INTERFERENCE OF COULOMB AND STRONG INTERACTIONS AT HIGH ENERGY

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It is demonstrated that the Bethe formula for the relative phase shifts of the Coulomb and hadron parts of the scattering amplitudes of charged particles is valid at high energies and fixed momentum transfers. The proof is based on factorization of the electromagnetic correction to the strongly-interacting amplitude in the region $k_\perp^2 \ll \mu^2$. The existence of factorization is a consequence of gauge invariance. The Bethe formula is invalidated by the contribution from large values of $\mathbf{k}_\perp$. Such contributions arise, however, only in hadron theories with small coupling constants.

In this communication we consider the question of the interference of the Coulomb and nuclear interactions in the scattering of charged particles of high energy. This question was considered in a number of papers[1-3] with different results. We shall show that on the basis of general considerations the result of this interference coincides with that obtained by Bethe[1].

Let us consider the scattering of two charged hadrons of high energy with initial momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ and final momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$. The strongly-interacting amplitude of this process G with the electromagnetic interaction turned off will be represented by the diagram shown in Fig. 1.

The electromagnetic interaction in first-order perturbation theory can be taken into account by all possible insertions of the virtual photon line into the hadron amplitude (Fig. 1). The diagrams with the photon line connected to the external lines (Figs. 2a and 2b) and the diagrams with the photon line emitted and absorbed within the hadron amplitude (Fig. 2c) give, generally speaking, contributions of the same order.

The integration with respect to the momenta k of the intermediate photon is best carried out in terms of the Sudakov variables[4], where $\mathbf{k}_\perp$ is the projection of the momentum perpendicular to the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$\mathbf{k} = \mathbf{p}_1\beta + \mathbf{p}_2\alpha + \mathbf{k}_\perp; \quad k_\perp{}^2 = s\alpha\beta + k_\perp{}^2; \; s = (\mathbf{p}_1 + \mathbf{p}_2)^2.$$

It is shown in[5,6] that in the region of integration with respect to small $k_\perp^2 \ll \mu^2$, where μ is the characteristic hadron mass (pion mass), the entire contribution from the electromagnetic correction is separated in the form of a multiplier for the hadron amplitude. This multiplier is determined by the sum of diagrams of the type of Fig. 2a, whose photon line joins by all possible means the charged ends of the hadron amplitude, with the internal hadron amplitude taken on the mass shell over the charged lines that enter in it. This result is a consequence of the gauge invariance and is manifest in the fact that the contribution of the cuts over the mass variables $(\mathbf{p}_i + \mathbf{k})^2$ from the sum of all the diagrams of Figs. 2a, b, c turns out to be smaller

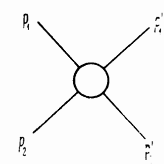

FIG. 1

by a factor $k_\perp/\mu$ than the contribution of the pole parts of Figs. 3a and b[6] 1).

We note that this result is analogous in form to the result for infrared photons[7], but at high energies the formulated statement is stronger than the corresponding statement for the infrared photons, since all that is required here is the smallness of $k_\perp^2$, whereas for the infrared photons it was required that all the components of the photon momenta be small.

The real part of the contribution of the s-photons joining the charged lines with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ or $\mathbf{p}_1'$ and $\mathbf{p}_2'$ cancels out at small values of t the contribution of the "u-photons," which join lines with momenta $\mathbf{p}_1$, $\mathbf{p}_2'$ or $\mathbf{p}_2$, $\mathbf{p}_1'$. The contribution of the "t-photons," which join lines with momenta $\mathbf{p}_1'$, $\mathbf{p}_1$ or $\mathbf{p}_2'$, $\mathbf{p}_2$ is small at small values of t. As a result, the contribution of the electromagnetic correction from the region of small $k_\perp^2$ is determined by the imaginary parts of the "s-photon" diagrams of Fig. 3 corresponding to the section of the lines 1, 2 or 3, 4, and can be written in the form

$$-\frac{i\alpha}{\pi}\int \frac{d^2k_\perp}{k_\perp{}^2 - \lambda^2} G(s, (\mathbf{q} - \mathbf{k}_\perp)^2); \qquad t = q^2 = (\mathbf{p}_1 - \mathbf{p}_1')^2 \qquad (1)$$

1)In the case of a four-point diagram, after averaging over the angles, there may appear in place of the intermediate momentum $\mathbf{l}_\perp$ (see formulas (11) and (12) of [6]) a momentum q, and therefore the contribution of the cuts contains a small quantity of the order of $\mathbf{k}_\perp/\mu$, and not $k_\perp/\mu^2$ as in the case of a three-point diagram. We note further that the condition $k^2 = k_\perp{}^2 + s\alpha\beta \ll \mu^2$, which was needed to prove this in [6], follows only from the requirement $k_\perp{}^2 \ll \mu^2$. Indeed, in the case when $s\alpha\beta \gg k_\perp{}^2$, the dependence on $k_\perp{}^2$ drops out from the integrand and a small quantity appears in terms of the phase volume

$$\frac{1}{s\alpha\beta}\int d^2k_\perp \sim \frac{k_\perp{}^2}{s\alpha\beta} \ll 1.$$

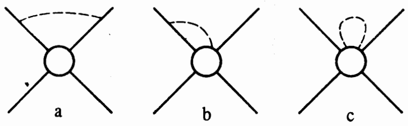

FIG. 2

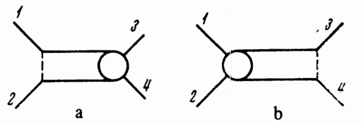

FIG. 3

This result does not depend on the character of the behavior of the hadron amplitude off the mass shell. However, the principal contribution is made by different Feynman diagrams for different forms of this behavior. When the hadron amplitude decreases with the mass, the main contribution is determined by the diagrams of Fig. 2c, and the contribution of the diagram 2a arises only from the region $k_\perp^2 \ll \mu^2/s$ and is small like $\sim \mu^2/s$ at any fixed $\lambda^2 \neq 0$.

When the hadron amplitude is independent of the mass variables, the contribution is determined by the diagram of Fig. 2a.

Since the hadron amplitude G decreases with increasing momentum transfer q^2, small $k_\perp^2 \sim q^2 \ll \mu^2$ are important in the integral (1). The contribution of large $k_\perp^2$ to the diagram of Fig. 4 is automatically cut off.

We do not see an essential contribution from photons with large $k_\perp^2 \sim \mu^2$ in diagrams with photon lines inside the hadron amplitude, with the exception of the diagrams corresponding to the electromagnetic renormalization of the position, and the residue of the vacuum pole of the hadron amplitude of Fig. 4, where the wavy line denotes the contribution of the singularity of the hadron amplitude. However, in contrast to formula (1), the diagrams of Fig. 4 give a purely imaginary contribution. The contribution from large $k_\perp^2 \gg \mu^2$ can arise only in a theory with a small coupling constant g^2, when the condition $g^2 \ln s \ll 1$ is satisfied, in the case when individual blocks of the Feynman diagram do not depend on the momentum transfer[8]. Thus, formula (1) determines the entire electromagnetic correction to the hadron amplitude.

If we take into account also the Coulomb interaction between the charged particles, we can write the total amplitude for the scattering of two charged hadrons in the form[2)]

$$A(s,t) = \frac{2\sqrt{\pi}\alpha}{q^2} + \frac{i\alpha}{\sqrt{\pi}} \int \frac{d^2k_\perp}{k_\perp^2 - \lambda^2} \frac{\alpha}{(\mathbf{q}-\mathbf{k}_\perp)^2} + G(s, q^2) \qquad (2)$$
$$+ \frac{i\alpha}{\pi} \int \frac{d^2k}{k_\perp^2 - \lambda^2} G(s, (\mathbf{q}-\mathbf{k}_\perp)^2)|_{\lambda^2 \to 0}, \quad \alpha = \frac{e_1 e_2}{4\pi\hbar c}.$$

The imaginary part of the amplitude (2) at positive and negative s determines the total cross section for particle-particle scattering (σ^+) and for particle-antiparticle scattering (σ^-)

$$\sigma^\pm = 8\pi\left\{ g^\pm(s,0) + \frac{\alpha}{\pi}\int \frac{d^2k_\perp}{k_\perp^2} f(s, k_\perp^2) + \frac{\alpha}{2\pi}\int_\delta \frac{d^2k_\perp}{k_\perp^2}\frac{\alpha}{k_\perp^2}\right\}, \qquad (3)$$

where

$$G(s, q^2) = f(s, q^2) + ig(s, q^2).$$

In (3) we assume that the Pomeranchuk theorem is satisfied for the hadron amplitude: $g^+ = g$. The integral in the last term, corresponding to the total cross section of the Coulomb interaction, is cut off at small momenta by the quantity δ. This means that it is necessary to subtract the Coulomb peak from the total cross section $\sigma^\pm$ up to momentum transfers $t = \delta$. The cutting off the integral in the second term of (3), generally speaking, is not required, since $f(s, k_\perp^2) \sim k_\perp^2$ at small $k_\perp^2$.

If we take for the hadron amplitude the contribution of the vacuum pole

$$G(s,t) = \beta \frac{-1 + e^{i\pi\alpha' t}}{\sin \pi\alpha' t} s^{\alpha' t},$$
$$g(s,t) = \beta s^{\alpha' t}, \qquad f(s,t) = -\frac{1}{2}\beta\pi\alpha' t s^{\alpha' t},$$

then we obtain (C is Euler's constant)

$$A(s,t) = \frac{2\sqrt{\pi}\alpha}{t}\left(1 - i\alpha \ln\frac{t}{\lambda^2}\right) + i\beta s^{\alpha' t}\left(1 - i\alpha\ln\frac{1}{\lambda^2\alpha' \ln s} + i\alpha C\right)$$
$$- \frac{1}{2}\beta\pi\alpha' t s^{\alpha' t} - \frac{i\alpha}{2}\beta\pi\frac{1}{\ln s}, \qquad (2')$$

$$\sigma^\pm - 4\alpha\int\frac{d^2k_\perp}{k_\perp^2}\frac{\alpha}{k_\perp^2} = 8\pi g(s,0) - 4\pi^2\alpha\beta\frac{1}{\ln s}. \qquad (3')$$

Thus we see from (3) that the presently observed difference between the cross sections of scattering of particles and antiparticles[9] cannot be explained with the aid of electromagnetic interactions[3)].

The validity of the foregoing statement can be verified with different models. Let us consider, for example, a model where the strongly-interacting amplitude G is taken to be a scalar square[11]. In this case the main contribution is made by the diagrams of Fig. 5. By direct calculation of the asymptotic form of the diagrams we can verify that the real parts of the diagrams a and d (a′ and d′) cancel out. The imaginary s-channel part of diagram d (d′) cancels out the imaginary part of the diagram a (a′) corresponding to division of the lines 5 and 7 (2′ and 4′). Diagrams b and c (b′ and c′) have a pure imaginary principal contribution[4)]. The imaginary parts of diagram b(c) corresponding to the sections of the lines 5, 1, 4 (2, 3,

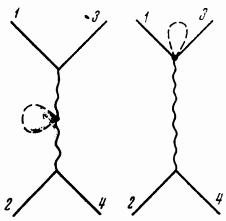

FIG. 4

[2)]The amplitude A(s, t) is normalized by the condition $|A|^2 = d\sigma_{el}/dt$.

[3)]The question of the possible violation of the Pomeranchuk theorem because of electrodynamic interactions was raised by Solov'ev [[10]] and by V. M. Shekhter.

[4)]These diagrams were incorrectly calculated in [[9]].

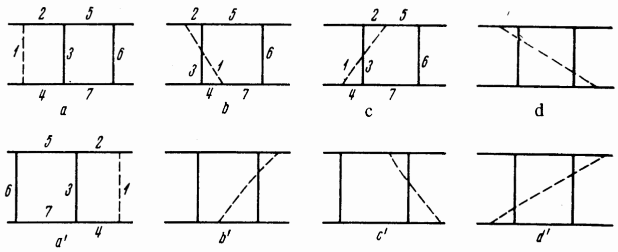

FIG. 5

7) and 5, 7 (5, 7) cancel each other; the imaginary part corresponding to the section 2, 3, 7 (5, 3, 4) cancels the imaginary part 2, 3, 7 (5, 3, 4) of diagram a. Analogously, everything stated above is valid for the diagrams a′, b′, and c′. These cancellations are the already mentioned consequence of gauge invariance. Thus, the resultant contribution coincides with the remaining uncancelled imaginary parts of diagrams and and a′ from the sections of lines 2 and 4, corresponding to the general result formulated above.

We note that it is possible to write out in similar fashion the contribution from any number of virtual photons. Summing over the photons with allowance for their identity, we obtain[12]

$$A(s,t)=\frac{1}{i\cdot 2\sqrt{\pi}}\int d^2\rho e^{iq\rho}\left[e^{2i\delta(\rho)}-1\right]+\int d^2\rho e^{iq\rho}e^{2i\delta(\rho)}A(s,\rho), \quad (4)$$

where

$$A(s,\rho)=\frac{1}{(2\pi)^2}\int d^2q A(s,q^2)e^{iq\rho},$$

$$\delta(\rho)=\frac{\alpha}{2\pi}\int\frac{d^2k_\perp}{k_\perp^2-\lambda^2}e^{i\rho k_\perp}, \quad k_\perp^2=-k_\perp^2<0.$$

The infrared divergences are separated in both terms in (4) in the form of an identical phase factor[8]. If we assume that we can introduce the concept of the nuclear potential for the hadron amplitude and use the quasiclassical approach, then

$$A(s,\rho)=\frac{1}{i\cdot 2\sqrt{\pi}}\left[e^{2i\delta_{\text{nuc}}(s,\rho)}-1\right].$$

In this case formula (4) leads to the answer obtained by Bethe[1].

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