QUASILINEAR DESTABILIZATION OF DISSIPATIVE PARAMETRIC INSTABILITY OF A PLASMA IN A WEAK HIGH-FREQUENCY ELECTRIC FIELD

A. Yu. KIRIĬ

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted June 4, 1970

Zh. Eksp. Teor. Fiz. 60, 955–964 (March, 1971)

Quasilinear action on parametric dissipative instability is studied and it is shown that the formation of a plateau on the velocity distribution function results in a decrease of Cerenkov dissipation and in a corresponding increase of the instability increment. It is also shown that during the destabilization time the plasma conductivity, which determines the energy growth of the excited plasma oscillations, increases considerably.

EARLIER investigations[1-5] pointed out the possibility of the development of parametric instability of a plasma situated in a high-frequency electric field. So long as the amplitudes of the oscillations produced in such an instability are small, the growth increments are described by a theory that is linear in these amplitudes. Allowance for nonlinear action of the excited oscillations on the indicated instability, analogous to the usual quasilinear theory, was undertaken by Silin[6]. He investigated stabilization of the parametric instability in a strong high-frequency field, when the velocity of the oscillations of the electrons in this field v_E exceeds their thermal velocity. Such a stabilization, due to the considerable growth of the electron temperature, can be set in correspondence with the hydrodynamic stage of stabilization of two-stream instabilities[7]. Unlike in[6], in the present paper we consider the quasilinear influence of the excited oscillations on the instability that develops in a relatively weak high-frequency field, namely we discuss the case of buildup of plasma and ion-acoustic oscillations under conditions when the frequency of the external field is close to the plasma frequency, and the conditions for the decay of the external wave with frequency ω_0 into plasma and ion-acoustic waves are satisfied approximately, while the growth increment γ of these oscillations does not exceed the usual damping decrement of the ion-acoustic oscillations γ_s[5]. The maximum increment of the buildup of the plasma and ion-acoustic oscillations is $\gamma_{max} \sim 1/\gamma_s$. It is known that the quasilinear action of the excited oscillations leads to the occurrence of a plateau on the velocity distribution function in the vicinity of the phase velocity of the excited oscillations. If the contribution made to the decrement γ_s by the ion-ion collisions and by the Cerenkov effect on the ions is small compared with the contribution of the Cerenkov damping by the electrons, then the formation of the plateau on the electronic distribution function in the region of the phase velocity of the ion-acoustic oscillations leads to a decrease of the decrement γ_s and a corresponding increase of the increment γ_{max}. Such a growth of the increment γ_{max} (in the case of sufficiently strong non-isothermy, when the contribution of the ion Cerenkov effect to the decrement γ_s remains small) is limited by the condition $\gamma_{max}(t) \le \gamma_s(t)$. As soon as this condition is violated because of the growth of γ_{max} and the decrease of γ_s, the maximum increment ceases to depend on γ_s and on the dissipative effects in general, and assumes a maximum value equal to $(\gamma_{max}(0)\gamma_s(0))^{1/2}$. When this value of γ_{max} is reached, the growth of the increment and the described quasilinear influence on the instability stop.

During the considered destabilization process, the work of the external field determines mainly the growth of the energy of the plasma oscillations, while the thermal energy changes relatively little. At the end of this stage, the conductivity causing the excitation of the plasma oscillations greatly exceeds the value corresponding to the pair collisions of the particles.

1. The dispersion properties of a plasma that are linear in the amplitudes of the excited longitudinal oscillations when it is situated in a relatively weak ($E_0^2 \ll 4\pi n_e T_e$) homogeneous high-frequency electric field

$$E(t) = E_0 \sin \omega_0 t$$

with a frequency close to the plasma frequency $\omega_p = (\omega_{Le}^2 + \omega_{Li}^2)^{1/2}$ (here $\omega_{L\alpha} = (4\pi e_\alpha^2 n_\alpha / m_\alpha)^{1/2}$ is the Langmuir frequency of the particles of species α) were investigated in[2-5]. Let us consider the case of parametric instability for sufficiently low frequencies of electron-ion collisions ν_{ei} and values of the detuning $\omega_0 - \omega_p$:

$$\nu_{ei} \ll 2\gamma_s(k_{st}), \tag{1.1}$$

$$\frac{\nu_{ei}\omega_s(k_{st})}{2\gamma_s(k_{st})\omega_{Le}}, \quad \frac{3}{8} k_{st}^2 r_{De}^2 \frac{\nu_{ei}^2}{\gamma_s^2(k_{st})} < \frac{\omega_0 - \omega_p}{\omega_p} \leqslant \frac{3}{2} k_{st}^2 r_{De}^2. \tag{1.2}$$

Here the collision frequency is

$$\nu_{ei} = \frac{4}{3} \frac{\sqrt{2\pi}\, e^2 e_i^2 n_i}{m_e^{1/2} T_e^{3/2}} \ln \frac{r_D}{r_{min}}, \tag{1.3}$$

the wave number k_{st} is determined by the condition of equality of the contributions made to the linear damping decrement of the plasma oscillations by the electron-ion collisions and by the Cerenkov effect on the electrons, and with logarithmic accuracy we have

$$k_{st} r_{De} = [2 \ln (\omega_{Le} / \nu_{ei})]^{-1/2},$$

$\omega_s = k\omega_{Li} r_{De} \equiv k v_s$ is the usual frequency of the ion-acoustic oscillations, $r_{De} = (T_e / 4\pi n_e e^2)^{1/2}$ the Debye radius of the electrons, $\gamma_s(\mathbf{k})$ the linear damping decrement of the ion-acoustic oscillations:

$$\gamma_s(\mathbf{k}) = -\omega_s r_{De}^2 \sum_{a=e,i} \frac{2\pi^2 e_a^2}{m_a} \int d\mathbf{v}_a \delta(\omega_s(k) - \mathbf{k}\mathbf{v}_a) \mathbf{k} \frac{\partial f_a(\mathbf{v}_a, t)}{\partial \mathbf{v}_a}, \quad (1.4)$$

with the contribution made to $\gamma_s(\mathbf{k})$ by the ion-ion collisions in the detuning region (1.2) negligible[5]. As is well known[5], the maximum of the growth increment of the plasma and ion-acoustic oscillations

$$\gamma = \frac{(\mathbf{k}\mathbf{E}_0)^2}{16\pi n_e T_e k^2} \frac{\omega_0 \omega_s^2 \Delta\omega_0 \gamma_s}{[(\Delta\omega_0)^2 - \omega_s^2]^2 + 4\gamma_s^2 (\Delta\omega_0)^2} - \tilde{\gamma}, \quad (1.5)$$

$$\Delta\omega_0(k) = \omega_0 - \omega_p(1 + {}^3/_2 k^2 r_{De}^2), \quad \tilde{\gamma} \approx {}^1/_2 \nu_{ei},$$

is reached, when (1.1) and (1.2) are satisfied, under the conditions

$$\Delta\omega_0(k_0) = \omega_s(k_0), \quad (1.6)$$

corresponding to the decay of the external wave into a plasma wave and an ion-acoustic wave. It is determined from (1.5) at not too large intensities E_0 of the high-frequency field, by the relation

$$\gamma_{max} = \frac{E_0^2}{64\pi n_e T_e} \frac{\omega_0 \omega_s(k_0)}{\gamma_s(k_0)} - \frac{1}{2}\nu_{ei} < \gamma_s(k_0). \quad (1.7)$$

We shall subsequently consider such intensities E_0 at which $\gamma_{max} \gg \nu_{ei}/2$, and, consequently, we can neglect the contribution of the electron-ion collisions to the increment (1.7). If the contribution of the Cerenkov effect on electrons dominates in expression (1.4) for γ_s, as is the case in the case of sufficiently strong non-isothermy, when

$$(|e_i|T_e / |e|T_i) > \ln(e_i^2 T_e^3 m_i / e^2 T_i^3 m_e),$$

then the influence of the growing oscillations on the instability increment (1.7) can be taken into account with the aid of an equation obtained by Silin[6] for the electron distribution function $f_e(\mathbf{v}, t)$:

$$\frac{\partial f_e(\mathbf{v}, t)}{\partial t} = \frac{\partial}{\partial v_\alpha} D_{\alpha\beta}(\mathbf{v}, t) \frac{\partial f_e(\mathbf{v}, t)}{\partial v_\beta}, \quad (1.8)$$

$$D_{\alpha\beta}(\mathbf{v}, t) = \frac{e^2}{m_e^2} \int \frac{d\mathbf{k}}{(2\pi)^3} k_\alpha k_\beta \cdot$$
$$\times \left\{ |\varphi_s(\mathbf{k}, t)|^2 \left[\pi\delta(\omega_s(k) - \mathbf{k}\mathbf{v}) - \gamma(\mathbf{k}, t) \frac{\partial}{\partial\omega_s} \frac{P}{\omega_s - \mathbf{k}\mathbf{v}} \right] + |\varphi_p(\mathbf{k}, t)|^2 \left[\pi\delta(\omega_{p1}(k) - \mathbf{k}\mathbf{v}) - \gamma(\mathbf{k}, t) \frac{\partial}{\partial\omega_{p1}} \frac{P}{\omega_{p1} - \mathbf{k}\mathbf{v}} \right] \right\}, \quad (1.9)$$

$$\omega_{p1} = \omega_p(1 + {}^3/_2 k^2 r_{De}^2).$$

Formula (1.9) for the tensor $D_{\alpha\beta}$ was obtained with account taken of relations (1.6) and (1.7). In expression (1.9) the integration is carried out over the region of positive values of the increment $\gamma(\mathbf{k})$ (1.5). The amplitudes of the potentials φ_s and φ_p of the growing ion-acoustic and plasma oscillations depend in the usual manner on the time[5]:

$$\frac{|\varphi_s(\mathbf{k}, t)|^2}{|\varphi_p(\mathbf{k}, t)|^2} = 16 \frac{\gamma^2}{\omega_0^2} \frac{k^4 r_{De}^4}{(\mathbf{k}\mathbf{r}_E)^2}, \quad \mathbf{r}_E = \frac{e\mathbf{E}_0}{m_e \omega_0^2},$$

$$|\varphi_a(\mathbf{k}, t)|^2 = |\varphi_a(\mathbf{k}, 0)|^2 \exp\left[2\int_0^t dt' \gamma(\mathbf{k}, t') \right]. \quad (1.9a)$$

The contribution made to the tensor $D_{\alpha\beta}$ (1.9) by the plasma oscillations is appreciable only at electron velocities exceeding the electron thermal velocity $v_{Te} = (T_e/m_e)^{1/2}$, whereas the contribution of the ion-acoustic oscillations is appreciable only for a longitudinal velocity component $|v_z| < v_{Te}$ (the z axis is directed along the electric field $\mathbf{E}_0$). Therefore the change of the distribution function $f_e(\mathbf{v}, t)$ occurs in these velocity regions independently, and since we are interested in the velocity region $|v_z| < v_{Te}$, we retain in (1.9) only the terms connected with the interaction of the electrons with the ion-acoustic oscillations. In addition, the only term significant in (1.9) is

$$\pi\delta(\omega_s(k) - \mathbf{k}\mathbf{v}),$$

which corresponds to resonant interaction between the electrons and the ion sound.

We shall henceforth assume the initial distribution of the electron velocities to be isotropic and Maxwellian, and the quantity $|\varphi_s(\mathbf{k}, 0)|^2$ to be independent of the direction of the wave vector $\mathbf{k}$. In this case the cylindrical symmetry (with the z axis along the vector $\mathbf{E}_0$) of the velocity distribution function f_e remains unchanged, i.e., $f_e(\mathbf{v}, t) = f_e(v_z, v_\perp, t)$, since the increment $\gamma(\mathbf{k})$ (1.5) does not depend on the angle of that component of the wave vector $\mathbf{k}$ which is perpendicular to $\mathbf{E}_0$. Integrating (1.8) with respect to the angle of the velocity component $v_\perp$ perpendicular to $\mathbf{E}_0$, we obtain

$$\frac{\partial f_e(v_z, v_\perp, t)}{\partial t} = \frac{\partial}{\partial v_z} D_{zz} \frac{\partial f_e}{\partial v_z} + \frac{\partial}{\partial v_\perp} D_{\perp\perp} \frac{\partial f_e}{\partial v_\perp} + \frac{\partial}{\partial v_z} D_{z\perp} \frac{\partial f_e}{\partial v_\perp} + \frac{\partial}{\partial v_\perp} D_{\perp z} \frac{\partial f_e}{\partial v_z}. \quad (1.10)$$

The components of the tensor $D_{\alpha\beta}$ are here of the form

$$D_{zz} = \frac{e^2}{m_e^2} \int \frac{d\mathbf{k}}{(2\pi)^3} |\varphi_s(\mathbf{k}, t)|^2 k_z^2 g[k_\perp^2 v_\perp^2 - (\omega_s - k_z v_z)^2], \quad (1.11)$$

$$D_{z\perp} = D_{\perp z} = \frac{e^2}{m_e^2} \int \frac{d\mathbf{k}}{(2\pi)^3} |\varphi_s(\mathbf{k}, t)|^2 k_z \frac{\omega_s - k_z v_z}{v_\perp} g[k_\perp^2 v_\perp^2 - (\omega_s - k_z v_z)^2], \quad (1.12)$$

$$D_{\perp\perp} = \frac{e^2}{m_e^2} \int \frac{d\mathbf{k}}{(2\pi)^3} |\varphi_s(\mathbf{k}, t)|^2 \frac{(\omega_s - k_z v_z)^2}{v_\perp^2} g[k_\perp^2 v_\perp^2 - (\omega_s - k_z v_z)^2], \quad (1.13)$$

and the function g is given by

$$g[x] = x^{-1/2}\eta(x), \quad \eta(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}.$$

The kinetic equations (1.8) and (1.10) are applicable at sufficiently large values of the quantity

$$2\int_0^t dt' \gamma(\mathbf{k}, t') > 1;$$

since no account is taken in them of the contributions of the paired collisions of the particles. Since the increment γ (1.5) is proportional to $\cos^2\theta \equiv k_z^2/k^2$, it is obvious that the largest asymptotic contribution to the components of the tensor $D_{\alpha\beta}$ (1.11)—(1.13) comes from the vicinity of the values $|\cos\theta| = 1$; it is easy to see here that the components $D_{\perp\perp}$ and $D_{z\perp}$ are small compared with D_{zz} (1.11). Thus, we can retain only the first term in the right-hand side of (1.10), and the problem of diffusion in velocity space reduces to a one-dimensional problem, where the transverse component of the electron velocity $v_\perp$ plays the role of a parameter.

We note, however, that unlike the usual one-dimensional diffusion problem, the condition for Cerenkov interaction of electrons with ion-acoustic oscillations $\omega_s(k) = \mathbf{k} \cdot \mathbf{v}$ can be satisfied not only when $|v_z| \approx v_s$ and $|\mathbf{k}_\perp \cdot \mathbf{v}_\perp| \ll \omega_s$, but also when $|k_z v_z|$

$\approx |k_\perp v_\perp|$. The possibility of resonance $\omega_S = \mathbf{k}\cdot\mathbf{v}$ with $|v| \gg v_S$ makes it necessary that a plateau be formed on the distribution function $f_e(v_Z)$ in the velocity region $|v_Z| < v_\perp \cdot \max|k_\perp/k_Z|$ in order to decrease significantly the electron contribution $\gamma_{S,e}$ to the decrement γ_S(1.4).

To find the width of the diffusion region in v_Z space, let us calculate the diffusion coefficient D_{ZZ}. We note here that the increment (1.5) reaches a maximum at $|\cos\theta \approx 1$ and $k \approx k_0$ (see (1.6)), and the characteristic width δk of the wave-number region in which $\gamma \approx \gamma_{max}$ is equal to

$$\delta k(t) = \gamma_s(t)(v_s + 3v_{Te}k_0 r_{De})^{-1} \ll k_0. \tag{1.14}$$

Recognizing also that the width of the diffusion region exceeds the phase velocity of the ion-acoustic oscillations v_S, let us calculate asymptotically the integral with respect to k in (1.11), as a result of which we obtain for the diffusion coefficient D_{ZZ}

$$D_{zz}(v_z, v_\perp, t) = \frac{e^2}{m_e^2}\frac{k_0^3}{4\pi}|\varphi_s(k_0,0)|^2\left[2\psi(t)\int_0^t dt'\frac{\gamma_{max}(t')}{\delta k^2(t')}\right]^{-1/2} \times\frac{1}{v_\perp}\exp\left[\psi(t) - \frac{v_z^2}{v_\perp^2}\psi(t)\right], \tag{1.15}$$

$$\psi(t) \equiv 2\int_0^t dt'\gamma_{max}(t') > 1.$$

From this expression we see that the characteristic region of the diffusion a, where $D_{ZZ}(v_Z)$ is not small, is given by

$$a = v_\perp\psi^{1/2}(t) \gtrsim |v_z|. \tag{1.16}$$

Since the dependence of the coefficient D_{ZZ} (1.15) on v_Z is connected with asymptotic integration with respect to the angle θ in expression (1.11), it is easy in accord with (1.16), to estimate the quantity $\max|k_\perp/k_Z| = \psi^{-1/2}$. The diffusion process considered below actually consists of two stages. In the first stage there occurs an exponential growth of the coefficient $D_{ZZ}(t)$, and the decrement $\gamma_{S,e}(t)$, and, consequently, the increment γ_{max} change insignificantly. During the second stage there occurs diffusion of particles in velocity space, an appreciable decrease of the decrement $\gamma_{S,e}(t)$, and an increase of $\gamma_{max}(t)$. The increase of $\psi(t)$ during the second stage turns out to be smaller than the corresponding contribution from the first stage, making it possible to regard the quantities $\psi(t)$ and $a(t)$ (1.16) as slowly varying functions of the time when the diffusion process is considered.

Expression (1.15) for the diffusion coefficient can be simplified by putting approximately

$$D_{zz}(v_z, v_\perp, t) = (1 - v_z^2/a^2)D(v_\perp, t), \quad |v_z| \leqslant a,$$
$$D_{zz}(v_z, v_\perp, t) \equiv 0, \quad |v_z| > a, \tag{1.17}$$
$$D(v_\perp, t) = \frac{1}{v_\perp}\frac{e^2}{m_e^2}\frac{k_0^3}{4\pi}|\varphi_s(k_0,0)|^2 e^{\psi(t)}\left[2\psi(t)\int_0^t dt'\frac{\gamma_{max}(t')}{\delta k^2(t')}\right]^{-1/2}.$$

When $|v_Z| < a$, expression (1.17) represents the first terms of the expansion of $D_{ZZ}(v_Z)$ (1.15) in terms of the parameter v_Z^2/a^2; when $|v_Z| > a$, the diffusion coefficient decreases exponentially. Thus, the approximation (1.17) describes well the dependence of the diffusion coefficient D_{ZZ} (1.15) on the longitudinal velocity component v_Z.

The solution of the one-dimensional ($D_{Z\perp} = D_{\perp\perp} = 0$) diffusion problem (1.10) with $a \approx$ const and with the diffusion coefficient (1.17) is

$$f_e(v_z, v_\perp, t) = \frac{1}{2a}\int_{-a}^{a} dv_z' f_e(v_z', v_\perp, 0)\sum_{n=0}^{\infty}(-1)^n\frac{(4n+1)(2n)!}{2^{2n}(n!)^2} \times P_{2n}\left(\frac{v_z - v_z'}{a}\right)\exp\left[-\frac{2n(2n+1)}{a^2}\int_0^t dt' D(v_\perp, t')\right]. \tag{1.18}$$

Here $f_e(v_Z, v_\perp, 0)$ is the distribution function of the electrons at the initial instant of time, and $P_n(x)$ is a Legendre polynomial. The term with $n = 0$ in formula (1.18) is the average particle density, which occurs when

$$a^2 < \int_0^t dt' D(v_\perp, t')$$

as a result of formation of a plateau on the distribution function $f_e(v_Z)$ when $|v_Z| < a$. The term with $n = 1$ describes in this case the dependence of the distribution function on the velocity v_Z.

The expression for the electronic contribution $\gamma_{S,e}$ to the damping decrement of the ion-acoustic oscillations (1.4), when account is taken of the inequalities $a \gg v_S$ and $|k_\perp/k_Z| \ge \psi^{-1/2} \ll 1$, can be readily written in the following form:

$$\gamma_{s,e}(\mathbf{k}, t) = -\pi n_e^{-1}\omega_s^2(k)v_{Te}^2 \times\int_{-\infty}^{\infty} dv_z\int_0^{\infty} dv_\perp v_\perp(k_\perp^2 v_\perp^2 - k_z^2 v_z^2)^{-1/2}\eta(k_\perp^2 v_\perp^2 - k_z^2 v_z^2) \times\left[\frac{\partial^2 f_e(v_z, v_\perp, t)}{\partial v_z^2} - \frac{v_z}{v_\perp}\frac{\partial^2 f_e(v_z, v_\perp, t)}{\partial v_z\,\partial v_\perp}\right]. \tag{1.19}$$

Substituting in this expression the distribution function (1.18), neglecting the exponentially small terms with $n > 1$ in the sum (1.18), and assuming the initial distribution function $f_e(v_Z, v_\perp, 0)$ to be Maxwellian, we obtain

$$\gamma_{s,e}(\mathbf{k}, t) = \gamma_{s,e}(k, 0)\psi(t)\frac{15}{2}\int_0^{\infty}\frac{dx}{x}\left[-\frac{1}{2}x^2 - \frac{G(t)}{x^3}\right],$$
$$x \equiv \frac{v_\perp}{v_{Te}}, \quad \gamma_{s,e}(k, 0) = \sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{\omega_{Le}}\omega_s(k), \tag{1.20}$$
$$G(t) = 6v_{Te}^{-2}\psi(t)\int_0^t dt' D(v_{Te}, t') > 1.$$

Here $D(v_\perp, t)$ is determined by formula (1.17). In deriving (1.20) we have neglected the dependence of $\gamma_{S,e}(k, t)$ on the angle θ between the vectors $\mathbf{k}$ and $\mathbf{E}_0$, a dependence connected with the longitudinal component of the wave vector k_Z, since $|k_\perp/k_Z| \ll 1$ and $|k_Z| \approx k$. In addition, it was assumed that $a < v_{Te}$.

Asymptotic integration with respect to the transverse velocity component $v_\perp$ in formula (1.20) with $G > 1$ leads to the following expression:

$$\gamma_{s,e}(k, t) = \gamma_{s,e}(k, 0)3^{4/5}\sqrt{\frac{5\pi}{2}}G^{-1/5}(t)\psi(t)\exp\left[-\frac{5}{2}3^{-3/5}G^{2/5}(t)\right]. \tag{1.21}$$

Differentiating expression (1.7) for the maximum increment $\gamma_{max}(t)$ with respect to the time and taking into account the exponential character of the dependence of the decrement $\gamma_{S,e}(t)$ on $G(t)$, we obtain for $\gamma_{max}(t)$ the equation

$$\frac{\partial\gamma_{max}(t)}{\partial t} = \gamma_{max}(t)(3G(t))^{-3/5}\frac{\partial G(t)}{\partial t}. \tag{1.22}$$

Taking into account the slow dependence of ψ on the time and using expressions (1.20) and (1.17) for G(t), and also assuming an equilibrium density for the initial noise

$$|\varphi_s(k,\ 0)|^2 = 4\pi T_e / k^2,$$

we can easily obtain from (1.22) the following equation for the increment:

$$\frac{\partial\gamma_{max}(t)}{\partial t} = \frac{3^{2/5}}{2\pi}\frac{k_0 v_{Te}}{n_e r_{De}^3} G^{-2/5}(t)\psi^{4/5}(t)e^{\psi(t)}\gamma_{max}(t)\left[2\int_0^t dt'\frac{\gamma_{max}(t')}{r_{De}^2\delta k^2(t')}\right]^{-4/5}. \quad (1.23)$$

To solve this equation it is convenient to use the dimensionless variables

$$\tau \equiv 2t\gamma_{max}(0),$$

$$\frac{\gamma_{max}(t)}{\gamma_{max}(0)} \equiv \Gamma(\tau), \qquad \psi(\tau) = \int_0^\tau d\tau'\Gamma(\tau'). \quad (1.24)$$

Taking into account expression (1.14) for $\delta k(t)$, we obtain from (1.23) with the aid of the substitution (1.24), the following equation for $\Gamma(\tau)$:

$$\frac{d\Gamma(\tau)}{d\tau} = \frac{1}{2}\xi\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\Gamma(\tau)\left[\int_0^\tau d\tau'\Gamma^3(\tau')\right]^{-4/5}\exp\left[\int_0^\tau d\tau'\Gamma(\tau')\right], \quad (1.25)$$

$$\xi(\tau) = \frac{3^{2/5}}{2\pi n_e r_{De}^3}\frac{k_0 r_{De}}{3k_0 r_{De} + (\omega_{Li}/\omega_{Le})}G^{-2/5}(\tau)\psi^{1/5}(\tau). \quad (1.26)$$

We solve Eq. (1.25) assuming that the pre-exponential factor

$$\Gamma(\tau)\left[\int_0^\tau d\tau'\Gamma^3(\tau')\right]^{-4/5} \approx 1. \quad (1.27)$$

Under such an assumption, and also when account is taken of the relatively slow dependence of $\xi(\tau)$ (1.26) on τ, the solution of (1.25) takes the form

$$\Gamma(\tau) = \alpha\frac{1+\alpha+(1-\alpha)e^{\alpha\tau}}{1+\alpha-(1-\alpha)e^{\alpha\tau}},$$

$$\alpha \equiv \left[1 - \xi\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\right]^{1/2}. \quad (1.28)$$

In the derivation of this equation it was assumed that the quantity defined in (1.26) is $\xi \ll \gamma_{max}(0)/\gamma_{s,e}(0)$, and accordingly we obtain from (1.28)

$$\Gamma(t) = \frac{\gamma_{max}(t)}{\gamma_{max}(0)} = \frac{1+(\xi\gamma_{s,e}(0)/4\gamma_{max}(0))\exp(2\gamma_{max}(0)t)}{1-(\xi\gamma_{s,e}(0)/4\gamma_{max}(0))\exp(2\gamma_{max}(0)t)} \quad (1.29)$$

Since the instability is dissipative so long as $\gamma_{max}(t) < \gamma_s(t)$, it is clear that expression (1.29) is valid when $t \le t_D$, where the destabilization time t_D is defined by

$$\frac{\gamma^2_{max}(t_D)}{\gamma^2_{max}(0)} = \frac{\gamma_{s,e}(0)}{\gamma_{max}(0)} \quad (1.30)$$

and with logarithmic accuracy we have

$$t_D = \frac{1}{2\gamma_{max}(0)}\ln\frac{4\gamma_{max}(0)}{\xi(t_D)\gamma_{s,e}(0)}. \quad (1.31)$$

By that time the maximum increment reaches, as seen from (1.30), the value

$$\gamma_{max}(t_D) = (\gamma_{s,e}(0)\gamma_{max}(0))^{1/2} = \left(\omega_0\omega_s(k_0)\frac{E_0^2}{64\pi n_e T_e}\right)^{1/2}, \quad (1.32)$$

which does not depend on the dissipative effects, and the quasilinear influence on the instability, which was considered above, stops. With the aid of (1.25) and (1.30) we can easily verify that when $t = t_D$ we have

$$e^{\psi(t_D)} = \frac{1}{\xi(t_D)}, \quad (1.33)$$

and the parameter G(t) which enters in the expression (1.26) for ξ reaches (with logarithmic accuracy), as can be seen from (1.21) and (1.30), the value

$$G(t_D) = 3^{3/2}5^{-5/2}\left[\ln\left(\frac{5\pi}{2}3^{2/5}\psi^2(t_D)\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\right)\right]^{5/2}. \quad (1.34)$$

Formula (1.33), with account taken of (1.9a), makes it possible to calculate the noise level of the ion-acoustic oscillations at the instant $t = t_D$.

The assumptions (1.27) and $\xi \ll \gamma_{max}(0)/\gamma_{s,e}(0)$ used above lead to the joint inequality

$$\ln\frac{1}{\xi(t_D)} \ll \frac{\gamma_{s,e}(0)}{\gamma_{max}(0)} \ll \frac{1}{\xi(t_D)}. \quad (1.35)$$

When considering the diffusion problem it was assumed that the diffusion region a (1.16) is smaller than the thermal velocity of the electrons. Since the maximum contribution to the integral (1.19) comes from the vicinity of values $v_\perp \approx v_{Te}(3G)^{1/5}$, the characteristic transverse velocity turns out, in accord with (1.33), to be of the order of

$$v_\perp \approx v_{Te}\left[\frac{3}{5}\ln\left(\frac{5\pi}{2}3^{2/5}\psi^2(t_D)\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\right)\right]^{1/2}.$$

Therefore the condition $a < v_{Te}$ is satisfied as a result of the inequality (1.35).

2. Let us calculate the expression for the high-frequency conductivity which determines the work of the external field and the corresponding growth of the energy of the plasma oscillations. As shown in[6], the energy balance equation of the electrons and of the excited oscillations is

$$\frac{d}{dt}\left\{\frac{3}{2}n_e T_e + \int\frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{8\pi}[|\varphi_s(\mathbf{k},t)|^2 + |\varphi_p(\mathbf{k},t)|^2]\right\} = \langle \mathbf{E}(t)\mathbf{j}(t)\rangle. \quad (2.1)$$

The averaging in the right-hand side of the equation is carried out here over the period of the external field. For the quantity determining the work of the high-frequency field it is necessary to use the expression ($\gamma \ll \omega$)

$$\langle \mathbf{E}(t)\,\mathbf{j}(t)\rangle = -i\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}\left(\frac{e\mathbf{k}\mathbf{E}_0}{2m_e\omega_0}\right)[\delta\varepsilon_i'(\omega,\mathbf{k})]^2|\varphi_s(\mathbf{k},t)|^2\cdot$$
$$\times\sum_{n=-\infty}^{\infty}\frac{R_n}{1+R_n}J_n(\mathbf{k}\mathbf{r}_E)\left[\frac{1}{1+R^*_{n+1}}J_{n+1}(\mathbf{k}\mathbf{r}_E) + \frac{1}{1+R^*_{n-1}}J_{n-1}(\mathbf{k}\mathbf{r}_E)\right], \quad (2.2)$$

which differs from that obtained by Silin (see formula (4.14) of[6]) in that it takes consistent account of the imaginary part of the quantities R_n:

$$R_n = \delta\varepsilon_e'(\omega+n\omega_0,\mathbf{k}) + i\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\delta\varepsilon_e'(\omega+n\omega_0,\mathbf{k})$$
$$+ i\delta\varepsilon_e''(\omega+n\omega_0,\mathbf{k}) + J_0^2(\mathbf{k}\mathbf{r}_E)\Big[\delta\varepsilon_i'(\omega+n\omega_0,\mathbf{k})$$
$$+ i\gamma(\mathbf{k},t)\frac{\partial}{\partial\omega}\delta\varepsilon_i'(\omega+n\omega_0,\mathbf{k}) + i\delta\varepsilon_i''(\omega+n\omega_0,\mathbf{k})\Big];$$
$$n \neq 0,\ R_0 = \delta\varepsilon_e(\omega+i\gamma,\mathbf{k}).$$

Here $\delta\varepsilon_a(\omega,\mathbf{k}) = \delta\varepsilon_a'(\omega,\mathbf{k}) + i\delta\varepsilon_a''(\omega,\mathbf{k})$ is the contribution of the particles of species a to the usual linear longitudinal dielectric constant

$$\varepsilon(\omega,\mathbf{k}) = 1 + \sum_a\delta\varepsilon_a(\omega,\mathbf{k}),$$

and J_n is a Bessel function of order n.

In the case of the dissipative instability considered above, the amplitude of the oscillations of the electrons in the high-frequency field $r_E = eE_0/m_e\omega_0^2$ is much smaller than the wavelengths of the excited oscillations, and we therefore expand the Bessel function in terms of the quantity $\mathbf{k}\cdot\mathbf{r}_E$ and, retaining only the larger terms in the sum over the harmonics (2.2), we confine ourselves to $n = 0, \pm 1$. The largest contribution to the right-hand side of (2.2) is made by the imaginary parts of the resonant denominators $1 + R_{\pm 1}$, and (2.2) takes the form

$$\langle \mathbf{E}(t)\mathbf{j}(t)\rangle \approx \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{k^2}{4\pi}|\varphi_p(\mathbf{k},t)|^2\gamma(\mathbf{k},t). \qquad (2.3)$$

Here $\gamma(\mathbf{k}, t)$ $(\gamma \gg \tilde{\gamma})$ is determined by formula (1.5), in which it is necessary to neglect the contribution of the electron-ion collisions. From (2.3), (1.9a), and (1.33) we can see that by the time of termination of the quasilinear destabilization t_D (1.31) the high-frequency conductivity, defined by the formula

$$\sigma = 2E_0^{-2}\langle \mathbf{E}(t)\mathbf{j}(t)\rangle,$$

reaches a value

$$\sigma = \frac{\omega_{Le}}{64(5\pi)^{3/2}k_0 r_{De}}\left[\ln\frac{1}{\xi(t_D)}\right]^{-3/2}\left[\ln\left(\frac{5\pi}{2}3^{8/5}\psi^2(t_D)\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\right)\right]^{3/5}. \qquad (2.4)$$

We note that the wave number k_0 in accordance with the limitation (1.2) and expression (1.6) can be relatively large. Thus, for example, at a detuning

$$\frac{\omega_0 - \omega_p}{\omega_p} \approx \frac{3}{4}\left[\ln\frac{\omega_p}{\nu_{ei}}\right]^{-1}$$

the wave number is $k_0 \approx r_{De}^{-1}[2\ln(\omega_p/\nu_{ei})]^{-1/2}$ and we obtain from (2.4) for the high-frequency conductivity

$$\sigma = \omega_{Le}\frac{\sqrt{2}}{64(5\pi)^{3/2}}[\ln(n_e r_{De}^3)]^{-1}\left[\ln\left(\frac{5\pi}{2}3^{8/5}\frac{\gamma_{s,e}(0)}{\gamma_{max}(0)}\right)\ln^2(n_e r_{De}^3)\right]^{8/5}. \qquad (2.5)$$

It is easily seen from (2.1) and (2.3) that the work of the high-frequency field goes mainly to the increase of the energy of the high-frequency plasma oscillations, as is characteristic of the considered case of decay parametric instability.

CONCLUSIONS

Quasilinear action on the parametric dissipative instability that develops in a weak high frequency field at $\omega_0 \approx \omega_p$ leads to formation of a plateau on the electron distribution function in the region of velocities lower than thermal, and to a corresponding growth of the increment up to a value that does not depend on the dissipative processes. During the destabilization time, the high-frequency conductivity (2.4) increases appreciably in comparison with the value $\sigma = \nu_{ei}/4\pi$, which is connected with the electron-ion collisions. Here, however, the characteristic growth time of the temperature remains much larger than t_D, making it possible to regard the electron temperature as constant during the quasilinear stage.

We note that a quasilinear destabilization analogous to that considered above will be realized also in the case when the decrement γ_s (1.4) is smaller than the damping decrement of the plasma oscillations and the intensity of the high-frequency field E_0 differs little from the threshold value. The increase of the increment corresponds here to a decrease in the value of $E_{0,thr}$, which is proportional to γ_s (see formula (4.25) of[5]). Finally, at sufficiently large detunings, when the linear damping decrement $\tilde{\gamma}$ (1.5) of the high-frequency oscillations is connected with the Cerenkov effect on electrons, the increase of the increment can be due to the formation of a plateau on the electron distribution function in the velocity region $v = \omega/\hat{p}/k_0$; such a situation corresponds to allowance for the second term in the curly brackets of formula (1.9) for the diffusion coefficient.

In conclusion, I am grateful to V. P. Silin for valuable advice and to N. E. Andreev for a discussion of the results.

[1] V. P. Silin, Interaction of a strong High-frequency Electromagnetic Field with a Plasma (A Survey of Phenomena in Ionized Gases), IAEA, Vienna, 1968, p. 205.

[2] D. F. Dubois and M. V. Goldman, Phys. Rev., 164, 207 (1967).

[3] D. F. Dubois and M. V. Goldman, Phys. Rev. Lett., 19, 1105 (1967).

[4] K. Nishikawa, J. Phys. Soc., Japan, 24, 916, 1152 (1968).

[5] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 1024 (1969) [Sov. Phys.-JETP 30, 559 (1970)].

[6] V. P. Silin, ibid. 57, 183 (1969) [30, 105 (1970)].

[7] V. D. Shapiro and V. I. Shevchenko, ibid. 42, 1515 (1962) [15, 1053 (1962)].

Translated by J. G. Adashko
101