds to the non-existence of a "stationary" solution.

The real part of (12) gives

$$K' \operatorname{Re} H_{\mathbf{k}}(\omega) = |\varepsilon_{\mathbf{k}}(\omega)|^{-2}\{\operatorname{Re} H_{\mathbf{k}}^0(\omega) + 2\varepsilon_{\mathbf{k}}''(\omega) \operatorname{Im} H_{\mathbf{k}}(\omega)\}.$$

By virtue of (17) this equation always has the solution:

$$\operatorname{Re} H_{\mathbf{k}}(\omega) = K|\varepsilon_{\mathbf{k}}(\omega)|^{-2}\{\operatorname{Re} H_{\mathbf{k}}^0(\omega) + 2\varepsilon_{\mathbf{k}}''(\omega) \operatorname{Im} H_{\mathbf{k}}(\omega)\}.$$

When (19) is satisfied, i.e., in the case of stability, it is unique.

We now formulate by means of the operators K and K′ a method to obtain successive approximations for the higher correlation functions. The Fourier transform of Eq. (8) with respect to $\mathbf{r}_1 \ldots \mathbf{r}_s$ has, after integration over τ the form

$$\tilde{g}_s(y_1 \ldots y_s) - \left(\sum_{i=1}^{s} \mathbf{k}_i \mathbf{v}_i - i\delta\right)^{-1} n \sum_{i=1}^{s} \tilde{\varphi}_{\mathbf{k}_i} \mathbf{k}_i \cdot$$
$$\times \frac{\partial f(\mathbf{p}_i)}{\partial \mathbf{p}_i} \int d\mathbf{p}_i \tilde{g}_s(y_1 \ldots y_s) = M_s(y_1 \ldots y_s), \tag{21}$$

where

$$y_i \equiv \{\mathbf{k}_i, \mathbf{p}_i\}, \quad \tilde{g}_s(y_1 \ldots y_s) \equiv \int \prod_{i=1} (d\mathbf{r}_i e^{-i\mathbf{k}_i \mathbf{r}_i}) g_s(x_1 \ldots x_s),$$

while M_s is the Fourier transform of the right-hand side of (8). We introduce the functions $\tilde{g}_{s,\alpha}$, $1 \leq \alpha \leq s$:

$$\tilde{g}_{s,\alpha}(z_1 \ldots z_\alpha, y_{\alpha+1} \ldots y_s) \equiv \int \prod_{i=1} \{d\mathbf{p}_i \delta(\mathbf{k}_i \mathbf{v}_i - \omega_i)\} \tilde{g}_s(y_1 \ldots y_s),$$
$$z_i \equiv \{\mathbf{k}_i, \omega_i\}$$

and similarly $M_{s,\alpha}$. According to (21) $\tilde{g}_s$ can be expressed in terms of $\tilde{g}_{s,1}$. Multiplying both sides of (21) by $\prod_{i=1}^{\alpha} \delta(\mathbf{k}_i \cdot \mathbf{v}_i - \omega_i)$ and integrating over $\mathbf{p}_1 \ldots \mathbf{p}_\alpha$ we get equations connecting $\tilde{g}_{s,\alpha}$ with $\tilde{g}_{s,\alpha+1}$:

$$\tilde{g}_{s,\alpha}(z_1\ldots z_\alpha)+\frac{1}{\pi}\Big(\sum_{i=1}\omega_i+\sum_{i=\alpha+1}\mathbf{k}_i\mathbf{v}_i-i\delta\Big)^{-1}$$

$$\times\sum_{i=1}\varepsilon_{\mathbf{k}_i}''(\omega_i)\int_{-\infty}^{\infty}d\omega_i\tilde{g}_{s,\alpha}(z_1\ldots z_\alpha)=M_{s,\alpha}(z_1\ldots z_\alpha)$$

$$+\Big(\sum_{i=1}^{\alpha}\omega_i+\sum_{i=\alpha+1}^{s}\mathbf{k}_i\mathbf{v}_i-i\delta\Big)^{-1}n\sum_{i=\alpha+1}^{s}\tilde{\varphi}_{\mathbf{k}_i}\mathbf{k}_i\frac{\partial f(\mathbf{p}_i)}{\partial\mathbf{p}_i}\int_{-\infty}^{\infty}d\omega_i\tilde{g}_{s,\alpha+1}(z_1\ldots z_\alpha,z_i).\tag{22}$$

When $\alpha = s$ the second term on the right-hand side of (22) is not present so that we find for $\tilde{g}_{s,\alpha}$ an expression in terms of the right-hand side of (22) and we can then successively determine $\tilde{g}_{s,s}$, $\tilde{g}_{s,s-1}\ldots\tilde{g}_s$. In order to do this we note that the operator K' by virtue of the dispersion relation (15) has the property that

$$K'\Big\{\omega\psi(\omega)+\frac{1}{\pi}\varepsilon_{\mathbf{k}}''(\omega)\int_{-\infty}^{\infty}d\omega\psi(\omega)\Big\}=\omega K'\psi(\omega).$$

Therefore, multiplying both sides of (22) by $(\sum_{i=1}^{\alpha}\omega_i+\sum_{i=\alpha+1}^{s}\mathbf{k}_i\cdot\mathbf{v}_i-i\delta)$ and acting with the operator $\prod_{i=1}^{\alpha}K_i'(\omega_i)$ we obtain on the left-hand side

$$\Big(\sum_{i=1}^{\alpha}\omega_i+\sum_{i=\alpha+1}^{s}\mathbf{k}_i\mathbf{v}_i-i\delta\Big)\prod_{i=1}^{\alpha}K_i'(\omega_i)\tilde{g}_{s,\alpha}(z_1\ldots z_\alpha,y_{\alpha+1}\ldots y_s).$$

By virtue of (19) this operation does not introduce redundant solutions and we have in an obvious way

$$\tilde{g}_{s,\alpha}(z_1\ldots z_\alpha,y_{\alpha+1}\ldots y_s)=\prod_{i=1}K_i(\omega_i)\Big(\sum_{i=1}^{\alpha}\omega_i+\sum_{i=\alpha+1}^{s}\mathbf{k}_i\mathbf{v}_i-i\delta\Big)^{-1}$$

$$\times\prod_{i=1}^{\alpha}K_i'(\omega_i)\Big(\sum_{i=1}^{\alpha}\omega_i+\sum_{i=\alpha+1}^{s}\mathbf{k}_i\mathbf{v}_i-i\delta\Big)\{\ldots\},\tag{23}$$

where $\{\ldots\}$ is the right-hand side of Eq. (22). We have considered here δ to be a finite quantity as equations of the kind (21) but with a finite (complex) quantity δ occur and with F_1 containing a small component which changes fast in space and time.

4. KINETIC EQUATION FOR AN ELECTRON PLASMA

Using the results of the preceding section for pair correlations we consider a "stationary" kinetic equation for a stable spatially uniform plasma. We write Eq. (3) for s = 1 in the form

$$\frac{\partial f(\mathbf{p},t)}{\partial t}=-n\frac{\partial}{\partial\mathbf{p}}\int\frac{d\mathbf{k}}{(2\pi)^3}\,\mathrm{Im}\,\tilde{h}_{\mathbf{k}}(\mathbf{p},t)\tilde{\varphi}_k\mathbf{k},\tag{24}$$

$$\tilde{\varphi}_k=\frac{4\pi e^2}{k^2}.$$

After a few transformations we have from (10), (12), (18), (16), and (16′)

$$\mathrm{Im}\,\tilde{h}_{\mathbf{k}}(p)=|\varepsilon_{\mathbf{k}}(\mathbf{kv})|^{-2}\,\mathrm{Im}\,\{\varepsilon_{\mathbf{k}}^*(\mathbf{kv})\tilde{h}_{\mathbf{k}}^0(\mathbf{p})$$

$$-n\tilde{\varphi}_k\mathbf{k}\frac{\partial f(\mathbf{p})}{\partial\mathbf{p}}\int_{-\infty}\frac{d\omega H_{\mathbf{k}}^{0*}(\omega)}{\omega-\mathbf{kv}-i\delta}\Big\}.\tag{25}$$

The approximation of a weak long-range interaction (Balescu-Lenard-Guernsey collision integral) introduced by Bogolyubov[1] corresponds to the substitution into (25) of the asymptotic expression for $\tilde{h}_{\mathbf{k}}^0$ as $k\to0$:

$$k^2\tilde{h}_{\mathbf{k}}^0(\mathbf{p})\to k^2\tilde{\varphi}_k\int d\mathbf{p}'\frac{\mathbf{k}(\partial/\partial\mathbf{p}'-\partial/\partial\mathbf{p})f(\mathbf{p})f(\mathbf{p}')}{\mathbf{k}(\mathbf{v}'-\mathbf{v})-i\delta}+O(k).\tag{26}$$

Different constructions of the collision integral as a combination of the results of three approximations—strong short-range interaction, weak long-range interaction, and weak short-range interaction—proposed by Hubbard[7] and later by other authors (see the review [8]) correspond to splitting off on the right-hand side of (25) the function $\mathrm{Im}\,\tilde{h}_{\mathbf{k}}$ and taking in the remaining part the asymptotic expression (26).

We now change in the kinetic equation (24), (25) to the limit as $n\to0$. Writing $|\epsilon_{\mathbf{k}}(\mathbf{k}\cdot\mathbf{v})|^{-2}$ in the form

$$-[2i\,\mathrm{Im}\,\varepsilon_{\mathbf{k}}(\mathbf{kv})]^{-1}[\varepsilon_{\mathbf{k}}^{-1}(\mathbf{kv})-\varepsilon_{\mathbf{k}}^{*-1}(\mathbf{kv})],$$

we verify easily, using (26), that the terms in Im $\{\ldots\}$ proportional to n in (25) give a finite contribution which vanishes as $n\to0$ to the integral over $\mathbf{k}$ in (24). We write the remaining part of the expression for $\mathrm{Im}\,\tilde{h}_{\mathbf{k}}$ in the form

$$\frac{\mathrm{Im}\,\tilde{h}_{\mathbf{k}}^0(\mathbf{p})}{|\varepsilon_{\mathbf{k}}(\mathbf{kv})|^2}=\frac{k}{2}\frac{d}{dk}\Big\{-\ln\Big|\frac{\varkappa_{\mathbf{q}}^2(\mathbf{qv})}{\varkappa_{\mathbf{q}}^2(\mathbf{qv})+k^2}\Big|$$

$$+\frac{\mathrm{Re}\,\varkappa_{\mathbf{q}}^2(\mathbf{qv})}{\mathrm{Im}\,\varkappa_{\mathbf{q}}^2(\mathbf{qv})}\arg\varepsilon_{\mathbf{k}}(\mathbf{kv})\Big\}\mathrm{Im}\,\tilde{h}_{\mathbf{k}}^0(\mathbf{p}),$$

$$-\pi<\arg\varepsilon_{\mathbf{k}}(\mathbf{kv})<\pi,$$

where

$$\varkappa_{\mathbf{q}}^2(\mathbf{qv})\equiv4\pi ne^2\int\frac{d\mathbf{p}'}{\mathbf{qv}-\mathbf{qv}'+i\delta}\mathbf{q}\frac{\partial f(\mathbf{p}')}{\partial\mathbf{p}'},\quad\mathbf{q}\equiv\mathbf{k}/k,\tag{27}$$

and integrate by parts in (24), using (26) and the fact that as $k\to\infty$ we have $k^2\tilde{h}_{\mathbf{k}}^0\to0$; dropping quantities which are of higher order than the first in n we get

$$\frac{\partial f(\mathbf{p},t)}{\partial t}=n\frac{\partial}{\partial\mathbf{p}}\Big\{\int\frac{d\mathbf{k}}{(2\pi)^3}\ln\Big|\frac{k}{\varkappa_{\mathbf{q}}(\mathbf{qv})}\Big|\tilde{\varphi}_k\frac{d}{dk}[k^2\,\mathrm{Im}\,\tilde{h}_{\mathbf{k}}^0(\mathbf{p})]\mathbf{q}$$

$$-e^4\int d\mathbf{p}'\int do_{\mathbf{q}}\gamma_{\mathbf{q}}(\mathbf{qv})\mathrm{Q}(\mathbf{p},\mathbf{p}',\mathbf{q})\Big\},\tag{28}$$

where

$$\gamma_{\mathbf{q}}\equiv|\mathrm{Im}\,\varkappa^2|^{-1}\,\mathrm{Re}\,\varkappa^2\,\mathrm{arcctg}\{|\mathrm{Im}\,\varkappa_{\mathbf{q}}^2|^{-1}\mathrm{Re}\,\varkappa_{\mathbf{q}}\},$$

$$0\leqslant\mathrm{arcctg}\,\{\ldots\}<\pi,$$

$$\mathrm{Q}(\mathbf{p},\mathbf{p}',\mathbf{q})\equiv\delta(\mathbf{qv}-\mathbf{qv}')\Big[\mathbf{q}\Big(\frac{\partial}{\partial\mathbf{p}}-\frac{\partial}{\partial\mathbf{p}'}\Big)f(\mathbf{p})f(\mathbf{p}')\Big]\mathbf{q}.$$

The first term on the right-hand side of (28) is responsible for the "dynamically screened" direct Coulomb interaction of two particles while the second one can be interpreted as the result of the exchange of plasma waves.[1)] We emphasize that the kinetic equation (28)

[1)]The difference in the meaning of the terms on the right-hand side of (28), which is clear already from the fact that the first one depends only on the modulus of the "dynamical screening length" κ^{-1} while the second one depends only on Re $\kappa^2\,|\mathrm{Im}\,\kappa^2|^{-1}$, becomes physically clear if we write $|\epsilon|^{-2}$ as Re ϵ^{-1} + Re $\kappa^2(\mathrm{Im}\,\kappa^2)^{-1}$ Im ϵ^{-1} (the first and second terms give, respectively, as $n\to0$, the first and second terms in (28): The regions were $|\epsilon|\ll1$ corresponding to plasma oscillations are only important in the integral in the second term—the contribution of the first one from those regions is close to the principal value integral. We note that $|\mathrm{Im}\,\epsilon^{-1}|$ has as a function of k a maximum for $k=|\kappa|$ and decreases as k^2 for small and as k^{-2} for large values of k. The contribution from the second term is thus finite when we take instead of Im $h_{\mathbf{k}}^0$ its asymptotic value (26) (Balescu's collision integral); it is then equal to the second term in (28). The term Re ϵ^{-1} gives, however, in the Balescu integral a divergence.

is valid for any functional form of $H_1(p)$.

If $H_1(p) = p^2/2m$, we can reduce Eq. (28) to the following explicit form (see Appendix 1):

$$\frac{\partial f(\mathbf{p},t)}{\partial t} = ne^4\frac{\partial}{\partial \mathbf{p}}\int d\mathbf{p}'\int do_{\mathbf{q}}\left\{\ln\frac{m^2w^4}{|\varkappa_{\mathbf{q}}^2(\mathbf{qv})|e^4} - \gamma_{\mathbf{q}}(\mathbf{qv}) - 2C\right\}Q(\mathbf{p},\mathbf{p}',\mathbf{q}) + Rf(\mathbf{p}), \quad (29)$$

where $\mathbf{w} \equiv \mathbf{v}' - \mathbf{v}$,

$$-Rf(\mathbf{p}) \equiv 4ne^4\int d\mathbf{p}'\int_0^{mw} d\Delta \ln\frac{\Delta}{mw}\frac{d}{d\Delta}\int\frac{do_\Delta}{\Delta}\cdot\delta\left(\mathbf{w}\Delta + \frac{\Delta^2}{m}\right) \times \{f(\mathbf{p}-\Delta)f(\mathbf{p}'+\Delta) - f(\mathbf{p})f(\mathbf{p}')\}.$$

The first term on the right-hand side of (29) is the lowest ("Fokker-Planck") approximation in terms of the momentum transfer Δ when there are binary collisions while the term containing the logarithm corresponds to the Landau collision integral.[9] The second term having a purely binary character gives corrections of higher order in Δ.

When there are several kinds of particles present (a, b, ...) we must supplement the operation $ne^2\int dp'$ in (27) and (29) by a summation over the kinds of particles, m must be replaced by $2m_am_b(m_a + m_b)^{-1}, \ldots$, and e^4 by $e_a^2e_b^2, \ldots$.[2)] We note that the correction term does not contribute to the exchange of momentum and energy between the components of the plasma.

Hubbard[7] obtained an equation (in somewhat different form) which is close to (29) but is the same only with logarithmic accuracy; he started from the intuitively justified idea that in the lowest approximation the collision integral must be equal to the Boltzmann collision integral with a correction due to the polarization of the medium which is equal to the difference between the Balescu-Lenard-Guernsey and Landau collision integrals.[3)] The logarithmic divergences cancel one another but a "correct" cancellation requires clearly that all integrals should be written in the same representation. This requirement was not satisfied in [7] and this clearly explains the difference between the equation obtained there and (29) (we show in Appendix 3 that when this requirement is satisfied the above-mentioned combination of collision integrals gives the correct result).

APPENDIX 1

We denote the first term on the right-hand side of (28) by J and shall consider it to be the limit as $\eta \to 0$ of the integral over k with the lower limit of integration equal to η. We integrate by parts over k and in the remaining integral over k we change to an integration over the coordinates. We obtain

$$J(\mathbf{p}_1) = n\lim_{\eta\to 0}\frac{\partial}{\partial \mathbf{p}_1}\left\{\int d\mathbf{r}_2 h^0(\mathbf{p}_1,\mathbf{r}_1,\mathbf{r}_2)\frac{\partial}{\partial \mathbf{r}_1}\varphi_\eta(|\mathbf{r}_2-\mathbf{r}_1|) + e^4\int d\mathbf{p}_2\int do_{\mathbf{q}}\ln\frac{\eta^2}{|\varkappa_{\mathbf{q}}^2(\mathbf{qv})|}Q(\mathbf{p}_1,\mathbf{p}_2,\mathbf{q})\right\}, \quad (A.1)$$

where

$$\varphi_\eta(r) \equiv \int_{k<\eta}\frac{dk}{(2\pi)^3}\tilde{\varphi}_k\cos(\mathbf{kr}) = \frac{2}{\pi}\frac{e^2}{r}\int_{\eta r}^{\infty}\frac{dx}{x}\sin x.$$

We write the first term on the right-hand side of (A.1) in the form

$$n\int d\mathbf{p}_2\left\{\int_{\rho<\rho_0} d\mathbf{r} + \int_{\rho>\rho_0} d\mathbf{r}\right\}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)g_2^0(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2)\frac{\partial}{\partial \mathbf{r}_1}\varphi_\eta(r),$$

$$\rho \equiv w^{-1}|[\mathbf{rw}]|, \quad \mathbf{w} \equiv \mathbf{v}_2 - \mathbf{v}_1, \quad \mathbf{r} \equiv \mathbf{r}_2 - \mathbf{r}_1, \quad (A.2)^*$$

where $\rho_0 \to \infty$ in such a way that $\eta\rho \to 0$. The first integral is finite for $\eta = 0$. In the second integral for g_2^0 we can take the asymptotic value as $\rho \to \infty$:

$$g_2^0(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2) \to \int_{-\infty}^0 d\tau\frac{\partial}{\partial \mathbf{r}_1}\frac{e^2}{|\mathbf{r}_2-\mathbf{r}_1+\tau\mathbf{w}|}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)f(\mathbf{p}_1)f(\mathbf{p}_2).$$

By virtue of the linear connection between v and p we get from symmetry considerations

$$ne^2\frac{\partial}{\partial \mathbf{p}_1}\int d\mathbf{p}_2\int\frac{d\rho}{w}\int_{-\infty}^{\infty}dz\frac{\partial\varphi_\eta(r)}{\partial\rho}\int_{-\infty}^{z}dz' \times\left\{\frac{\partial}{\partial\rho}[\rho^2+(z')^2]^{-1/2}\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right)f(\mathbf{p}_1)f(\mathbf{p}_2)\right\},$$

$$\rho\mathbf{w} = 0, \quad z^2 + \rho^2 = r^2.$$

Making the transition

$$\int_{\rho>\rho_0}\frac{d\rho}{w}\psi(\rho) = \int_{\rho_0}^{\infty}\rho\, d\rho\int do_{\mathbf{q}}\delta(\mathbf{q}w)\psi(\rho\mathbf{q}), \quad q \equiv 1,$$

we get after integration over z′ and ρ

$$2ne^2\int_0 dz\varphi_\eta(\sqrt{\rho_0^2+z^2})\frac{\partial}{\partial \mathbf{p}_1}\int d\mathbf{p}_2\int do_{\mathbf{q}}Q(\mathbf{p}_1,\mathbf{p}_2,\mathbf{q}).$$

We can write the integral over z in the form

$$\frac{2e^2}{\pi}\int_{\eta\rho_0}^{\infty}\frac{dx}{x}\int_x^{\infty}\frac{dy}{y}\sin y + \frac{2e^2}{\pi}\int_1^{\infty}dx\left(\frac{1}{\sqrt{x^2-1}} - \frac{1}{x}\right)\int_{\eta\rho_0 x}\frac{dy}{y}\sin y.$$

Integrating by parts in the first integral over x, we get as $\eta\rho_0 \to 0$: $[-\ln(\eta\rho_0) - C + \ln 2]e^2$, C = 0.577.... We have thus for J the following equation:

$$J(\mathbf{p}_1) = n\lim_{\rho_0\to\infty}\left\{e^4\frac{\partial}{\partial \mathbf{p}_1}\int d\mathbf{p}_2\int do_{\mathbf{q}}\left[\ln\left|\frac{2}{\rho_0\varkappa_{\mathbf{q}}(\mathbf{qv})}\right|^2 - 2C\right] \times Q(\mathbf{p}_1,\mathbf{p}_2,\mathbf{q}) + B(\mathbf{p}_1,\rho_0)\right\}, \quad (A.3)$$

where $B(\mathbf{p}_1, \rho_0)$ denotes the integral over the region $\rho < \rho_0$ in (A.2).

Since $[H_2; g_2^0] = -[\varphi_{12}; f(\mathbf{p}_1)f(\mathbf{p}_2)]$, we have, as $\partial f/\partial \mathbf{r} \equiv 0$,

$$B(\mathbf{p}_1,\rho_0) = \int d\mathbf{p}_2\int_{\rho<\rho_0} d\mathbf{r}\,\mathbf{w}\frac{\partial}{\partial \mathbf{r}}\{S_{-\infty}^{(2)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{r}_1,\mathbf{r}_2)f(\mathbf{p}_1)f(\mathbf{p}_2)\}$$

$$= \int d\mathbf{p}_2\, w\int_{\rho<\rho_0} d\rho\,\{f(\mathbf{p}_1-\Delta)f(\mathbf{p}_2+\Delta) - f(\mathbf{p}_1)f(\mathbf{p}_2)\},$$

where $\mathbf{p}_1 - \Delta$ and $\mathbf{p}_2 + \Delta$ are the momenta of the particles before the collision while after the collision they have momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

Because of the momentum conservation law

$$\mu^{-1}\Delta = \mathbf{w}' - \mathbf{w}, \quad (A.4)$$

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2) In this case one must when solving the equations for the pair correlation functions $(g_{ab}(x_a, x_b), \ldots)$ after changing to the Fourier representation introduce instead of $\tilde{h}_k$ and H_k, respectively, the functions $\tilde{h}'_a(\mathbf{k}, \mathbf{p}_a) \equiv \Sigma\, n_b e_b \int d\mathbf{p}_b \tilde{g}_{ab}(\mathbf{k}, \mathbf{p}_a, \mathbf{p}_b)$ and $H'_k(\omega) \equiv \sum_a n_a e_a \int d\mathbf{p}_a \times \delta(\mathbf{kv}_a - \omega)\tilde{h}'_a(\mathbf{k}, \mathbf{p}_a)$ where the summation is over all kinds of particles. The equations obtained for these functions are analogous to (10) and (12), respectively.

3) See Appendix 2 for a comparison between the results of the present paper and of [7].

*$[\mathbf{rw}] \equiv \mathbf{r} \times \mathbf{W}$.

where w′ is the difference of the particle velocities before the collision and μ the reduced mass. As w′ = w, it follows from (A.4) that

$$\Delta \mathbf{w} + \Delta^2/2\mu = 0, \quad \Delta^2 = 2\mu^2(1-\cos\chi)w^2, \qquad (A.5)$$

where $\chi(\rho) \equiv \mathbf{w}\mathbf{w}'$ is the "scattering angle." By virtue of (A.4) and (A.5) and of the spherical symmetry we can write $B(\mathbf{p}_1, \rho_0)$ in the following way:

$$B(\mathbf{p}_1, \rho_0) = \int d\mathbf{p}_2 w^2 \int_0^{\rho_0} \rho\, d\rho \int do_{\mathbf{q}} \delta\left(\mathbf{q}\mathbf{w} + \frac{\Delta}{2\mu}\right)$$

$$\times \{f(\mathbf{p}_1 - \Delta\mathbf{q}) f(\mathbf{p}_2 + \Delta\mathbf{q}) - f(\mathbf{p}_1) f(\mathbf{p}_2)\}, \qquad (A.6)$$

where **q** is a three-dimensional unit vector. In the Coulomb case

$$\Delta(\rho) = 2\mu w(1 + \mu^2 w^4 e^{-4}\rho^2)^{-1/2}. \qquad (A.7)$$

One sees easily that as $\Delta \to 0$ the integral over do_q in (A.6) up to terms of order Δ^4 is equal to

$$\frac{1}{2}\Delta^2\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right) \int do_{\mathbf{q}} Q(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}).$$

Therefore, forming the function

$$A(\rho, w) \equiv \int_0^{\rho} \rho'\, d\rho'\, \Delta^2(\rho', w) = -4\frac{e^4}{w^2} \ln \frac{\Delta(\rho, w)}{2\mu w} \qquad (A.8)$$

and in (A.6) integrating by parts over ρ we have

$$B(\mathbf{p}_1, \rho_0)_{\rho_0 \to \infty} \to \frac{1}{2}\frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{p}_2 w^2 A(\rho_0 w) \int do_{\mathbf{q}} Q(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q})$$

$$- \int d\mathbf{p}_2 w^2 \int_0^{\rho_0} d\rho\, A(\rho, w) \frac{\partial}{\partial \rho} \int \frac{do_\Delta}{\Delta} \delta\left(\mathbf{w}\Delta + \frac{\Delta^2}{2\mu}\right)$$

$$\times \{f(\mathbf{p}_1 - \Delta) f(\mathbf{p}_2 + \Delta) - f(\mathbf{p}_1) f(\mathbf{p}_2)\}. \qquad (A.9)$$

In the Coulomb case the first term diverges logarithmically as $\rho_0 \to \infty$, while the second one is finite. We get (29) by substituting into (28) the result obtained from (A.3), (A.9). (A.8), and (A.7) for the first term on the right-hand side.

APPENDIX 2

The kinetic equation in the Fokker-Planck approximation was considered in [7] in the form

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial p_i}(D_i f) + \frac{1}{2}\frac{\partial^2}{\partial p_i \partial p_j}(D_{ij} f)$$

(here and henceforth a_i, a_j, ... are the Cartesian components of the vector a). The following expression was obtained for the diffusion coefficient (Eq. (6.9) of [7] in the case of a one-component plasma in our notation):

$$2ne^4 \int d\mathbf{p}' f(\mathbf{p}') \left\{\pi \frac{3w_i w_j - w^2 \delta_{ij}}{w^3} + 2 \int_{k < mw^2/2e^2} \frac{d\mathbf{k}\, d\omega\, k_i k_j}{k^4 |\varepsilon_{\mathbf{k}}(\omega)|^2} \delta(\omega + \mathbf{k}\mathbf{v}) \delta(\omega + \mathbf{k}\mathbf{v}')\right\}.$$

The second term in the braces takes, after integration over ω and k and dropping of quantities which vanish for n = 0, the form

$$\int do_{\mathbf{q}} q_i q_j \delta(\mathbf{q}\mathbf{w}) \left[\ln \frac{m^2 w^4}{4|\varkappa_{\mathbf{q}}^2(\mathbf{q}\mathbf{v})|e^4} - \gamma_{\mathbf{q}}(\mathbf{q}\mathbf{v})\right].$$

One sees easily from Eq. (29) from the present paper that

$$D_{ij} = 2ne^4 \int d\mathbf{p}' f(\mathbf{p}') \int do_{\mathbf{q}} q_i q_j \delta(\mathbf{q}\mathbf{w}) \left[\ln \frac{m^2 w^4}{|\varkappa_{\mathbf{q}}^2(\mathbf{q}\mathbf{v})|e^4} - \gamma_{\mathbf{q}}(\mathbf{q}\mathbf{v}) - 2C\right]$$

with a clear difference with Hubbard's result. The differences in the results for the D_i are similar.

APPENDIX 3

We consider the expression

$$-n\frac{\partial}{\partial \mathbf{p}} \int \frac{d\mathbf{k}}{(2\pi)^3} \operatorname{Im} \{\tilde{h}_{\mathbf{k}}^0(\mathbf{p}) - \tilde{h}_{\mathbf{k}}^{00}(\mathbf{p}) + |\varepsilon_{\mathbf{k}}(\mathbf{k}\mathbf{v})|^{-2} \tilde{h}_{\mathbf{k}}^{00}(\mathbf{p})\} \varphi_k \mathbf{k},$$

where $\tilde{h}_{\mathbf{k}}^{00}(\mathbf{p})$ is the asymptotic expression (26). Separately the terms in the braces give, respectively, the Boltzmann, Landau, and Balescu-Lenard-Guernsey collision integrals. We write this expression as the limit as $\eta \to 0$ of the integral over the region $k > \eta$. Splitting off the two last terms and integrating over k we get

$$n \lim_{\eta \to 0} \frac{\partial}{\partial \mathbf{p}} \left\{-\int_{k<\eta} \frac{d\mathbf{k}}{(2\pi)^3} \operatorname{Im} \tilde{h}_{\mathbf{k}}^0(\mathbf{p}) \varphi_k \mathbf{k} + e^4 \int d\mathbf{p}' \int do_{\mathbf{q}} \left[\ln \frac{\eta^2}{|\varkappa_{\mathbf{q}}^2(\mathbf{q}\mathbf{v})|} - \gamma_{\mathbf{q}}(\mathbf{q}\mathbf{v})\right] Q(\mathbf{p}, \mathbf{p}', \mathbf{q})\right\}.$$

One sees easily that this expression is equal to the result of substituting (A.1) into (28).

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