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THEORY OF ELECTRON PHOTOPRODUCTION IN A STRONG ELECTROMAGNETIC FIELD WITH INCLUSION OF THE FINAL-STATE INTERACTION

A. M. BRODSKII and Yu. Ya. GUREVICH

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A generalization of a well known method of taking into account final-state interactions has been used to discuss the production of charged particles with interaction in the final state with the time-dependent field of a strong electromagnetic wave. It is shown that, in the threshold approximation, inclusion of this interaction determines the form of the photoproduction as a function of the field intensity. For the one-dimensional case in which there are no additional external fields in the final state, a nonsingular integral equation is obtained which can be solved by standard methods, and the limiting case of a weak magnetic field is discussed. Perturbation theory is developed for the more general case in which additional external stationary fields, particularly a Coulomb field, are present. The results are compared with experimental curves for photoemission from metal surfaces and with earlier calculations.

1. INTRODUCTION

In recent years a number of papers have been published on the theory of electron emission in the field of a monochromatic high-intensity electromagnetic wave.[1-4] However, in view of the complex nature of the phenomenon considered, the theory developed up to the present time cannot be considered exhaustive; furthermore, the theory does not explain even a number of qualitative effects relating the dependent of the photoproduction cross sections and photocurrents on the intensity of the radiation.[5] In the present article we will discuss the question of how to take into account the interaction with the electromagnetic field in the final state. This will be done by generalizing the corresponding quantum-mechanical theory developed for description of interactions in the final state which are not time dependent.[6] Systematic inclusion of the interaction in the final state is only one of the additional effects arising in particle production in the transition from a weak external electromagnetic field, i.e., a field in which the particles can be considered free in the final state, to a field in which it is necessary to take into account the final-state interaction. However, in those cases in which use of the threshold approximation is justified and resonance phenomena are unimportant, the effect of the emitted electron's interaction with the final-state field can turn out to be decisive (if, of course, we exclude such effects as incoherence, macroscopic heating, and so forth).

In the case of a weak field the threshold approximation has turned out to be particularly effective in discussing the one-dimensional problem of electron photoemission in the visible and ultraviolet parts of the spectrum at metal-vacuum and metal-dielectric boundaries.[7] In this connection, all of the discussion will be carried on for the one-dimensional problem of photoemission from a metal surface under the influence of laser radiation, with attentional also to the fact that this phenomenon has been studied experimentally by a number of workers.[8-10] At the same time, the analysis of the one-dimensional electron-production problem made in this work permits explanation of a number of basic features of the problem being considered and can be generalized to the three-dimensional case, and also to the case of production of other charged particles.

The discussion will begin in Section 2 with an illustrative formulation of the problem for a model example. Then in Section 3 we will develop the general theory of threshold production in the field of a strong electromagnetic wave with detailed consideration of the case in which other, time-independent, external fields are absent in the final state. This limiting case of the theory can be applied directly to description of photoemission at the boundary between a metal and a polar medium or between a metal and a dielectric with a rather high dielectric permittivity. In Section 4 we develop the perturbation theory for the case corresponding to presence in the final state of additional external fields and, in particular, the Coulomb field. Finally, in Section 5 we formulate the principal results and compare them with results of earlier work.

2. FORMULATION OF THE PROBLEM

In order to explain the general approach adopted in this work we will consider first the scheme for solving the model problem of photoemission of electrons with initial energy E_0 from a semi-infinite potential well given by the potential $U(x) = -U_0$ for $x \leq -\delta$ ($U_0 > 0$, $\delta > 0$); $U(x) = 0$ for $x > -\delta$, where $-\delta$ is the location of the arbitrary boundary of the metal (see Fig. 1). Just this model is usually used[4,5] in discussing photoemission from a metal (we note that in the case of production of particles in a weak field with energy near threshold the results for a potential $U(x)$ of the form indicated and potentials with smeared edges should agree[7]). Before the external electromagnetic field is turned on, the wave function of the initial electron with $E_0 < 0$, which has the form $\psi_0(x, t) = \psi_0 \exp\{-iE_0t/\hbar\}$, is localized in the region $x < -\delta$, not taking into account

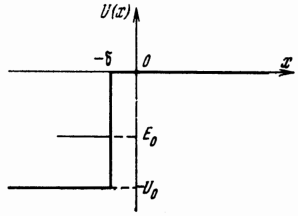

FIG. 1. Shape of the model potential U(x).

the tail which is exponentially drawn out with increasing x. By appropriately using $\delta \sim \hbar/\sqrt{m|E_0|}$, we can neglect the contribution from the region $x > 0$ in the integrals which follow, which contain in the integrand the source wave function $\psi_0(x, t)$.

We will represent the interaction with the external electromagnetic field[1)] in the form $U_{\mathscr{E}}(x) \sin \omega t$, where $U_{\mathscr{E}}(x)$ is broken down formally into two terms, the latter of which produces the interaction in the final state, which is not ordinarily taken into account in consideration of photoproduction in perturbation theory terms of lowest order in the applied field. Specifically, everywhere setting $e = m = \hbar = 1$, we will write

$$U_{\mathscr{E}}(x) = \tilde{U}_{\mathscr{E}}(x)\theta(-x) + \mathscr{E}x\theta(x) , \qquad (1)$$

where, by definition, $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x \geq 0$. Here $\mathscr{E}$ is the amplitude of the electric field of the wave. We will not choose a specific form for $\tilde{U}_{\mathscr{E}}(x)$, which is determined by the complex interaction of the field with the electrons and the medium, but will only choose a calibration in such a way that $\tilde{U}_{\mathscr{E}}(0) = 0$; in addition, evidently, $\tilde{U}_{\mathscr{E}}(x) \propto \mathscr{E}$. The notation of the second term in (1) corresponds to use of the dipole approximation, which is well satisfied for the wavelengths discussed (lasers).

In the case of a weak field, when the contribution of the second term in (1), generally speaking, is discarded, the electron wave function, after adiabatically turning on the interaction with the electromagnetic field outside the production region for $x > 0$, is represented in the form

$$\psi(E_0, x, t) = \psi_0(x)e^{-iE_0t} + \sum_n M_n(\mathscr{E}) \exp\{-i(E_0 + n\omega)t + ix\sqrt{2(E_0 + n\omega) + i\varepsilon}\} \qquad (2)$$

$(\epsilon \to +0)$, where $M_n(\mathscr{E})$ is the sum of the perturbation theory matrix elements of the direct and multistage transitions to the state with energy $E_0 + n\omega$,[2)] and ω is the frequency of the electric field of the wave. Here

$$M_n(\mathscr{E}) = \mathscr{E}^n \sum_{m=0}^{\infty} \alpha_m(n, \omega)\mathscr{E}^{2m},$$

where the coefficients $\alpha_m(n, \omega)$ do not depend explicitly on $\mathscr{E}$. As follows from simple estimates, and also from model calculations[11] for $\omega \sim U_0$, for not too great dis-distance from the threshold

$$|M_{n+1}/M_n|^2 \approx \Delta/U_0, \qquad (3)$$

where $\Delta \equiv \mathscr{E}^2/4\omega^2$ is the average kinetic energy of the pulsations of a classical electron in the external magnetic field. The ratio of two successive terms in the sum for the quantity $M_n(\mathscr{E})$ also has order Δ/U_0. In most physically realized situations Δ does not exceed 0.1 eV (the value achievable if the energy of the quanta is of the order 1 eV for a field intensity $\mathscr{E} \sim 10^7$ V/cm), while a reasonable depth of the model well is of the order 10 eV; accordingly, in constructing the current from $\psi_0(E_0, x, t)$ in Eq. (2) we can limit ourselves to one term with $n = n_0$, where n_0 is the first value of n, beginning with which $E_0 + n\omega > 0$, and also it is not necessary to take into account corrections associated with α_m for $m > 0$.

In particular, without inclusion of these corrections,

$$M_1(\mathscr{E}) = \frac{1}{W[\psi^+; \psi^-]} \int_{-\infty}^{\infty} \psi^-(E_0 + \omega, x)\tilde{U}_{\mathscr{E}}(x)\psi_0(x)\,dx, \qquad (4)$$

where $W[\psi^+, \psi^-]$ is the Wronskian of the solutions ψ^+ and ψ^-. The general form of the function $\psi^-(E_0 + \omega, x)$—the solution of the stationary Schrödinger equation without the electromagnetic field with energy $E_0 + \omega$—is determined by its behavior outside the region of action of the second term of the potential (1) for $x < 0$.

In the case being considered of the problem of the photoeffect from a metal, the function $\psi^-(x)$ should contain as $x \to -\infty$ only waves traveling from the region of production (for production in a spherically symmetric atom the analog of $\psi^-(x)$ is a regular radial function). The solution $\psi^+(E_0 + \omega, x)$, which enters into the Wronskian, satisfies the same equation as $\psi^-(x)$ and contains only waves traveling from the metal, normalized according to Eq. (2), for $x \to \infty$. In the threshold approximation in the absence of external fields for $x > 0$ the matrix elements $M_n(\mathscr{E})$ can be considered as constants[7,12] which do not depend on energy. Use of the threshold approximation is justified if the following inequalities are satisfied:

$$E_0 + n\omega < U(x) \quad \text{for} \quad x < -\delta,$$
$$|d \ln \psi^+/dx|_{x=0}\delta < 1.$$

In the simplest interpretation of the threshold approximation,[7,12] it follows from the first inequality that since for $x < -\delta$ the quantity $E_0 + n\omega$ enters into the equation for the functions $\psi^\pm$ only in a sum with the much larger quantity $U(x)$, then $\psi^\pm$ for $x < -\delta$ depends on changes of $E_0 + n\omega$ which are small in comparison with $U(x) \sim U_0$ only through the boundary condition for $x = 0$, and the second inequality permits us to justify the connection with a skip across the interval $[-\delta, 0]$, independent of details of the variation of the potential in this interval.

Further inclusion of the external fields, which we call inclusion of the final-state interaction, consists[12] in essence simply of replacing in the second term of Eq. (2) of the functions $\exp\{ix[2(E_0 + n\omega) + i\epsilon]^{1/2}\}$ by the corresponding Jost functions[13] in the external stationary field being discussed. The suggested generalization of this approach to the case of a periodically time-dependent external field is the replacement in Eq. (2) of the time-dependent functions

$$\exp\{-i(E_0 + n\omega)t + ix[2(E_0 + n\omega) + i\varepsilon]^{1/2}\}$$

by the functions $F(E_0 + n\omega, x, t)$, which for $x > 0$ satisfy the equation

[1)] In the case being discussed of a large number of photons, the external electromagnetic field can be considered classical.

[2)] We recall that on the physical sheet $i\sqrt{2E + i\epsilon} = i\sqrt{2E}$ for $E > 0$ and $i\sqrt{2E + i\epsilon} = -\sqrt{2|E|}$ for $E < 0$, where in both cases the arithmetic value of the root appears ont he right.

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2}\frac{\partial^2}{\partial x^2}-\mathscr{E}x\sin\omega t\right)F(E_0+n\omega,x,t)=0, \quad (5)$$

contain for $x \to \infty$ in the current averaged over the period of the variable field $2\pi/\omega$ only waves leaving the metal surface or attenuated waves, and for $x = 0$ are equal to $\exp\{-i(E_0 + n\omega)t\}$. Here the value of $\psi^-(E_0 + n\omega, x)$ for $x < 0$ does not change. In a strong field the current averaged over the period, constructed from $F(E_0 + n\omega, x, t)$, depends on the amplitude and frequency of the electromagnetic field, the origin of this dependence also being in this case the inclusion of the effect of the final-state interaction. As a result, if in the sum in Eq. (2) we leave only one term, then the current $\bar{j}_n$ averaged over the period of the field is

$$\bar{j}_n=\frac{\omega}{2\pi}|M_n(\mathscr{E})|^2\int\limits_{-\pi/\omega}^{\pi/\omega}\frac{i}{2}\left[F(E_0+n\omega,x,t)\frac{\partial F^*(E_0+n\omega,x,t)}{\partial x}-F^*(E_0+n\omega,x,t)\frac{\partial F(E_0+n\omega,x,t)}{\partial x}\right]dt \quad (6)$$

the value of the integral in Eq. (6) depending on the external field intensity $\mathscr{E}$.

We will point out that it is sufficient to determine the function $F(E, x, t)$ with accuracy to the constant term, since a compensating constant appears in the matrix element from the conditions of normalization of the Green's function. For the normalization of F chosen here by providing a boundary condition at $x = 0$, the quantity independent of energy in the threshold approximation is the matrix element M_n and not its product with the value of the Jost function at zero as is obtained for the standard definition.[13] We note further that this conclusion of the possibility of replacement of the final-state functions in Eq. (2) by F can be obtained also in terms of formal scattering theory in exactly the same way as is done in considering the final-state interaction with decomposition of the potential $\tilde{U}_{\mathscr{E}}$ into parts, one of which acts in the region of the potential, and the other outside it.

If the problem of finding $F(E, x, t)$ is, in essence, without a model, then determination of M_n requires that the detailed structure of the source be given. However, in many cases and, in particular, in emission from a metal, the matrix elements M_n, as in the case of a weak field, can be assumed in the threshold approximation[3)] to be constants proportional to $\mathscr{E}^n$. Thus, with an accuracy to the set of constant terms (the same as in the case of the weak field) the additional dependence of the current on the field and its energy dependence are given by the form of the functions $F(E, x, t)$.

3. PRODUCTION OF PARTICLES IN THE ABSENCE OF STATIONARY LONG-RANGE INTERACTIONS

Before we turn to determination of the universal dependence of the emission current on the external field, due to the final-state interaction, let us generalize the preceding discussion. In Sec. 2 we assumed that the electrons produced in the emission (which are going away to $+\infty$) actually exist in the metal as true particles moving in some potential well. In reality these model assumptions turn out to be not absolutely necessary—in practice it is required only that there exist an electronic wave function which is a certain function of time only at the boundary of the production region $\psi(E_0, 0, t)$ and for $x > +0$; here E_0 can be considered simply a parameter which gives the boundary condition. Then for $x > 0$ the function $\psi(E_0, x, t)$, if we can neglect the effect on the source of the field acting in the final state,[4)] is a functional of $\psi(E_0, 0, t)$ of the form

$$\psi(E_0,x,t)=\int\limits_{-\infty}^{\infty}K(x,t;0,t')\psi_0(E_0,0,t')dt' \quad (7)$$

where $K(x, t; 0, t')$ satisfies for $x > 0$ the equation

$$\left[i\frac{\partial}{\partial t}+\frac{1}{2}\frac{\partial^2}{\partial x^2}-\mathscr{E}x\sin\omega t\right]K(x,t;0,t')=0 \quad (8)$$

and the condition $K(0, t; 0, t') = \delta(t - t')$, and also the nonrelativistic causality condition[5)]

$$\delta\psi(E_0, x, t)/\delta\psi_0(E_0, 0, t')=0 \text{ for } t<t',$$

from which

$$K(x, t; 0, t')=0 \text{ for } t<t'. \quad (9)$$

If $\psi(E_0, 0, t)$ can be represented in the form of the series (2), then we reach a conclusion similar to that obtained at the end of the preceding section, that in order to take into account the final-state interaction it is necessary to define functions

$$\int\limits_{-\infty}^{\infty}K(x,t;0,t')\exp\{-i(E_0+n\omega)t'\}\,dt',\quad n=0,1,\ldots, \quad (10)$$

which, as follows from the property of $K(x, t; 0, t')$, are identical to the functions $F(E_0 + n\omega, x, t)$ introduced above. From Eq. (10) it also follows that the product

$$F(E_0+n\omega,\ x,\ t)\exp\{i(E_0+n\omega)t\}$$

is a finite analytical function of E in the complex half-plane $\operatorname{Im} E > 0$, since, in view of Eqs. (9) and (10), this product contains in its Fourier integral expansion only damped exponentials for $\operatorname{Im} E > 0$.

Following the work of Keldysh,[1] we will now use the fact that a well known[14, 15] set of solutions exist for Eq. (8), which can be written in the form

$$\psi_v(w,x,t)=\exp\left\{-iwt+i\frac{\sqrt{8\Delta}\,(w-\Delta)}{\omega}\sin\omega t-i\frac{\Delta}{2\omega}\sin 2\omega t+ix(\sqrt{2(w-\Delta)}-2\sqrt{\Delta}\cos\omega t)\right\}, \quad (11)$$

where w is a parameter which takes on any complex values. The real part of w we will designate, like the energy, by the letter E. Accordingly,

$$K(x,t;0,t')=\frac{1}{2\pi}\int\limits_{-\infty+i\varepsilon}^{\infty+i\varepsilon}g(E')\psi_v(E',x,t)\psi_v(E',0,-t')dE', \quad (12)$$

where $g(E')$ is a weighting function which does not have singularities on the physical sheet for $\operatorname{Im} w > 0$ and which is determined from the condition that $K(x, t; 0, t')$ approach $\delta(t - t')$ at $x = 0$. In the limit of a weak field, in the absence of constant external fields, $g(E') = 1$.

3) If separated levels are present in the spectrum $\Psi_0(x)$, the latter can be taken into account by the customary means for threshold theory.

4) The effect on the source, obviously, is negligible if the inequality $e\mathscr{E}\delta \ll U_0$ is satisfied.

5) The relativistic generalization of this condition is obvious: the functional derivatives must vanish for space-time points separated by a space-like interval.

For $|E'| \to \infty$ the value of $g(E')$ in the general case must approach unity, since for $|w| \to \infty$ the function $\psi_V(w, x, t)$ ceases to depend on the field.

The choice of integration contour in Eq. (12) corresponds to preservation only of waves leaving the metal and waves which are damped, and at the same time assures that $K(x, t; 0, t')$ goes to zero for $t < t'$, since then it is possible to close the integration contour by an infinite semicircle in the half-plane Im $w > 0$. Correspondingly, from the requirement that $K(x, t; 0, t')$ vanish for $t < t'$, and taking into account the notation of Eq. (12), the form of the functions (11), and the limitation mentioned above on the rise of the function $g(w)$, we can obtain the result that $g(w)$ does not have a singularity for Im $w > 0$ and that $K(x, t; 0, t')$ can contain only departing and attenuated waves. Combining Eqs. (10) and (12), we see that the desired functions $F(E, x, t)$ are given for $x > 0$ by the integral

$$F(E,x,t)=\int_{-\infty}^{\infty}K(x,t;0,t')e^{-iEt'}dt'=\int_{-\infty+i\varepsilon}^{\infty+i\varepsilon}g(E,E')\psi_v(E',x,t')dE'. \quad (13)$$

Here the function

$$g(E,E')=g(E')\int_{-\infty}^{\infty}\psi_v(E',0,-t')e^{-iEt'}dt' \quad (14)$$

is given by the boundary condition at $x = 0$. Since, according to Eq. (11), the function $\psi_V(E', 0, -t')$, after separation of the factor $\exp iE't'$, becomes a periodic function of time, the function $g(E, E')$ for real E and E' must have the form[6)]

$$g(E,E')=\sum_{k=-\infty}^{\infty}B_k(E)\delta(E+k\omega-E'), \quad (15)$$

from which we obtain

$$F(E,x,t)=\sum_{k=-\infty}^{\infty}B_k(E)\psi_v(E+k\omega,x,t). \quad (16)$$

Setting $x = 0$ in Eq. (16) and multiplying the right and left sides by $\psi_V(E + n\omega, 0, -t)$, we obtain, after averaging over t in the interval $[-\pi/\omega, \pi/\omega]$, the basic equation of Ref. 16:

$$B_n(E)=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\varphi\exp\left\{in\varphi+2p_n i\frac{\sqrt{\Delta}}{\omega}+i\frac{\Delta}{2\omega}\sin 2\varphi\right\} - \sum_k{}' B_k J_{k-n}\left(\frac{4\sqrt{\Delta}(k-n)}{p_k+p_n}\right), \quad (17)$$

where $p_n \equiv \sqrt{2}(E + n\omega - \Delta)^{1/2}$, and the prime on the summation sign indicates that terms with $k = n$ should be omitted in it.

In view of the properties of Bessel functions with integral index,

$$J_q(qz)=J_{-q}(-qz),$$
$$J_q(qz)\approx\frac{1}{\sqrt{2\pi q}}\exp\left\{q\left(1+\ln\frac{z}{2}\right)\right\} \quad (18)$$
$$(q\ll 1,\ |z|<1).$$

Further, keeping in mind that with increasing k the quantity $4\sqrt{\Delta}/(p_k + p_n) \to 0$, we find, with inclusion of (18), that the kernel of Eq. (17)—which is of the so-called kernel type[17]—is completely continuous. Accordingly, Eq. (17) uniquely defines B_n for any values of the parameters. In particular, as follows from (18), for the condition $\sqrt{\Delta/\omega} \ll 1$ Eq. (17) can be solved by successive iterations, beginning with $n = 0$, since in this case the corresponding kernel is majorized by a "good" difference kernel.

Concerning ourselves only with the n-th term in the sum of the form of (2), we find that the emission current averaged over the period of the field is

$$\bar{j}_n=|M_n|^2\sum_{k=k_0}^{\infty}p_k|B_k(E_0+n\omega)|^2, \quad (19)$$

where k_0 is the minimal integer for which $p_k^2 = 2[e_0 + \omega(k + n) - \Delta] > 0$. In the case of greatest practical interest $\sqrt{\Delta/\omega} \ll 1$, the terms in the series of Eqs. (17) and (19), as can be seen from (18), decrease rapidly, so that in (19) we can limit ourselves to the first term with $k = k_0$, and in the sum occurring in solution of (17) by iterations, we can limit ourselves to the interval $0 \le k \le k_0 - 1$, which transforms (17) to a simple recurrence relation. In particular, for $\sqrt{\Delta/\omega} \ll 1$, we obtain $B_0 = 1$, $B_1 = \sqrt{2|E_0|\Delta/\omega}$.

As follows from the equations presented, the interaction of an emitted electron with the field outside the metal leads first of all to an effective increase of the threshold for photoemission by an amount Δ. Further, the equations obtained permit rigorous relations to be found between the components of the photocurrent, which are proportional to different powers of the light intensity $J \infty \mathcal{E}^2$. Here the limitation to only n-photon] transitions inside and at the surface of metal in construction of j corresponds to the assumption that the contribution to the current of all remaining transitions in the region $x < 0$ is relatively small. If it is necessary to take into account the additional components in the current corresponding to excitation of electrons by absorption of a different number of photons inside the metal, it is sufficient to consider the superposition of solutions of the type indicated, joined at the boundary with $M_n \exp\{-i(E_0 + n\omega)t\}$ for different values of n with appropriate obvious modification of Eq. (19). It should be noted that here, for example, the possibility arises of destructive interference. Analysis of the dependence of photocurrent on intensity, which, as follows from the foregoing, can have an extremely complex nature, especially for multiphoton transitions, should permit, in discussion of specific experiments, important information to be obtained on the behavior of the normal component of the electric vector of the electromagnetic wave at the metal boundary.

Up to the present time, experiments on production in a strong field at the boundary between a metal and a polar medium, where the equations presented are applicable, have been carried out[10] only for values of $\sqrt{\Delta/\omega} \ll 1$ with a sensitivity which does not permit the effects discussed to be observed. Appearance of distinct qualitative features in the functional dependence of photocurrent on light intensity, with deviation from simple power laws, should occur, and in particular when the parameter $\sqrt{\Delta/\omega}$ becomes comparable with unity.

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6)Equation (15) follows also directly from the condition of invariance of K in Eq. (13) relative to the shift $t \to t + 2\pi n/\omega$, where n is an integer. [16]

4. PERTURBATION THEORY IN THE PRESENCE OF ADDITIONAL, TIME-INDEPENDENT FINAL-STATE INTERACTIONS

In the preceding discussion it has been assumed that outside the production region no external fields are acting except the field of the electromagnetic wave. One is frequently interested in the more general case in which outside the production region for $x > 0$ there is an additional, time-independent potential $V(x)$, which decreases as $x \to \infty$. A particularly important example of this type is the long-range Coulomb potential $V_\alpha(x) = -\alpha/x$, which corresponds in photoemission into a vacuum to action of image forces, and in photoproduction in atoms to the field of the parent ion at large distances. In this connection it should be recalled that the long-range Coulomb forces in the case of a weak field qualitatively change the behavior of the photoproduction cross section.[7, 18]

In the presence of external fields, the general course of the discussion presented above does not change; only the principal equation (8) for $K(x, t; 0, t')$ is replaced for $x > 0$ by the equation

$$\left[i\frac{\partial}{\partial t}+\frac{1}{2}\frac{\partial^2}{\partial x^2}-V(x)-\mathscr{E}x\sin\omega t\right]K(x,t;0,t')=0. \quad (20)$$

Here the construction of the current in the form of Eq. (19) is hindered by the absence in the general case of a set of solutions similar to the set of wave functions (11). However, in the parameter ranges of greatest interest for description of experiments with use of lasers, the electromagnetic field in Eq. (20) can be considered as a perturbation and we look for a function $F(E, x, t)$, connected with the new expression for $K(x, t; 0, t')$ by Eq. (13), immediately in the form of a series in the parameter $\sqrt{\Delta/\omega}$, where Δ can as before be interpreted as the electron energy shift (renormalization), averaged over the period, in the field of the electromagnetic wave as $x \to \infty$.

We will discuss briefly the method for construction of this series for the function $F(E, x, t)$. For reasons similar to those used in the preceding section after Eq. (14), the function $F(E, x, t)$ can be represented in the form of a series:

$$F(E,x,t)=e^{-iEt}\sum_k F_k(E,x)e^{-ik\omega t}. \quad (21)$$

Here the function $F_k(E, x)$, as a consequence of Eq. (20), satisfies the infinite system of coupled equations

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)-(E+k\omega)\right]F_k=-\frac{\mathscr{E}x}{2i}(F_{k+1}-F_{k-1}), \quad (22)$$

where k takes on all integral values, and for $x = 0$ with an accuracy to an arbitrary phase shift,

$$F_k(E,\ x)=\delta_{k,0}. \quad (23)$$

We will then introduce into the discussion the Jost solutions (13), $f(\pm p, x)$, which satisfy the equation

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)-E\right]f(\pm p,x)=0,\quad p\equiv\sqrt{2E+i\varepsilon}. \quad (24)$$

These solutions are chosen from the definition of $f(\pm p, x)$ such that for $x \to \infty$ the function $f(\pm p, x) = \exp \pm ipx$. Here[13]

$$W[f(p,x)f(-p,x)]\equiv\frac{\partial f(p,x)}{\partial x}f(-p,x)-f(p,x)\frac{\partial f(-p,x)}{\partial x}=2ip$$
$$[f(\pm p,x)]^*=f(\mp p,x)\quad\text{for}\quad E>0,$$
$$[f(\pm p,x)]^*=f(\pm p,x)\quad\text{for}\quad E<0. \quad (25)$$

If we take into account Eqs. (24) and (25), we can write for $F_k(E, x)$ instead of (22) the system of integral equations

$$F_k(E,x)=\delta_{k,0}\frac{f([2(E-\Delta)+i\varepsilon]^{1/2},x)}{f([2(E-\Delta)+i\varepsilon]^{1/2})}+\sum_{k'}\int_0^\infty\mathscr{K}_{kk'}(x,x')F_k(x')\,dx'. \quad (26)$$

Here $f([2(E-\Delta)+i\epsilon]^{1/2}) \equiv f([2(E-\Delta)+i\epsilon]^{1/2}, x|_{x=0}$ is the Jost function, $\mathscr{K}_{kk'}(x, x')$ is the complete kernel of the system of integral equations, determined, as follows from Eq. (22), by the relation

$$\mathscr{K}_{kk'}(x,x')\equiv G(E+k\omega-\Delta,x,x')\left[\frac{\mathscr{E}x}{2i}(\delta_{k-1,k'}-\delta_{k+1,k'})+\Delta\delta_{kk'}\right],$$

where $G(E, x, x')$ is a Green's function in the half-space $x > 0$. The function G satisfies for $x > 0$ the equation

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)-E\right)G(E,x,x')=\delta(x-x') \quad (27)$$

and for $x > x'$ contains only waves leaving the metal surface or damped waves, and approaches zero for $x = 0$, $x' > 0$. The function $G(E, x, x')$ is expressed explicitly through solution of Eq. (24). Specifically, introducing the discontinuous solution of Eq. (24) $\varphi(p, x)$ (which is a linear combination of $f(p, x)$ and $f(-p, x)$) such that $\varphi(p, x)|_{x=0} = 0$, $\partial\varphi/\partial x|_{x=0} = 1$ are satisfied, we can write[7)]

$$G(E,x,x')=\frac{2}{f(\sqrt{2E+i\varepsilon})}[f(p,x)\varphi(p,x')\theta(x-x')+f(p,x')\varphi(p,x)\theta(x'-x)]. \quad (28)$$

Introduction into (27) of the shift Δ, which is simply an energy renormalization, can be motivated here by the need of reducing the kernek $\mathscr{K}_{kk'}$ to a nonsingular form, which is achieved by choice of Δ such that the components of terms diagonal in kk' which lead to a singularity, arising in taking the trace of an iteration of the kernel, are compensated by the term containing Δ in the preceding iteration. Accordingly, the quantity Δ should be of the second order in the field.

The current averaged over the period, constructed from the functions $F(E, x, t)$, as a consequence of (21) and (23) and of the condition for conservation of the current averaged over the period, which follows from (20), is

$$\overline{j(E)}=\sum_k\frac{i}{2}\left[F_k\frac{\partial F_k^*}{\partial x}-F_k^*\frac{\partial F_k}{\partial x}\right]=\frac{i}{2}\left[F_0\frac{\partial F_0^*}{\partial x}-F_0^*\frac{\partial F_0}{\partial x}\right]_{x=0} \quad (29)$$

Using Eqs. (26) and (28), and taking into account (26) and (29), we obtain

$$\overline{j(E)}=\left[\frac{\sqrt{2(E-\Delta)}}{f([2(E-\Delta)+i\varepsilon]^{1/2})}\theta(E-\Delta)+\operatorname{Im}\mathscr{N}(E)\right], \quad (30)$$

---

7) In the simplest case $V(x) = 0$ we have $\varphi(p, x) = p^{-1}\sin px$.

where

$$\mathcal{N}(E)=2\int_0^\infty dx'\left\{\frac{f([2(E-\Delta)+i\varepsilon]^{1/2},x')}{f([2(E-\Delta)+i\varepsilon]^{1/2})}\left[\frac{\mathscr{E}x'}{2i}(F_{-1}-F_1)+\Delta F_0\right]\right\} \quad (31)$$

Limiting ourselves to the first iterative approximation for F_0 and $F_{\pm1}$ in (26), we find approximately by means of (28)

$$\mathcal{N}(E)=2\int_0^\infty dx'\,dx''\left\{\frac{f([2(E-\Delta)+i\varepsilon]^{1/2},x')}{f([2(E-\Delta)+i\varepsilon]^{1/2})}\right.$$

$$\times\left(\frac{\mathscr{E}^2x'x''}{4}[G(E-\omega-\Delta;x',x'')-G(E+\omega-\Delta;x',x'')]+\Delta\right)$$

$$\left.\cdot\frac{f([2(E-\Delta)+i\varepsilon]^{1/2},x'')}{f([2(E-\Delta)+i\varepsilon]^{1/2})}\right\}. \quad (32)$$

We cannot analyze in detail here the singularities in expressions (30)–(32) for $E-\Delta<0$ in those cases when the Jost function f occurring in the denominator approaches zero and resonance effects arise.[13] In these cases the simple use of the perturbation theory set forth is not justified, and the necessary corrections lead to appearance of a width in the resonance. The physical interpretation of this phenomenon is evident: if $f(p)=0$, then states exist in the external field with negative energy, a substantial part of which are located in the region $x>0$ and therefore are easily "captured" by the electromagnetic field. The nature of the expression for $\operatorname{Im}\mathcal{N}$ is determined by the behavior of the integrands in the vicinity of zero and also depends substantially on the form of the potential V(x) entering into Eq. (20).

In the case of rapidly falling potentials, $\operatorname{Im}\mathcal{N}\sim\sqrt{\Delta/\omega}$ and the results agree with those found in Sec. 3. In the case of a Coulomb potential[8)] $V_\alpha(x)=-\alpha/x$, the situation changes. In view of the singularities in behavior of the functions $f(\pm p, x)$ in the vicinity of zero, which are expressed for this potential in terms of the Whittaker function,[18] the value of $\operatorname{Im}\mathcal{N}$ turns out, as estimates show, to be greater in order of magnitude by $(me^2/\hbar^3\omega)^{3/2}$ times than for $V_\alpha(x)=0$. The long-range Coulomb field generally has the property of "amplifying" the perturbation action, which appears also in the case being discussed. In addition, it is important that for the case of a Coulomb field the function $f(\sqrt{2E}+i\epsilon)$ has in the vicinity of $E=0$ for $E<0$ an infinite number of zeros (a cluster point), as the result of which a substantial additional contribution to the total current should occur.

On the basis of the above, evaluation of (32) in order of magnitude gives

$$\mathcal{N}(E)\sim\frac{e^6m}{\omega\hbar^4}\sqrt{\frac{\Delta}{\omega\hbar}},$$

from which it follows that the effect of the interaction of the electron in the final state with a variable field with the presence also of the Coulomb interaction can become appreciable already at a field strength $\mathscr{E}\sim10^5$ V/cm. We note that the change in the nature of the dependence of the current on the external field intensity, which has been observed experimentally and which is the same for different metals, can be explained on the basis of the reasoning given above. In Fig. 2 we have shown the experimentally measured behavior of the photoemission current I from different metals in in vacuum as a function of the energy J of the incident light flux. It is evident from the figure that for an intensity $J\sim J^*$ a noticeable change in the dependence of I on J occurs.

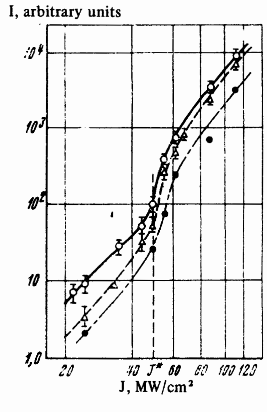

FIG. 2. Photoemission current I as a function of the intensity of a ruby laser with $\hbar\omega=1.78$ eV for different metals (according to the data of Farkas et al. [9]): O–Au, Δ–Ag, ●–Ni. For $J<J^*$ we have $I\propto J^3$, and for $J>J^*$ the nature of the deviation is given by Eq. (30).

5. CONCLUSION

The general scheme of the discussion given above reduces to the following: the matrix element and the interaction in the final state with the time-dependent field of an electromagnetic wave are discussed individually. Here a qualitative difference arises in comparison with the case of an interaction in the final state which is independent of time. In the latter case the asymptotic wave functions of the final state are uniquely determined from the conditions of energy conservation and the choice of only diverging waves. In the field of the electromagnetic wave the condition of energy conservation is replaced by the condition of joining with the time-dependent source function at the production boundary. As a result we obtain asymptotically in the final state, for example in the situation described in Sec. 3, a whole packet of wave functions of the form (11) with coefficients determined by Eq. (17). The limitation to only one such function[2,3] corresponds, generally speaking, to the unjustified retention, for example in the right-hand part of Eq. (17), of only the first term with the source. Keeping this single term is particularly inadequate in the approach to the limit $\omega\to0$, when the remaining terms in (17) begin to play such an important role. In general the iterations of Eq. (17), as in the more general method of perturbation theory developed in Sec. 4, are an expansion in a series in the parameter $\sqrt{\Delta/\omega}$, so that the obtaining in the limit $\omega\to0$ of a final-state interaction with a constant field, i.e., of definite combinations of Airy functions, is mathematically far from trivial and requires a rearrangement with regularization of the corresponding series.

Another difference in the approach presented is that here it is most convenient to study the analog of the threshold approximation, and not the Born approximation, of the problem of production in a weak field, which for the parameters which are actually achieved experimentally, apparently has greater interest.

[8)] It must be kept in mind that the potential $V_\alpha(x)$ always has a finite source region, and $V_\alpha(x)=-\alpha/x$ only for x larger than some finite dimension of order δ.

The discussion which we have given shows that the formula for the theory of production in a weak field is valid for the conditions that $\sqrt{\Delta/\omega} < 1$ and the final energy $E_0 + n\omega$ turns out to be greater than Δ. In most cases which have been realized up to the present time, both of these conditions have been satisfied. At the same time, the second condition is always violated directly at threshold. For this reason, for example, in the case of presence of an additional Coulomb field, the current corresponding to absorption of a definite number of photons exactly at the threshold does not go to zero with a step, but only decreases rapidly. We will also point out that fulfillment of the second of the conditions mentioned is brought about not only by the value of the field intensity—its violation can be detected already with the comparatively low intensity and high accuracy of near-threshold measurements.

We note in conclusion that the one-dimensional discussion can be extended to description of charged-particle production in a spherically symmetric potential. Here it is necessary to make only one remark. In the present work in all of the equations we have selected the Coulomb gauge, which permits direct expression of the interaction in terms of the intensity of the electric field. This fact is important since, in spite of the fact that the complete series of perturbation theory are gauge-invariant, the individual terms of these series, beginning with the second, do not possess this property. The situation with the gauge is complicated in particular because of the use of approximate wave functions. All of this leads, as has been shown, for example, by Nikishov and Ritus,[2] to the paradoxical possibility of obtaining results which depend substantially on the gauge. In our opinion, the optimal procedure is description of the interaction with the electromagnetic field in a formalism which does not depend on the gauge, where the potentials are replaced by integrals over the trajectory, involving the field intensities $F_{\mu\nu}$.[19,20] While in the one-dimensional case this formalism leads to use of the Coulomb gauge, in the three-dimensional case we obtain somewhat more complicated but completely defined expressions which are given in the references cited.

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[1] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [Sov. Phys.-JETP **20**, 1307 (1965)].

[2] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **50**, 255 (1966); **52**, 1707 (1967) [Sov. Phys.-JETP **23**, 168 (1966); **25**, 1135 (1967)].

[3] A. M. Perelomov, V. S. Popov, and M. V. Terent'ev, Zh. Eksp. Teor. Fiz. **50**, 1393 (1966); **51**, 309 (1966) [Sov. Phys.-JETP **23**, 924 (1966); **24**, 207 (1967)].

[4] F. V. Bunkin and M. F. Fedorov, Zh. Eksp. Teor. Fiz. **48**, 1341 (1965) [Sov. Phys.-JETP **21**, 896 (1965)].

[5] A. D. Gladun and P. P. Barashev, Usp. Fiz. Nauk **98**, 493 (1969) [Sov. Phys.-Uspekhi **12**, 490 (1970)].

[6] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[7] A. M. Brodskiĭ and Yu. Ya. Gurevich, Zh. Eksp. Teor. Fiz. **54**, 213 (1968) [Sov. Phys.-JETP **27**, 114 (1968)].

[8] E. M. Logothetis and P. L. Hartman, Phys. Rev. Lett. **18**, 581 (1967).

[9] Gy. Farkas, I. Kertesz, Zs. Naray, and P. Varga, Phys. Lett. **25A**, 572 (1967).

[10] L. I. Korshunov, V. A. Benderskiĭ, V. I. Gol'danskiĭ, and Ya. M. Zolotovitskiĭ, ZhETF Pis. Red. **7**, 55 (1968). [JETP Lett. **7**, 42 (1968)].

[11] I. Adawi, Phys. Rev. **134**, A788 (1964).

[12] A. M. Brodskiĭ and Yu. Ya. Gurevich, Izv. AN SSSR, ser. fiz. **33**, 388 (1969) [Bull. USSR Acad. Sci., Phys. Ser., p. 363].

[13] V. De Alfaro and T. Regge, Potential Scattering (Russ. Transl., Mir, 1966).

[14] D. M. Volkov (Wolkow), Z. Physik **94**, 250 (1935).

[15] L. S. Brown and T. W. B. Kibble, Phys. Rev. **133**, A705 (1964).

[16] A. M. Brodsky and Yu. Ya. Gurevich, Phys. Lett. **32A**, 145 (1970).

[17] Functional Analysis, Edited by S. G. Crane (Russ. Transl., Nauka, 1964).

[18] R. Newton, Theory of Scattering of Waves and Particles (Russ. Transl., Mir, 1969).

[19] J. Schwinger, In the collection Noveĭshee razvitie kvantovoĭ elektrodinamiki (Recent Developments in Quantum Electrodynamics), IIL, 1954, p. 254.

[20] S. Mandelstam, Phys. Rev. **175**, 1580 (1968).

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