RESONANT INELASTIC EFFECTS IN A PLASMA IN A STRONG FIELD

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The rate of energy loss and other kinetic coefficients in a plasma located in strong electric and magnetic fields are calculated in the Born approximation with allowance for the inelastic collisions of the electrons with the neutral particles. It is shown that the dependence of the kinetic coefficients on the external field has a resonant character. Resonant oscillations of the rate of energy loss and of the electric conductivity are investigated with the aid of correlation functions in a strong homogeneous field, which are derived in the appendix.

In the case of elastic collisions, the dependence of the collision integral of charged particles in a plasma on the external-field intensity comes into play only in very strong fields (see below), and does not have a resonant character. Allowance for inelastic collisions of the electrons with the ions and neutrals leads to the appearance of resonant terms in the expressions for the collision integrals and different kinetic coefficients. The resonances appear when the frequencies of the inelastic transitions coincide with the frequency of the transitions between the Landau levels of the electron in the magnetic field (cyclotron frequency and frequency of the overtones), and also with the frequency of the oscillations of the electrons in an alternating electric field. As shown in[1], in the Born approximation, to take into account the contribution of inelastic collisions to the kinetic coefficients, it is necessary to use the correlation functions of the colliding particles. We shall consider below transport and relaxation processes in a plasma in a strong external field with the aid of the correlation functions of charged particles.

1. RELAXATION AND TRANSPORT PROCESSES IN A STRONG MAGNETIC FIELD

With the aid of the correlation functions Φ, obtained in closed form (see the Appendix), it is possible to study the influence of strong electric and magnetic fields on the frequencies of the elastic and inelastic collisions, on the rate constants, on the energy losses, and on other kinetic characteristics of the plasma. For example, when account is taken of only elastic collisions, we obtain for the rate of transfer of transverse energy from the electrons (temperatures $T_\parallel$ and $T_\perp$, kinetic-energy operator $\hat{K}_e = \hat{K}_\parallel + \hat{K}_\perp$, mass m) and the neutrals (T, K, M) in a magnetic field $H = mc\omega$ (we put $\hbar = k = e = 1$) in the Born approximation

$$-\overline{\frac{d\hat{K}_\perp}{dt}} = n_0 \int_{-\infty}^{\infty} dt \langle [\hat{K}_\perp \hat{V}(t)] \hat{V}^*(0) \rangle$$
$$= -in_0 \int d^3q\, |V_q|^2 \int_{-\infty}^{\infty} dt\, \Phi_0 \Phi_\parallel \frac{d\Phi_\perp}{dt}. \qquad (1.1)$$

If we assume that $\gamma = m/M \to 0$, then we can easily obtain from (1.1) ($K_0(x)$ is the Macdonald function)

$$-\overline{\frac{d\hat{K}_\perp}{dt}} = Z_0 \varphi^2 \operatorname{sh}\varphi \frac{T_\perp{}^2}{\overline{T}} \int \frac{dx\, \psi(x)}{\sqrt{\eta^2 - 1}} \sum_{n=1}^{\infty} \frac{n \operatorname{sh}(n\varphi(T_\perp - \overline{T})/\overline{T})}{(\eta + \sqrt{\eta^2 - 1})^n}$$
$$\times K_0\left(n\varphi \frac{T_\perp}{\overline{T}} \sqrt{1 + 8\mu x T}\right),$$
$$Z_0 = 16\pi n_0 \mu \sqrt{2\pi\mu\overline{T}}\, \overline{V}, \quad \eta = \operatorname{ch}\varphi + 2mx\omega \operatorname{sh}\varphi, \quad \varphi = \frac{\omega}{2T_\perp},$$
$$\frac{\overline{T}}{\mu} = \frac{T}{M} + \frac{T_\parallel}{m}, \quad \mu = \frac{mM}{m+M} \approx m. \qquad (1.2)$$

On going over from (1.1) to (1.2) we have assumed that for the square of the modulus of the Fourier component of the collision potential we can use the integral representation

$$|V_q|^2 = \overline{V} \int dx\, \psi(x) e^{-xq^2}. \qquad (1.1')$$

Calculation of the sum of the contributions of the multiquantum transitions between the Landau levels simplifies greatly for $\psi(x) = \delta(x)$ (collisions of electrons with homonuclear molecules, see[1]):

$$-\overline{\frac{d\hat{K}_\perp}{dt}} = Z_0 \varphi^2 \frac{T_\perp{}^2}{\overline{T}} \sum_{n=1}^{\infty} n e^{-n\varphi} \operatorname{sh}\left(n\varphi \frac{T_\perp - \overline{T}}{\overline{T}}\right) K_0\left(n\varphi \frac{T_\perp}{\overline{T}}\right). \qquad (1.3)$$

The Schlohmilch sums (1.2) and (1.3) can be calculated in the limit of weak ($\varphi \to 0$) and strong ($\varphi \to \infty$) fields. Thus, as $\varphi \to 0$, replacing the summation in (1.3) by integration, we get

$$-\overline{\frac{d\hat{K}_\perp}{dt}} = Z_0 \overline{T} [Y(\Delta) - Y(1)], \quad \Delta = \frac{2\overline{T} - T_\perp}{T_\perp},$$
$$Y(\Delta) = \frac{1}{\Delta^2 - 1} \left[\frac{\Delta}{\sqrt{\Delta^2 - 1}} \ln(\Delta + \sqrt{\Delta^2 - 1}) - 1\right]. \qquad (1.3')$$

In a strong field, the energy loss is exponentially small:

$$-\overline{\frac{d\hat{K}_\perp}{dt}} = \frac{1}{4} Z_0 \omega \sqrt{\frac{\pi\omega}{\overline{T}}} e^{-2\varphi} \operatorname{sh}\left(\varphi \frac{T_\perp - \overline{T}}{\overline{T}}\right), \quad \varphi \gg 1. \qquad (1.3'')$$

As $T_\perp \to \overline{T}$ we have

$$\overline{\frac{d\hat{K}_\perp}{dt}} = \frac{\overline{T} - T_\perp}{\tau_\perp}, \quad \frac{1}{\tau_\perp} = Z_0 \varphi^3 \sum_{n=1}^{\infty} n^2 e^{-n\varphi} K_0(n\varphi) \qquad (1.4)$$

and

$$\frac{1}{\tau_\perp} \to \frac{4}{15} Z_0, \quad \varphi \to 0. \qquad (1.4')$$

For Coulomb collisions, results analogous to (1.3′) and (1.4′) were obtained by Kogan[2] with the aid of the Landau collision integral. The Coulomb relaxation at $\varphi \neq 0$ was investigated by Silin[3] in the classical limit, corresponding to the real correlation function

$$\Phi_\perp(q,t) = \exp\left[-\frac{q_\perp^2 T_\perp}{m\omega^2}(1-\cos\omega t)\right].$$

In our case it is easy to describe the Coulomb relaxation by replacing Φ_0 in (1.1) by $\Phi_\parallel \Phi_\perp$ for heavy particles, corresponding to consideration of cyclotron rotation of not only the electrons but also the ions (transverse diffusion with allowance for the Debye polarization was taken into account in similar fashion in[4]).

The longitudinal relaxation is described in analogy with (1.1)—(1.4). The elastic loss of energy by heavy particles $\overline{d\hat{K}/dt}$ are determined by the relation

$$\overline{\frac{d}{dt}(\hat{K}_\parallel + \hat{K}_\perp + \hat{K})} = 0. \qquad (1.5)$$

A curious feature of relaxation in a strong magnetic field with allowance for inelastic collisions of the electrons with the neutrals is the resonant character of the dependence of different kinetic coefficients on the field intensity. Let us examine with the aid of (A.8) the oscillations of the rate of inelastic energy loss and of the electric conductivity, and let us confine ourselves to an approximation of an almost isothermal plasma, i.e., we assume that the translational (T) and the internal (T_1) temperatures of the heavy particles coincide and that $T_\parallel = T_\perp \to T$. From the expansion ($\hat{H}$ is the internal-energy operator)

$$-\overline{\frac{d\hat{H}}{dt}} = \frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k!}\left(\frac{\Delta T}{T^2}\right)^k \overline{(\Delta\varepsilon)^{k+1}} = n_0\int_{-\infty}^{\infty} dt\,\langle[\hat{H}\hat{V}(t)]\,\hat{V}^*(0)\rangle \qquad (1.6)$$

in the limit as $\Delta T \to 0$ we get ($\Delta\epsilon$ is the transferred energy)

$$\frac{1}{\tau} = \frac{\overline{\Delta\varepsilon^2}}{2T^2} = -\frac{n_0}{2T^2}\int d^3q\int_{-\infty}^{\infty} dt\,\exp\left[-\frac{q^2}{2M}(it+t^2T)\right] \times \Phi_\parallel\Phi_\perp\frac{d^2}{dt^2}\langle\hat{V}_q(t)\,\hat{V}_q^*(0)\rangle. \qquad (1.7)$$

In the general case the correlator of the collision operator is of the form (see[1])

$$\langle\hat{V}_q(t)\,\hat{V}_q^*(0)\rangle = \sum_{\nu\nu'} V_{\nu\nu'}(q)\,(\exp[-\varepsilon_\nu/T - it\omega_{\nu'\nu}] + \exp[-\varepsilon_{\nu'}/T + it\omega_{\nu'\nu}]),\quad \omega_{\nu'\nu} = \varepsilon_{\nu'} - \varepsilon_\nu. \qquad (1.8)$$

From (1.7) and (1.8) we obtain for the reciprocal time of inelastic relaxation

$$\frac{1}{\tau} = \frac{2\pi m n_0}{T}\sqrt{\frac{2\pi\mu}{T}}\,\varphi\,\mathrm{sh}\,\varphi\sum_{\nu\nu'}\bar{V}_{\nu\nu'}\exp\left(-\frac{\varepsilon_\nu+\varepsilon_{\nu'}}{T}\right)\int dx\,\psi_{\nu\nu'}(x) \times\iint_0^\infty\frac{du\,dv}{\sqrt{u(u+8av)}}\exp\left[-v(2a+2\mu Tx+\mathrm{ch}\,\varphi) - u\left(\frac{1}{4}+2\mu xT\right)\right] \times\left\{I_0(v)\exp\left[-\frac{\omega_{\nu'\nu}^2}{4T^2(u+8av)}\right] + 2\sum_{n=1}^{\infty}I_n(v)\exp\left[-\frac{(\omega_{\nu'\nu}-n\omega)^2}{4T^2(u+8av)}\right]\right\},$$
$$a = {}^1\!/_4\gamma\varphi\,\mathrm{sh}\,\varphi. \qquad (1.9)$$

In the approximation $\psi_{\nu\nu'}(x) = \delta(x)$, the contribution of the resonant term ($\omega_{\nu'\nu} = n\omega$) describing the transitions between the Landau levels of magnetic oscillators and the internal degrees of freedom of the structure particles is equal to

$$\frac{1}{\tau_{\nu\nu'}} = 2n_0\left(\frac{2\pi m}{T}\right)^{3/2}\sqrt{\frac{\mu}{m}}(n\omega)^2\varphi\,\mathrm{sh}\,\varphi\,\bar{V}_{\nu\nu'}\exp\left(-\frac{\varepsilon_\nu}{T}-n\varphi\right)g_n(a,b),$$
$$g_n(a,b) = \int_0^\infty dv\,e^{-v(a+b)}K_0(av)I_n(v),\quad b = \mathrm{ch}\,\varphi. \qquad (1.10)$$

In the limit as $\gamma \to 0$ we can obtain from (1.10) (see[5], 4.16.28 and[6], 8.831.1 and 6.611.4)

$$g_n(a,b) \to \frac{e^{-nA\,\mathrm{rch}\,b}}{\sqrt{b^2-1}}\ln\left(\frac{2b}{a}\right),\ a\to 0,\ \frac{b}{a}\to\infty. \qquad (1.10')$$

In the absence of a magnetic field the contribution of the transitions between levels with energies ϵ_ν and $\epsilon_{\nu'}$ to the total inelastic-relaxation time

$$\sum_{\nu\nu'}\frac{1}{\tau^0_{\nu\nu'}} = \frac{1}{\tau_0} = -\frac{n_0}{2T^2}\int d^3q\int_{-\infty}^{\infty} dt \times\exp\left\{-\frac{q^2}{2\mu}(it+t^2T)\right\}\frac{d^2}{dt^2}\langle\hat{V}_q(t)\,\hat{V}_q^*(0)\rangle \qquad (1.11)$$

is given by

$$\frac{1}{\tau_{\nu\nu'}^0} = 4n_0\left(\frac{2\pi\mu}{T}\right)^{3/2}\frac{\omega_{\nu'\nu}^2}{T}\bar{V}_{\nu\nu'}K_1\left(\frac{\omega_{\nu'\nu}}{2T}\right)\exp\left\{-\frac{\varepsilon_\nu-\varepsilon_{\nu'}}{2T}\right\}; \qquad (1.12)$$

therefore ($m \approx \mu$) we have the ratio

$$\frac{1}{\tau_{\nu\nu'}}:\frac{1}{\tau^0_{\nu\nu'}} \approx \frac{e^{-n\varphi}}{4nK_1(n\varphi)}\ln\left(\frac{8\,\mathrm{cth}\,\varphi}{\gamma\varphi}\right). \qquad (1.13)$$

It follows from (1.13) that in strong fields ($\varphi > 1/n$) the resonant processes become quite effective.

By way of an example let us consider the loss of internal energy by homonuclear molecules in a magnetic field corresponding to resonant transitions between rotational levels with $l = 0$ and $l' = 2$. In place of (1.8) we have (Q—quadrupole moment, B—rotational constant, Z—partition function)

$$\langle\hat{V}_q(t)\,\hat{V}_q^*(0)\rangle = \frac{2n_0Q^2}{45\pi Z}\sum_{l=0}^{\infty}\sum_{|l-l'|=0,2}(2l+1)(2l'+1)\left|\begin{pmatrix} l & l' & 2\\ 0 & 0 & 0\end{pmatrix}\right|^2 \times\exp\left\{-\frac{B}{T}l(l+1)+it\omega_{ll'}\right\} = \frac{2n_0Q^2}{45\pi Z}[e^{-6itB}+e^{6B(it-1/T)}+\ldots], \qquad (1.14)$$

whence

$$\frac{1}{\tau_{\nu\nu'}} = \frac{16}{5}n_0 m\sqrt{\frac{2\pi m}{T}}Q^2T\left(\frac{B}{T}\right)^2 e^{-3B/T}\varphi e^{-n\varphi}\ln\left(\frac{8\,\mathrm{cth}\,\varphi}{\gamma\varphi}\right),\quad n\varphi = 3\frac{B}{T}. \qquad (1.15)$$

The contribution of the resonant term (1.15) can reach a value equal to the total loss of the internal energy of the homonuclear molecules in the absence of the magnetic field (see[1])

$$\frac{1}{\tau_0} = \frac{64}{15}n_0\mu\sqrt{\frac{2\pi\mu}{T}}Q^2B. \qquad (1.16)$$

Thus, in a hydrogen plasma, for the main resonance ($n = 1$) $1/\tau_{\nu\nu'}$ amounts to ~ 30% of $1/\tau_0$. We note, however, that in the case of H_2 molecules the value of the magnetic field corresponding to $n = 1$ is quite large ($H_{max} > 10^6$ Oe). For dipole molecules with small B it follows from the resonance condition $H = 2Bmc/n$ that $H_{max} > 10^4$ Oe, which apparently is accessible to observation. In this case, using the connection between the dipole correlator

$$\langle \hat{d}(t)\hat{d}^*(0)\rangle = \frac{\pi q^2}{2}\langle \hat{V}_q(t)\hat{V}_q{}^*(0)\rangle$$

and the imaginary part of the polarizability $\alpha''(\omega)$ (see, e.g.[7]) we obtain in place of (1.14)—(1.16) (d is the dipole moment)

$$\langle \hat{d}(t)\hat{d}^*(0)\rangle = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\omega\alpha''(\omega)e^{-i\omega t}}{1-e^{-\omega/T}}$$
$$= \frac{d^2}{3Z}\sum_{l=0}^{\infty} e^{-Bl(l+1)/T}[le^{2itBl}+(l+1)e^{-2itB(l+1)}], \qquad (1.17)$$

whence in the absence of a field

$$-\frac{\overline{d\hat{H}}}{dt} = 8n_0\sqrt{\frac{2\mu}{\pi\bar{T}}}\int_{-\infty}^{\infty}\frac{d\omega\,\omega\alpha''(\omega)}{1-e^{-\omega/T}}e^{-\omega/2\bar{T}}K_0\left(\frac{\omega}{2\bar{T}}\right),$$
$$\frac{\bar{T}}{\mu} = \frac{T}{M}+\frac{T_e}{m}, \qquad (1.18)$$

and further in analogy with (1.9)—(1.13).

Let us proceed to consider the contribution of inelastic collisions to the electric conductivity in a magnetic field. For a semiconductor plasma the oscillations of the transverse component of the electric conductivity tensor σ_{xx} in a magnetic field, as a consequence of inelastic scattering by phonons (the magnetophonon effect), was considered in[8,9]. The magnetokinetic effect of the resonant $\sigma_{ik}(H)$ dependence as a result of inelastic collisions of electrons with atoms and molecules in a gas plasma is described in analogy with (1.6)—(1.10). In the case of an isothermal plasma (the generalization to the many-temperature case $T_\| \neq T_\perp \neq T \neq T_1$ is obvious) we have for the transverse electric conductivity, in accordance with Kubo (X is the coordinate of the center of the Landau oscillator)

$$\sigma_{xx} = \frac{1}{2T}\int_{-\infty} dt\langle \dot{X}(t)\dot{X}(0)\rangle = \frac{\pi c^2}{TH^2}\int d^3q q_y{}^2$$
$$\times\int_{-\infty}^{\infty} dt\exp\left\{-\frac{q^2}{2M}(it+t^2T)\right\}\Phi_\|\Phi_\perp\langle \hat{V}_q(t)\hat{V}_q{}^*(0)\rangle. \qquad (1.19)$$

For elastic scattering (for simplicity we consider the case $|V_q|^2 = V$) at $\varphi \gg 1$ the contribution made to the electric conductivity by multi-quantum transitions between Landau levels is exponentially small and (see (1.10), $\mu \approx m$)

$$\sigma_{xx} = -\frac{8\pi a^2 Z_0}{m\gamma^2\omega^2}\frac{d}{db}g_0(a,b). \qquad (1.20)$$

We note that the logarithmic divergences characteristic of the scattering by immobile centers (see[8]) are missing from (1.19) and (1.20), since the thermal motion of the heavy particles is taken into account.

Calculation of the contribution of the inelastic collisions to the electric conductivity yields for

$$\sigma_{xx} = \sum_{\nu\nu'}(\sigma_{xx})_{\nu\nu'}$$

an expression analogous to (1.9). The resonant term is given by

$$(\sigma_{xx})_{\nu\nu'} = -\frac{32\pi a^2 Z_{\nu\nu'}}{m\gamma^2\omega^2}\exp\left\{-\frac{\varepsilon_\nu+\varepsilon_{\nu'}}{2T}\right\}\frac{d}{db}g_n(a,b),$$
$$Z_{\nu\nu'} = 16\pi n_0 m\sqrt{2\pi mT}\bar{V}_{\nu\nu'}. \qquad (1.21)$$

Resonances similar to (1.10)—(1.21) are possessed by the diffusion coefficient $D = \sigma_{xx}T/n_0$, the thermal conductivity, and a few other kinetic coefficients.

2. RESONANCE EFFECTS IN AN ELECTRIC FIELD

In the case of a constant electric field **E** we can disregard the field-intensity dependence of the rate constant of the inelastic collisions of the charged particles with the neutral ones $k_{\nu\nu'}(E)$, of the probability of the elastic collisions $W_{pp'}(E)$, and of the number of the collisions

$$Z(E) = \sum_{\nu\nu'}N_\nu k_{\nu\nu'}(E) = \int d^3p\,d^3p' f(p)W_{pp'}(E). \qquad (2.1)$$

Thus, for the number of collisions

$$Z(E) = n_0\int_{-\infty}^{\infty} dt\langle \hat{V}(t)\hat{V}^*(0)\rangle \qquad (2.1')$$

we obtain from (A.11)

$$Z(E) = \frac{n_0}{T_e}(2\pi\mu T_e)^{3/2}\bar{V}\int dx\psi(x)\int_{-\infty}^{\infty}\frac{dt}{\Theta^{3/2}}\exp\left\{-\frac{E^2}{E_0{}^2}\frac{(t^2+it)^2}{\Theta}\right\},$$
$$E_0 = \sqrt{\frac{m}{\mu}}\left(\frac{\bar{T}}{\mathrm{Ry}}\right)^{3/2}E_{at},\quad E_{at} = \frac{e}{a_0{}^2},\quad \Theta = it+t^2\frac{\bar{T}}{T_e}+2\mu T_e x. \qquad (2.2)$$

In particular, at $\psi(x) = \delta(x)$ and $T = T_e$ we have

$$Z(E) = \frac{Z_0}{2}x^2e^{x^2/2}\left[K_1\left(\frac{x^2}{2}\right)-K_0\left(\frac{x^2}{2}\right)\right]\to Z_0\left(1-x^2\ln\frac{2}{x}+\ldots\right),$$
$$x = E/2E_0 \to 0. \qquad (2.3)$$

It follows from (2.2) and (2.3) that Z(E), $k_{\nu\nu'}$, $W_{pp'}$, etc. begin to depend on E in rather strong fields (although much weaker than E_{at}—the factor $(\mathrm{Ry}/T)^{3/2}$ can reach $\sim 10^3$).

A strong field after Druyvestein (see, e.g.,[10]) is much weaker than the field E_0, so that one can neglect the direct influence of the constant electric field on the number of collisions, the rate constant, the collision integral, etc.

Let us see how the kinetic characteristics of the plasma depend under conditions of inelastic resonance $\omega_{\nu'\nu} = n\Omega$ on the alternating electric field. In an alternating external field we have for the inelastic energy losses

$$-\frac{\overline{d\hat{H}}}{dt} = n_0\frac{\partial}{\partial t}\iint_0^t dt_1\,dt_2\langle[\hat{H}\hat{V}(t_1)]\hat{V}^*(t_2)\rangle, \qquad (2.4)$$
$$\hat{V}(t_1) = \exp\left[i\int_0^{t_1}dt(\hat{K}_e+\hat{K}+\hat{H})\right]\hat{V}\exp\left[-i\int_0^{t_1}dt(\hat{K}_e+\hat{K}+\hat{H})\right];$$

in a homogeneous field $E(t) = E\cos\Omega t$ the operator of the kinetic energy of the electrons is

$$\hat{K}_e = \frac{1}{2m}\left(\hat{\mathbf{p}}+\frac{1}{c}\mathbf{A}\right)^2,\quad \mathbf{A} = -\frac{c}{\Omega}\mathbf{E}\sin\Omega t. \qquad (2.4')$$

The scattering probability is equal to

$$W_{pp'} = n_0\frac{\partial}{\partial t}\iint_0^t dt_1\,dt_2\Phi_0(q,t_1-t_2)\langle\hat{V}_q(t_1-t_2)\hat{V}_q{}^*(0)\rangle$$
$$\times\exp\left[i(t_1-t_2)\frac{p^2-p'^2}{2m}+\frac{i}{mc}\int_{t_1}^{t_2}dt'\mathbf{qA}(t')\right]. \qquad (2.5)$$

Averaging over the period of the oscillations $t_0 = 2\pi/\Omega$, we obtain here in place of (2.5)

$$W_{pp'} = n_0\int_{-\infty}^{\infty}dt\Phi_0(q,t)\langle\hat{V}q(t)\hat{V}_q{}^*(0)\rangle J_0\left(\frac{2\mathbf{Eq}}{m\Omega^2}\sin\frac{\Omega t}{2}\right)\exp\left\{it\frac{p^2-p'^2}{2m}\right\}. \qquad (2.5')$$

For the internal-energy losses averaged over the period we have accordingly

$$-\frac{\overline{d\hat{H}}}{dt}=-in_0\int d^3q\int_{-t_0}^{t_0}dt\,J_0\left(\frac{2\mathbf{Eq}}{m\Omega^2}\sin\frac{\Omega t}{2}\right)\exp\left\{-\frac{q^2}{2\mu}(it+t^2T)\right\}\times\frac{d}{dt}\langle\hat{V}_q(t)\hat{V}_q^*(0)\rangle; \quad (2.6)$$

in the derivation of (2.6) we have assumed for the electrons a local equilibrium Maxwellian distribution with a temperature T_e.

As $E\to 0$ and $t_0\to\infty$ we obtain from (2.5′) and (2.6) the general expressions for the collision integral and the energy loss (see[1]). As $T_e\to T$ we obtain for the relaxation time τ_E ($t\to t/T$, $q^2\to 2\mu Tq^2$, $\varphi_1=\Omega/2T$)

$$\frac{1}{\tau_E}=-n_0\mu\sqrt{2\mu T}\int d^3q\int_{-\pi/\varphi_1}^{\pi/\varphi_1}dt\,\Phi(\mathbf{q},t)\frac{d^2}{dt^2}\langle\hat{V}_q(t)\hat{V}_q^*(0)\rangle. \quad (2.7)$$

The correlation function in an alternating field, which describes the translational degrees of freedom, is equal to

$$\Phi(\mathbf{q},t)=e^{-q^2(it+t^2)}J_0\left(\frac{2\sqrt{2\mu T}}{m\Omega^2}\mathbf{Eq}\sin\varphi_1 t\right), \quad (2.7')$$

and the correlation functions that describes the internal degree of freedom is given by (1.8).

From (2.7) and (2.7′) it follows that ($f=it+t^2+2\mu Tx$)

$$\frac{1}{\tau_E}=4n_0\sqrt{T}(2\pi\mu)^{3/2}\varphi_1^2\sum_{\nu\nu'}\overline{V}_{\nu\nu'}\left(\frac{\omega_{\nu\nu'}}{\Omega}\right)^2\left(\exp\left[-\frac{\varepsilon_\nu+it\omega_{\nu'\nu}}{T}\right]+\exp\left[-\frac{\varepsilon_{\nu'}-it\omega_{\nu'\nu}}{T}\right]\right)\int dx\psi_{\nu\nu'}(x)\int_{-\pi/\varphi_1}^{\pi/\varphi_1}\frac{dt}{f^{3/2}}\exp\left\{-4\frac{A}{f}\sin^2\varphi_1 t\right\}\cdot I_0\left(4\frac{A}{f}\sin^2\varphi_1 t\right),\quad A=\frac{4\mu T}{m\Omega}\left(\frac{\mathrm{Ry}}{\Omega}\right)^2\frac{E^2}{E_{\mathrm{at}}^2}. \quad (2.8)$$

Using the expansion

$$\Phi(q,t)=e^{-q^2(it+t^2)}\sum_{n=-\infty}^{\infty}J_n^2\left(\frac{\sqrt{2\mu T}Eq}{m\Omega^2}\right)e^{in\Omega t/T}, \quad (2.7'')$$

we obtain for the resonant term ($\alpha=(n\Omega-\omega_{\nu'\nu})/T\to 0$) in the approximation $\psi_{\nu\nu'}(x)=\delta(x)$

$$\frac{1}{\tau_{\nu\nu'}}=\frac{2n_0}{T}(2\mu T)^{3/2}(n\varphi_1)^2\overline{V}_{\nu\nu'}\int d^3qJ_n^2\left(\frac{\sqrt{2\mu T}}{m\Omega^2}\mathbf{Eq}\right)\cdot\int_{-\pi/\varphi_1}^{\pi/\varphi_1}dt\,e^{-q^2(it+t^2)}\left(e^{it\alpha-\varepsilon_\nu/T}+e^{-it\alpha-\varepsilon_{\nu'}/T}\right). \quad (2.9)$$

After calculating approximately the internal integral in (2.9) in the case $\varphi_1\ll 1$ and confining ourselves in the expansion of (2.9) in powers of the external field to the first non-vanishing term, we ultimately obtain ($n>0$, $\alpha>0$)

$$\frac{1}{\tau_{\nu\nu'}}:\frac{1}{\tau_{\nu\nu'}^0}\approx\frac{A^nF(\alpha)\,\mathrm{ch}\,n\varphi_1}{2(2n+1)(n!)^2n\varphi_1K_1(n\varphi_1)}, \quad (2.10)$$

where the function $F(\alpha)=\alpha^{n+1}K_{n+1}(\alpha/2)$ decreases rapidly with increasing α. It follows from (2.10) that contribution of resonances above the first order can be neglected. Near the main resonance ($n=1$), τ_E can differ from τ noticeably also for not too large values of E. For example, for rotational losses $\Omega\sim 10^{-3}$Ry and $E\sim 10^4$ V/cm.

Analogous results can be obtained for the frequency dependence of the electric conductivity and other kinetic coefficients. The quantum-kinetic equation and the kinetic coefficients in strong fields E and H, for a system of electrons and phonons, were considered recently in[11-14] (see also[15]), but the resonant situation was not discussed.

In conclusion we note that in the present paper we did not take into account the interaction of the external field with neutral particles (or the electron spin). The interaction of atoms and molecules with strong electric and magnetic fields can be taken into account by replacing the correlation function of the neutrals by

$$\Phi(\mathbf{q},t)=\langle e^{it(\hat{H}+\hat{U})}e^{i\mathbf{qr}}e^{-it(\hat{H}+\hat{U})}e^{-i\mathbf{qr}}\rangle, \quad (2.11)$$

where the energy of interaction with the external field is

$$U=-\mu\mathbf{H}-\mathbf{dE}+\frac{1}{6}Q_{\alpha\beta}\frac{\partial^2\varphi}{\partial x_\alpha\partial x_\beta}+\ldots \quad (2.11')$$

Correlation functions of the type (2.11) and (2.11′) can be used to obtain the collision integral and the kinetic coefficients in molecular gases in an external field (the Senftleben effect—see, e.g.[16,17]).

APPENDIX

CALCULATION OF THE CORRELATION FUNCTIONS IN A STRONG FIELD

We change over from the correlation function (see[1])

$$\Phi(\mathbf{q},t)=\frac{1}{Z}\sum_{\nu\nu'}\exp\left\{-\frac{\varepsilon_\nu}{T}+it\omega_{\nu\nu'}\right\}|(e^{i\mathbf{qr}})_{\nu\nu'}|^2,\quad Z=\sum_\nu e^{-\varepsilon_\nu/T}, \quad (\mathrm{A}.1)$$

to the real correlation function

$$\Phi(\mathbf{q},\tau)=\frac{1}{Z}\int d^3r_0\,d^3r_1e^{i\mathbf{q}(\mathbf{r}_0-\mathbf{r}_1)}|G_\tau(\mathbf{r}_0,\mathbf{r}_1)|^2,\quad \tau=t-\frac{i}{2T}. \quad (\mathrm{A}.2)$$

As is well known (see, e.g.,[18]), the single-particle Green's function

$$G_t(\mathbf{r}_0,\mathbf{r}_1)=\sum_\nu e^{-it\varepsilon_\nu}\psi_\nu^*(\mathbf{r}_0)\psi_\nu(\mathbf{r}_1) \quad (\mathrm{A}.3)$$

for a charged particle in a homogeneous electromagnetic field (and also in the case of a quadratic potential field $\varphi(r)$ and in the WKB approximation) is proportional to e^{iS}, where the classical action function is

$$S[\mathbf{r}(0),\mathbf{r}(t)]=\int_0^t dt'\left(\frac{mv^2}{2}+\frac{e}{c}\mathbf{Av}-e\varphi\right). \quad (\mathrm{A}.4)$$

Thus, by calculating the classical function S and putting $\overline{S}=S(\mathbf{r}_0,\mathbf{r}_1,t-i/2T)$, we can obtain the correlator

$$\Phi(\mathbf{q},\tau)=C\int d^3r_0\,d^3r_1\,e^{i\mathbf{q}(\mathbf{r}_0-\mathbf{r}_1)-2\,\mathrm{Im}\,\overline{S}}, \quad (\mathrm{A}.5)$$

where the constant C is determined from relations $\Phi(\mathbf{q},0)=\Phi(0,\tau)=1$ which follow from (A.1). In particular, in a homogeneous external field $\overline{S}=S(\mathbf{r}_0-\mathbf{r}_1,\tau)$ and

$$\Phi(\mathbf{q},\tau)=C\int d^3r\,e^{i\mathbf{qr}-2\,\mathrm{Im}\,\overline{S}}. \quad (\mathrm{A}.5')$$

For example, in a homogeneous magnetic field $\mathbf{H}(0,0,H)$, solving the classical equations of motion with the time τ and calculating the action from (A.4), we find[1)]

$$\operatorname{Im}\bar{S} = \operatorname{Im}\left[\frac{mz^2}{2\tau} + \frac{m\omega}{4}(x^2+y^2)\operatorname{ctg}\frac{\omega\tau}{2}\right]. \quad (A.6)$$

Inasmuch as

$$\operatorname{Im}\operatorname{ctg}\left(\omega\frac{t-i/2T}{2}\right) = \frac{\operatorname{sh}(\omega/2T)}{\operatorname{ch}(\omega/2T)-\cos\omega t},$$

we obtain from (A.5) and (A.6)

$$\Phi(q,\tau) = \exp\left[-\frac{q_\parallel^2 T_\parallel}{2m}\left(t^2+\frac{1}{4T_\parallel^2}\right) - \frac{q_\perp^2}{2m\omega}\frac{\operatorname{ch}(\omega/2T_\perp)-\cos\omega t}{\operatorname{sh}(\omega/2T_\perp)}\right]. \quad (A.7)$$

For greater generality we have assumed in (A.7) that the system is nonisothermal, corresponding to making in (A.6) the substitution $\tau \to \tau_\parallel = t - i/2T_\parallel$ for the thermostat of the longitudinal degrees of freedom and $\tau \to \tau_\perp = t - i/2T_\perp$ for the thermostat of the transverse degrees of freedom. Putting in (A.7) respectively $t = t + i/2T_\parallel$ and $t + i/2T_\perp$, we ultimately obtain ($\varphi = \omega/2T_\perp$)

$$\Phi(q,t) = \Phi_\parallel\Phi_\perp,$$
$$\Phi_\perp = \exp\left\{-\frac{q_\perp^2}{2m\omega}[(1-\cos\omega t)\operatorname{cth}\varphi + i\sin\omega t]\right\}$$
$$= \exp\left\{-\frac{q_\perp^2}{2m\omega}\operatorname{cth}\varphi\right\}\sum_{n=-\infty}^{\infty} e^{2in\varphi t - n\varphi} I_n\left(\frac{q_\perp^2}{2m\omega\operatorname{sh}\varphi}\right),$$
$$\Phi_\parallel = \exp\left[-\frac{q_\parallel^2}{2m}(it+t^2)\right] \quad (A.8)$$

In a homogeneous alternating electric field $\mathbf{E}(t)$ we have

$$\operatorname{Im}\bar{S} = \operatorname{Im}\left[\frac{mr^2}{2\tau} - \frac{1}{\tau}\int_0^\tau d\tau'\tau'(\mathbf{rE})\right]. \quad (A.9)$$

In particular, in a constant electric field

$$\Phi_E(q,t) = \exp\left[-\frac{q^2}{2m}(it+t^2T) - \frac{t}{2mT}\mathbf{qE} + \frac{it^2}{2m}\mathbf{qE}\right]. \quad (A.10)$$

In the case $\mathbf{E} \parallel \mathbf{H}$, the transverse and longitudinal degrees of freedom can be regarded independently, so that

$$\Phi(q,t) = \Phi_\perp\Phi_E(q_\parallel, t). \quad (A.11)$$

Finally, in the case of crossed fields, choosing $\mathbf{E}(0, E_y, E_z)$ and $\mathbf{A} = (1/2)\mathbf{H}\times\mathbf{r}$, we obtain

$$\bar{S} = \frac{mz^2}{2\tau} + \frac{\tau}{2}(yE_y+zE_z) + \frac{m\omega}{4}(x^2+y^2)\operatorname{ctg}\frac{\omega\tau}{2} + \frac{x}{\omega}E_y\left(1-\frac{\omega\tau}{2}\operatorname{ctg}\frac{\omega\tau}{2}\right), \quad (A.12)$$

whence (for simplicity we have changed over to dimensionless variables $t \to t/T$, $q^2 \to q^2 2mT$, $E \to ET\sqrt{T/2m}$)

$$\Phi(q,t) = \exp\left[-(t^2+it)(q_z^2 - iq_zE_z) - \frac{\operatorname{ch}\varphi - \cos(2\varphi t + i\varphi)}{2\varphi\operatorname{sh}\varphi}\times(q_x^2+q_y^2-iq_yE_y) + i\frac{(2t+i)\operatorname{sh}\varphi + \sin(2\varphi t+i\varphi)}{2\varphi\operatorname{sh}\varphi}q_xE_y\right]. \quad (A.13)$$

We note that by determining with the aid of the previously obtained correlation functions the polarization operator $\Pi(\mathbf{q}, \omega)$ in the single-loop approximation, we can generalize to the case of a nonisothermal plasma the dispersion equation in an external field, we can investigate the deformation of a Debye medium in a strong external field, etc. (see[19-21]).

[1)]In the particular case $T_\parallel = T_\perp$, the correlation function in a homogeneous magnetic field (A.8) was obtained earlier from other considerations [8,19].

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