SPIKE STRUCTURE OF SOLID STATE LASER EMISSION

G. N. VINOKUROV, N. M. GALAKTIONOVA, V. F. EGOROVA,* A. A. MAK, B. M. SEDOV, and Ya. I. KHANIN

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Experimental study of the kinetics of solid-state laser generation spectra in continuous and pulsed modes shows that spiking is due to the instability of resonator parameters. With stabilized resonator and pumping parameters the mode spectrum narrows down to 10^{-9} Å. Theoretical interpretation is based on the concept of resonator Q modulation due to time instability of resonator parameters. Analysis and numerical computations show that ordinary experimental conditions lead to intense irregular emission spikes.

CURRENTLY the most difficult problem in the theory of solid-state lasers is the interpretation of undamped spiking. The widely used Statz-de Mars^[1] equations fail to account for this phenomenon. Generalizations of these equations that would take mode field structure into account^[2, 3] also yield no such solutions. It was shown in ^[4, 5] that generation becomes unsta-

It was shown in ^[4, 5] that generation becomes unstable leading to undamped spiking when the resonator contains a medium with saturable absorption exceeding a certain critical value. However one can hardly assume that undamped self-modulation observed in all active solid-state laser media without exception^[6] can always be attributed to this effect.

Another well-known mechanism that relates spikes to nonlinear mode interaction in the active medium also fails to solve this problem. Analysis of a two-mode model^[7,8] shows that undamped spiking can occur only if the frequency interval between the modes does not exceed the bandwidth of each mode. Although this condition can be satisfied only by modes with different transverse indices we know that undamped spikes are also observed in lasers operating in a single transverse mode.^[9,10]

Numerical computation of generation kinetics taking spatial field inhomogeneities and medium polarization into account is given in $[^{11}]$. However, as in the case with $[^{7, 8}]$, the resulting spiking character of emission is due to multimode operation. Certain nonlinear effects such as self-focusing (see $[^{12}]$ for example) can also cause emission spiking.

A complete system of single-mode laser equations that takes phase relations between the field and medium polarization into account allows for periodic solutions^[13-15]. However, as is shown in ^[15], the conditions necessary for these solutions are not satisfied. It is shown in ^[16] that noise in a single-mode model while disrupting the regularity of the process causes faster damping of transient oscillations and faster achievement of steadystate operation.

All of the above theoretical papers presented laser models with constant parameters. Generally speaking such an idealization is not sufficiently justified by the experimental conditions. The overwhelming majority of experiments was performed with pulse-pumped lasers whose parameters varied in the generation process. Absorption of pump light is accompanied by heating (which is inhomogeneous as a rule) of the active medium

*Deceased.

causing a shift of the luminescence band, change in the index of refraction, and consequently deformation of the resonator. The resonator elements usually vibrate owing to external and internal sources (lamp flash and cooling of the active element that as a rule entails turbulent flow of the cooling agent).

Models of a laser with periodically varying Q of the resonator are considered in $^{17-19}$. When the modulation frequency is close to the natural frequency of the damped relaxation oscillations a very shallow Q modulation is sufficient to produce undamped spikes of large amplitude. What remains is the problem of the mechanism of periodic Q modulation under ordinary experimental conditions corresponding to the free-running mode when the resonator does not contain special modulating elements. This problem is considered in this paper.

It has been suggested^[20, 21] that the instability of resonator parameters can be the cause of spiking of laser emission.

The theoretical part of this paper shows that parameter variation under real experimental conditions is equivalent to a quasi-periodic modulation of the mode Q factors and that even when the Q modulation is periodic at frequencies of the order of the relaxation oscillation frequency there are long-term transients during which single-mode laser emission has irregular spiking. The experimental part shows, on the one hand, that pulsepumped individual modes generate spikes in all kinetic regimes of solid-state lasers without exception and, on the other hand, continuous pumping results in spikeless generation in single-mode and multi-mode regimes when pumping and especially resonator parameters are carefully stabilized.

1. THEORY

A Doppler shift of emission frequency occurs upon reflection from moving resonator elements and upon passage through a medium with time-dependent index of refraction. For emission at the frequency $\nu = 4$ $\times 10^{14}$ Hz and for a reflecting surface velocity of v = 10 cm/sec, $\Delta \nu = 2 \nu c/c = 0.3$ MHz.

A change in the medium temperature causes a frequency shift of the transmitted light (neglecting thermal expansion) amounting to

$$\Delta v_T = \frac{v \, n l_c}{c} \frac{dn}{dt} = v n \frac{l_c}{c} \frac{dn}{dT} \frac{dT}{dt}, \qquad (1)$$



FIG. 1. Diagram of a laser with flat mirrors. $a-l_0$ -optical length of the crystal; R'-Fresnel coefficient of reflection (amplitude) from crystal end face; R-amplitude coefficient of reflection of the mirrors; $1 \gtrsim R \gg R'$; b-effective shape of resonator with symmetric distortion in the form of "steps".

where l_c is layer thickness, n is the index of refraction, and T is temperature. For the numerical computation of $\Delta \nu_T$ in the case of ruby we used the data from $^{[22]}$ coinciding with our data with an accuracy up to the factor of 2-2.5: dn/dT = $1.5 \times 10^{-5} \text{ deg}^{-1}$, dT/dt = $2 \times 10^3 \text{ deg/sec}$. Setting $l_c = 10 \text{ cm}$ and n = 1.76 we obtain $\Delta \nu_T = 10 \text{ kHz}$.

We note that the change in the refraction index of the active medium can be caused by effects associated with the change in working level populations for example (see $\begin{bmatrix} 23 & 3 \end{bmatrix}$).

In adiabatic approximation the change in resonator length merely causes a shift of its natural frequencies. By virtue of the Doppler shift at the moving mirror, the emission of each mode follows the varying natural frequency. Since in the majority of solid-state lasers the luminescence linewidth is large, the shift of the natural mode frequency relative to the luminescence line center can be neglected. In this case the changing parameters of the resonator can affect the generation kinetics through a change in the Q factor and mode shape. We consider only the effect of Q variation.

We consider a laser with plane mirrors (Fig. 1a). Neglecting the transverse field structure we obtain an equation to determine natural frequencies and mode losses for $R \gg R'$:

$$R^{2} \exp\{2ik(2l+l_{0})\}\left[1+2R'(R^{-1}e^{-2ikl}+Re^{2ikl})\cos 2k\Delta l\right]=1, \quad (2)$$

where $k = \omega/c$ (the remaining notation follows Fig. 1a). The Q of the n-th mode is $Q_n = \operatorname{Re} k_n/2 \operatorname{Im} k_n$.

We can readily conclude from (2) that with a change of any one of the values l_0 , l, or Δl the relative oscillations of Q are confined to the range $1 \pm R'$. Usually $R' > 10^{-1}-10^{-2}$ and the Q oscillations are quite noticeable. The variation of Δl corresponds to translation of the crystal between the mirrors. The fundamental frequency of Q modulation is then equal to the Doppler shift upon reflection from the moving end faces as computed above.

For R' = 0 ("Brewster" end faces of the crystal or mirrors on the end faces) Q can change only because of a change in the effective shape of the resonator (from nonuniform heating of active medium, mirror skew, etc.). A monotonic variation of the effective curvature of mirrors in "stable" configuration resonators by itself does not cause oscillations of the mode Q factors. In this case the mirror surfaces continue to coincide with the equiphase surfaces of the modes. In practice, however, there is no such coincidence because of alignment errors as well as because of the asphericity caused by the thermal lens generation. Therefore monotonic variation of resonator shape can be accompanied by Q oscillations. Their magnitude can be evaluated by the following example.

Let the mirror deviation from sphericity have the form of symmetrical steps (see Fig. 1b) whose height h varies in time. If the integral equation of an unperturbed two-dimensional resonator has the form

$$R_{\gamma_0}V_0(x_2) = \int K_0(x_1, x_2) V_0(x_1) dx_1$$
(3)

(R is the amplitude coefficient of reflection of the mirror, $V_0(x)$ is field distribution over the mirror, $R\gamma_0$ is the eigenvalue, and $K_0(x_1, x_2)$ is determined by the resonator shape) the perturbation corresponding to the stepwise distortions is given in the form

$$\delta K(x_1, x_2) = K_0(x_1, x_2) \{ \exp[2ik(h(x_1, t) + h(x_2, t))] - 1 \}, \quad (4)$$

where

$$h(x,t) = \begin{cases} h(t) \text{ for } X_0 < |x| < X_0 + \Delta x \\ 0 \text{ for } |x| < X_0 \text{ II} |x| > X_0 + \Delta x. \end{cases}$$
(4a)

Considering the width Δx of a step a small parameter we can readily obtain a correction to the eigenvalue R γ_0 in adiabatic approximation of the first order of $\Delta x/a$:

$$R\gamma^{(1)} = 4R(\operatorname{Re}\gamma_0) \left[\exp\left(2ikh\right) - 1 \right] \int_{x_0}^{x_0 + \Delta x} V_0(x) V_0(x) dx$$

$$\approx 2R\gamma_0 \left(\Delta x/a \right) |V_0(X_0)|^2 \left[\exp\left(2ikh(t)\right) - 1 \right].$$
(5)

The mode Q is related to the corresponding eigenvalue of integral equation (3) by

$$Q^{-1} \coloneqq \pi (1 - |\gamma R|^2) / kL. \tag{6}$$

If we are interested in only one mode with least losses the kernel of the integral equation can always be predetermined by multiplying by a constant so that the eigenvalue under consideration of the unperturbed resonator is real. Denoting the Q of the unperturbed resonator by Q_0 we rewrite (6) in the form $Q^{-1} = Q_0^{-1}(1-\delta)$ where

$$\delta = \frac{R^2}{1 - \gamma_0^2 R^2} [2\gamma_0 \operatorname{Re} \gamma^{(i)} + |\gamma^{(i)}|^2].$$
(7)

After substituting (6) into (7) with an accuracy to terms $(\Delta x/a)^2$ we obtain

$$\delta = 4(\Delta x / a) |V_0(X_0)|^2 [\cos 2kh(t) - 1] R^2 \gamma_0^2 / (1 - R^2 \gamma_0^2).$$
 (8)

Setting $(\Delta x/a) |V_0(X_0)|^2 = 10^{-3}$ and $\gamma_0^2 R^2 = 0.8$, we find that monotonic variation of h(t) causes oscillations of resonator losses within the limits of $\approx 3\%$ at a Doppler shift frequency $(k/\pi)dh/dt$.

When the resonator aperture is small $\Delta x/a$ is comparable to unity and (5) and (8) are inapplicable. In this case however the smooth distortions can be regarded as small phase perturbations,^[24] i.e., we can assume that

$$\delta K(x_1, x_2) = 2ik[h(x_1, t) + h(x_2, t)]K_0(x_1, x_2), \qquad (9)$$

where $\varphi = \max \{2k[h(x_1, t) + h(x_2, t)]\} \ll 1$. Taking φ as a small parameter we can easily obtain a correction to the eigenvalue in the first order of perturbation theory

$$\frac{1}{i}\gamma^{(1)} = 2\varphi\gamma_0(2a)^{-1}\int_{-a}^a V_0^*(x) V_0(x)h(x,t)\varphi^{-1} dx \leq 2\varphi\gamma_0.$$
(10)

Substitution of (10) into (7) yields

$$\delta \leq 4\varphi^2 R^2 \gamma_0^2 / (1 - \gamma_0^2 R^2). \tag{11}$$

We note that small phase distortions cause changes in Q that are proportional only to the second power of the small parameter. As aperture decreases so does the coefficient $R^2 \gamma_0^2/(1 - R^2 \gamma_0^2)$. In the case of small oscillatory phase distortions, changes in Q should be expected only if the perturbation change is of a similar nature, i.e., if the mirrors vibrate. We can of course assume that such perturbations have audio frequencies of the order of several kHz.

Thus pulsed lasers can manifest noticeable resonator Q modulation due to inhomogeneous heating rate of the active medium and vibration. The single-mode generator can in this case be described by abbreviated equations:

$$\dot{x} = 0.5Gx[m-1-\delta_0 \exp(i\Omega \tau)], \quad \dot{m} = \alpha - m(|x|^2 + 1),$$
 (12)

where dimensionless quantities are introduced as follows: $\tau = t/T_1$; $G = 2\omega_t T_1/Q_0$, $\Omega = 2\pi \Delta \nu T_1$, m = $T_2 d_{12}^2 Q_0 (N_2 - N_1)/2\hbar$ is the ratio of population difference to the threshold value; α is the ratio of pump level to its threshold value; $x = E d_{12} \sqrt{T_1} T_2 / \hbar$ is the dimensionless complex structure of the field, T_1 and T_2 are relaxation times of the working transition; ω_t is its frequency, d_{12} is the ratio of loss oscillation amplitude to its stationary value. For the sake of simplicity it is assumed in (12) that losses vary harmonically. For $\delta_0 = 0$ (12) converts into ordinary rate equations.

Since the first equation in (12) contains explicit time the system has no stationary solutions. We can naturally assume that as $\delta_0 \rightarrow 0$ the solutions can approach the stationary solutions of an unperturbed system as closely as desired:

$$m_0 = 1, |x_0|^2 = \alpha - 1.$$
 (13)

Regarding loss modulation as a perturbation we seek a solution to (12) in the form

$$x = x_0 + x_1 e^{i\alpha\tau}, \quad m = m_0 + m_1 e^{i\alpha\tau}$$
 (14)

assuming that $|\mathbf{x}_1| \ll \mathbf{x}_0$, $|\mathbf{m}_1| \ll \mathbf{m}_0$.

Substituting (14) into (12) and neglecting nonlinear combinations of small quantities x_1 and m_1 , we find (see also ^[17])

$$x_{1} = -i \frac{G\delta_{0}x_{0}}{2} \frac{\Omega - i\alpha}{\Omega^{2} - G(\alpha - 1) - i\alpha\Omega}, \quad m_{1} = \frac{G\delta_{0}(\alpha - 1)}{\Omega^{2} - G(\alpha - 1) - i\alpha\Omega}.$$
(15)

Maximum deviations from stationary solution are reached when $\Omega = \Omega_0 = \sqrt{G(\alpha - 1)}$, i.e., when Ω coincides with the natural frequency of relaxation oscillations of the system about the equilibrium point. In this approach a perturbation is considered small only if the oscillation amplitudes determined by (15) are small. In numerical computation we set the characteristic value of the parameters $G = 10^5$, $\alpha = 10$, and $\Omega_0 = 10^3$, and obobtain the applicability conditions for (15):

$$\delta_0 \ll 2 \cdot 10^{-4} \text{ for } \Omega = \Omega_0,$$

$$\delta_0 \ll 0.2 \text{ for } \Omega = 0.1\Omega_0 \text{ and } \Omega = 10\Omega_0.$$
(16)

In the case of strong perturbations that fail to satisfy the condition of applicability of linear approximation (16) we solved (12) with a BÉSM-4 computer. Figure 2 shows the computed amplitude functions. As initial conditions we took equilibrium values of amplitude and in-



FIG. 2. Computed time dependencies of single-mode laser emission amplitude. G = 6×10^5 , T₁ = 1.5×10^{-3} sec, $\alpha = 2$ for a-e; for a, b, c, $\Delta \nu = 0.04$ MHz and $\delta_0 = 10^{-3}$, 2×10^{-3} , and 4×10^{-3} respectively; for d-f $\delta_0 = 4 \times 10^{-3}$ and $\Delta \nu = 2 \times 10^{-2}$, 10^{-2} , and 0.67 MHz respectively; g- $\alpha = 1.1$, $\Delta \nu = 2.4$ MHz, $\delta_0 = 2 \times 10^{-2}$.

version in the absence of perturbation. The perturbation phase was selected so as to avoid a jump in the Q factor when the perturbation is switched on. The computation was made with parameters typical of ruby lasers: $G = 6 \times 10^5$, $T_1 = 1.5 \times 10^{-3}$ sec. For $\alpha = 2$ the natural frequency of small relaxation oscillations is ν_0 = 0.08 MHz.

Figures 2 a-c show a sharp dependence of modulation depth on the quantity δ_0 . It is a typical feature that the emission over a large time interval has the form of random spikes that are not synchronized by periodic perturbation. In Fig. 2c the intensity between the spikes falls below the spontaneous noise level $(|\mathbf{x}|^2 < 10^{-8})$.

The time behavior of emission as a function of perturbation frequency is shown in Figs. 2 c-g. When the perturbation frequency deviates considerably from ν_0 the solutions are weakly modulated and have a fairly regular structure. In Figs. 2 f, g the perturbation frequency carrier is modulated by slower relaxation oscillations. The case of low-frequency modulation of Q is shown in Fig. 2e, where we see a converse effect of relaxation oscillations about a slowly changing equilibrium point. The curve in Fig. 2g corresponds to the case of partially transparent end face crystal in the resonator moving at 80 cm/sec. Modulation of such an emission was observed in ^[25].

Figures 2 c, d correspond to the case when the perturbation magnitude exceeds a frequency-dependent critical value^[18] necessary to excite a solution with deep modulation. Figure 2d shows a clear picture of the "swinging" process. It is typical that the pulse regularity in the clusters deteriorates as the swings increase until the clusters split into individual spikes.





2. EXPERIMENTS

We studied emission spectrum kinetics of pulsed lasers (ruby, neodymium glass, CaF₂:Sm²⁺, CaWO₄:Nd³⁺, YAG:Nd³⁺) and cw lasers (YAG:Nd³⁺, neodymium glass) subject to wide-range variation of temperature and pump and resonator parameters (the ratio of resonator length L to mirror curvature radius r varied from 0 to 2). Pumping was usually accomplished with straight lamps in elliptic or cylindrical reflectors (the ratio of maximum to minimum pump intensity over the cross section of the active element was at least 2, see [6] for example). In addition to oscillographic observations we studied the spectral composition of the emission (Fabry-Perot etalon whose resolution was not less than $\frac{1}{150}$ of an order, i.e., with maximum baselines reaching 10^{-4} Å). Young's interference pattern, and near and far fields of emission. Time scan in all cases was obtained with an SFR camera having a resolution of up to 2 $\times 10^{-8}$ sec. Furthermore, a frame-by-frame photography was used to investigate the spatial structure. In cw operation we studied beat spectra using an FÉU-28 photomultiplier and spectrum analyzers of the ASChKh (50 Hz-20 kHz), S4-8 (20 kHz-30 MHz), and S3-5 (30-250 MHz) types.

Basic Results. Pulsed Operation

Depending upon conditions, we observed all the known kinetic generation regimes: random, quasistationary, and regular spikes. The modes pulsated in time in all the observed regimes.

1. In a flat resonator, the quasistationary regime in

which integrated emission is smooth occurred at a temperature of the active medium lower than T ~ 80-100 °K (for CaF₂:Sm²⁺ $\lesssim 35$ °K); in a spherical resonator, at L/r = 1-2, although the "smoothest" emission pulse occurs for L/r = 1.9-1.98.¹⁾ Typical of a quasistationary regime at low temperatures is the large ratio of inhomogeneous broadening of the luminescence line to the homogeneous broadening; for the spherical resonator typical is excitation of modes with transverse indices m, n $\gtrsim 200-250$. A limitation of the resonator aperture that suppresses high-order modes results in the transition of the quasistationary regime into a random spiking regime.

Investigation of the kinetics and spatial coherence of emission showed that in the quasistationary regime individual modes are pulsing randomly in time and the modes replace each other (Fig. 3). Since many modes are excited simultaneously in this regime it is not possible to detect the structure in the near and far fields, and the degree of spatial coherence is low.

In the random spiking regime with multimode generation mode exchange occurs continuously from spike to spike and can be frequently observed in the course of a single spike. In each spike, except for the first, a small number of modes are excited and the spatial coherence of emission is high in each spike with the exception of

¹⁾In the neodymim glass laser to obtain a "smooth" pulse it was necessary either to cut off the ultraviolet pumping radiation or to use glass with enhanced Nd₂O₃ or cerium content (>10%), or glass containing Cr^{3+} ions in addition to Nd³⁺.



FIG. 4. a-Ruby laser emission, fragment of spectrum, $T = 300^{\circ}$ K, L = 175 cm, $l_c = 16$ cm, $\tau_1 = 0.2$, t = 10 cm. Fig. b shows enlarged section of spectrum in Fig. a (marked by arrows).

cases in which mode exchange takes place during the spike.

In the regular pulsation regime a very large number of modes are excited (the spectrum width in a ruby laser is ~ 0.6 Å) and individual modes cannot be resolved. Spatial coherence is close to zero.

2. In the irregular pulsation regime, in cases when more than one mode is excited in individual spikes, and also during quasistationary generation of ruby and $CaF_2:Sm^{2+}$ lasers, a high-frequency modulation of mode emission is observed within the limits of a spike (Fig. 4), provided equipment with high spectral and time resolution is available. Modulation frequencies of 3-60 MHz were observed experimentally and for the frequencies of $\gtrsim 20$ MHz the spectrograms always showed resolved modes whose frequency difference was close to the modulation frequency (see Fig. 4). The spectrogram in Fig. 4 shows that frequency modulation of mode emission accompanied the amplitude modulation.

3. With nonuniform pumping the selection of axial and transverse modes fails to eliminate emission spikes and merely causes some regularization of spikes. In particular a single TEM_{00} or TEM_{01} mode generation in flat resonator ruby, neodymium glass, and YAG:Nd³⁺ lasers also has a spiking nature; the suppression of all but one axial modes does not change the situation and merely reduces the number of spikes. At the same time in a neodymium glass laser, when pumping uniformity is improved, resonator length increased, and its diameter and losses decreased, the depth of modulation of the emission by spikes is significantly reduced and under certain conditions the emission becomes almost smooth (Fig. 5). In a ruby generator this effect is much less pronounced which is apparently due to the low quality of the crystals.

Continuous Generation

Pumping was performed with helical incandescent iodine lamps, and xenon and krypton arc lamps; the reflector was a silvered cylinder 40 mm in diameter. Threshold could be exceeded by a factor of 3 or 4. Active elements 3 mm in diameter were 30-50 mm long and were cooled with tap water. To secure single-mode generation of YAG:Nd³⁺ the length of the flat mirror (reflection coefficients of 100 and 98%) resonator was 25 cm so that the TEM₀₀ mode was generated with the ~3 mm diameter crystal. A single axial mode was separated by a plane parallel transparent glass plate 2.5 mm thick placed in the resonator near and at the angle of 1-2° to the semitransparent mirror.

Without special measures to stabilize the resonator and pump parameters (especially when the flow of the cooling liquid is turbulent) the YAG:Nd³⁺ and neodymium glass generation, whether in multimode or singlemode (YAG:Nd³⁺) operation, was always of a spiking irregular nature with the spike length depending on pump intensity and ranging from a few to $20-30 \ \mu$ sec.

In case of the YAG: Nd³⁺ laser measures were taken to eliminate all the possible causes of vibration and instability in the elements of the resonator and pump.^[26] Oscillations of pump intensity with dc-powered iodine lamp and electrical filtration were less than 0.5%. The active element with the pump system and the resonator elements were mounted on the same foundation fixed to the optical bench. The bench rail was mounted on a massive steel plate. Shock absorbers (sections of vacuum rubber tubes) were inserted between the rail and the plate and between the plate and its base. The shock absorbing system suppressed external vibrations whose frequencies exceeded a few Hz. The system design and water flow rate assured a laminar flow for crystal cooling (2 mm gap, flow rate of 25-50 cm/sec, Reynolds number ≤ 2000).

Oscilloscopic observations and the study of beat spectra showed that in both the single-mode and multimode operations a stationary spikeless generation took place (see also ^[20, 21]). In the case of a single-mode operation the generation spectrum width measured by recording "null" beats on a spectrum analyzer did not exceed



FIG. 5. Neodymium glass laser emission oscillogram (L = 200 cm, $\tau_1 = 0.1$, $l_c = 13$ cm, T = 300° K). a, b-elliptical cylinder reflector, c-hollow coaxial lamp with diffused reflector, a-d = 0.8 cm (multimode regime), b-d = 0.3 cm (single-mode regime).

50 Hz (10^{-9} Å) which corresponded to the instrument resolution of the ASChKh-1 spectrum analyzer. At the same time beat spectrum monitoring within the limits from 50 Hz to 250 MHz showed the absence of beats. However, oscilloscopic traces of laser emission sometimes showed a low-frequency emission modulation close to the sinusoidal that was apparently due to weak low-frequency perturbations of the resonator. In particular acoustic oscillations caused laser emission modulation with perturbation frequency (see also ^[27]). It should be noted that in stationary generation mode selection is significantly more stable than in spiking generation. Thus for example the above axial mode selector provided for a reliable selection of a single axial mode in stationary regime; under the same conditions but in a spiking regime a single-mode generation is unstable accompanied by drifts causing the excitation of two or three modes.

3. DISCUSSION OF RESULTS

The results of the above experiments and computations lead to the conclusion that instability of generator parameters is one of the basic causes of emission spiking and apparently the only cause in the case of singlemode generation. The analysis of a large number of published experimental papers on generation kinetics is fairly difficult to accomplish from this point of view because parameter stability was not monitored in the majority of these papers. Fairly interesting results were obtained in a series of papers^[28-30] on generation kinetics of $CaF_2:Dy^{2^+}$ crystals at high pumping levels. Although the results of this work cannot be interpreted adequately from the viewpoint discussed here, some of these results can be explained.

If we assume that acoustic crystal oscillations causing mirror vibrations are the source of mode Q oscillations in these experiments, then according to (11) the magnitude of relative Q modulation drops with decreasing mirror size. This can explain the fact that multimode spikeless generation was observed in ^[30] only for small mirror sizes and short crystals (vibration is greater in long crystals). The fact that such generation occurred only at sufficiently high pump levels can be attributed to the increased frequency of relaxation oscillations pulling it away from the perturbation frequency band and the increasing critical perturbation value^[18] necessary to excite spikes, on the one hand, and to the smoothing of the total intensity in the generation of a large number of modes, on the other.

We note that the viewpoint presented here supplies a convincing reason for the absence of spiking in the majority of gas lasers: the characteristic frequencies of perturbations in this case are much lower than the frequency of quickly damped (due to the relative smallness of G) relaxation oscillations and perturbations merely cause corresponding smooth variations of emission intensity (see also [16, 19]).

The above experiments show that individual modes in pulsed solid-state lasers generate spikes in all known kinetic regimes including the quasistationary regime.

The fact that the stabilization of laser parameters, mainly resonator parameters, led to multimode spikeless regime permits us to draw the conclusion that it is precisely the instability of the effective form of the resonator that is the cause of spiking in pulsed generation in the random pulsation regime. The laws of mode exchange observed in this regime can be attributed in equal measure to the nonstationary resonator parameters during generation and to spatial inhomogeneity of the medium gain.

The interaction of modes close in frequency considered in $[^{7,8},^{11}]$ can result, according to our experiments, in a high-frequency modulation of mode intensity within the limits of a spike, accompanied by a frequency shift of interacting modes. As we see from Fig. 4, for example, high-frequency modulation can be held responsible only for fine details of the total generation process.

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