

SOUND WAVE DISPERSION IN METALS LOCATED IN AN INCLINED MAGNETIC FIELD

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The angular dependence of the sound velocity  $s$  in metals located in an inclined magnetic field  $H$  is studied theoretically and experimentally. It is shown that a deviation effect occurs for  $\omega\tau_0 > 1$  ( $\omega$  is the sound frequency,  $\tau_0$  the relaxation time), i.e., the velocity  $s$  increases sharply near the angle  $\varphi = \varphi_{cr} = \arcsin(s/\bar{v}_{x\max})$  ( $\bar{v}_{z\max}$  is the maximal electron drift velocity along the vector  $H$ ,  $\varphi$  is the angle of deviation of the vector  $H$  from the direction  $k \cdot H = 0$ ,  $k$  is the sound wave vector). For  $\omega\tau_0 \lesssim 1$ , the line is characterized by a gradual decrease with increase in the angle from the initial value  $\varphi = 0$ . This effect affords a method for the determination of the velocities  $v_{z\max}$ , the deformation potential constants  $\Lambda_{ik}$  and the relaxation time  $\tau_0$  for a group of electrons in the vicinity of the reference points on the Fermi surface. The deformation potential tensor components are calculated on the basis of measurements of the sound velocity  $s$  at the maximum along the trigonal and bisector axes  $|\Lambda_{33}| = (1.9 \pm 0.2)$  eV and  $|\Lambda_{22}| = (3.7 \pm 0.2)$  eV.

THE presence of a magnetic field  $H$  has an important effect on the character of the interaction of sound waves with the conduction electrons in metals. In addition to the magneto-acoustic effects associated with absorption, the features of the interaction, as follows from dispersion relations of the Kramers-Kronig type, should produce a change in the sound frequency. Thus, oscillations of the phase velocity have been observed in strong magnetic fields under conditions in which gigantic quantum oscillations of the absorption are recorded.<sup>[1,2]</sup>

We have previously considered the deviation effect—the sharp increase in the sound absorption in metals upon change in the angle of inclination of the vector  $H$  relative to the direction of propagation of the sound.<sup>[3]</sup> The present work is devoted to a study of the dispersion of the sound velocity in inclined fields  $H$ . Although the deviation effect for the velocity was predicted in 1963,<sup>[4]</sup> until recently there have been no theoretical or experimental researches devoted to this problem. Communications have now appeared on its observation in Sb<sup>[5]</sup> and Ga<sup>[6]</sup>. In this connection, we thought it of interest to carry out a theoretical calculation of the angular dependence of the sound velocity and compare the results with the experimental data for Bi.

1. THEORY

Effective interaction of electrons with sound is achieved for their motion in phase with the wave,  $k \cdot v = \omega$  ( $k$ ,  $\omega$  are the wave vector and sound frequency,  $v$  the velocity of the electron). In strong magnetic fields, for which the inequality

$$kR \ll 1, \quad \Omega \gg \omega, \quad \nu \tag{1}$$

holds ( $\Omega$  and  $R$  are the cyclotron frequency and the radius of the electronic orbit;  $\nu = \tau^{-1}$ , is the relaxation time), this phase relation has the form

$$k\bar{v}_H \sin \varphi = \omega \tag{2}$$

because of averaging over the period of rotation ( $\bar{v}_H$  is the mean drift velocity of the electron along the

vector  $H$ ;  $\frac{1}{2}\pi - \varphi$  is the angle between the vectors  $k$  and  $H$ ). Since the electronic velocity  $v$  is much greater than the sound velocity in metals  $s$ , then, for low frequencies,

$$\omega < \nu \tag{3a}$$

condition (2) is satisfied for angles  $\varphi \approx 0$ . For high frequencies,

$$\omega > \nu \tag{3b}$$

the relation (2) is satisfied near the small angles  $\varphi = \varphi_{cr}$ :

$$\sin \varphi_{cr} = s / \bar{v}_{H\max} \tag{4}$$

consequently, when  $H$  and  $k$  are not mutually perpendicular, a decrease in the sound absorption and the sound velocity should be observed in case (3a). For high frequencies (3b), the picture is entirely different. For inclination of the vector  $H$  at small angles  $\varphi = \varphi_r$ , a sharp rise takes place in the absorption and the velocity in comparison with their values at  $\varphi = 0$ . Upon further increase in the angle of inclination  $\varphi$ , the values of the absorption and velocity decrease.

The dispersion equation describing the propagation of sound waves in metals should be introduced from the equations of lattice vibrations:

$$\rho u_i = \lambda_{ik} \frac{\partial^2 u_m}{\partial x_k \partial x_i} + f_i \tag{5}$$

$\rho$  is the density of the metal,  $\hat{\lambda}$  the tensor of the elastic moduli,  $u = u_0 \exp[ik \cdot r - i\omega't]$  the displacement vector,  $\omega'$  the excited sound frequency, and  $f_i$  the volume force density acting on the lattice from the electrons. The expression for  $f_i$  is obtained from Maxwell's equations and the kinetic equation for the electron distribution function in<sup>[7,8]</sup>. In<sup>[3]</sup>, it was shown that the deformation mechanism of interaction of the electrons with the sound plays a fundamental role in the deviation effect, and the contribution of variable electric fields does not have to be taken into account. Therefore, only the component

$$\frac{\partial}{\partial x_k} \int d^3p \Lambda_{ik}(p) F;$$

need be kept in the expression for the force (see<sup>[7]</sup>);  $\Lambda_{ik}(\mathbf{p})$  is the symmetric deformation potential tensor, which vanishes in averaging over the Fermi surface;  $F$  is the electron distribution function, which takes into account the deformation mechanism only. After substitution of the distribution function, Eq. (5) takes the form

$$\rho \ddot{u}_i = \lambda_{\alpha i m} \frac{\partial^2 u_m}{\partial x_k \partial x_l} + \frac{\partial}{\partial x_m} \int_{\epsilon_F} \frac{m d p_H}{\Omega} \times \int_0^{2\pi} dt \Lambda_{ik}(\mathbf{p}) \int_{-\infty}^t dt_1 \Lambda_{im}(\mathbf{p}) \dot{u}_{im} \exp \left[ \int_t^{t_1} \frac{v - i\omega' + ikv}{\Omega} dt_2 \right]. \quad (6)$$

Here  $\epsilon_F$  is the Fermi energy,  $t$  the dimensionless time of motion of the electrons about an orbit. We introduce the  $z$  axis, which is directed along the vector  $\mathbf{H}$ , the  $\zeta$  axis directed along the wave vector  $\mathbf{k}$ . The angle between them is  $\frac{1}{2}\pi - \varphi$ . It is easy to obtain the dispersion equation for longitudinal waves from the expression (6):

$$\omega'^2 + i\omega'\Phi(\omega') - \lambda_{\zeta\zeta\zeta} k^2 / \rho = 0, \quad (7)$$

$$\Phi(\omega', \varphi) = \frac{k^2}{\rho (2\pi\hbar)^3} \sum_{\alpha} \int_{\epsilon_F} \frac{m d p_z}{\Omega} \int_0^{2\pi} dt \Lambda_{\zeta\zeta}(\mathbf{p}, t) \times \int_{-\infty}^t dt_1 \Lambda_{\zeta\zeta}(\mathbf{p}, t_1) \exp \left[ \int_t^{t_1} \frac{v - i\omega' + ikv}{\Omega} dt_2 \right]; \quad (8)$$

$\alpha$  is the number of the group of carriers in the case of a multiconnected Fermi surface.

The absorption due to the electrons, as well as the velocity dispersion, is a relatively small effect. Therefore we can use the method of successive approximations in the solution of Eq. (7). As a result, we get

$$\omega' = \omega - i\Phi(\omega), \quad (9)$$

$\omega$  is the unperturbed frequency of lattice vibrations. The real part of the function  $\Phi$  determines the sound attenuation in the metal and the imaginary the change in frequency.

The calculations of the integrals over  $t$  and  $t_1$  are elementary:

$$\Phi = \frac{k^2}{\rho (2\pi\hbar)^3} \sum_{\alpha} \int_{p_z \min}^{p_z \max} \frac{\bar{\Lambda}_{\zeta\zeta}^2(\mathbf{p}_z) m d p_z}{v - i\omega + ik\bar{v}_z \sin \varphi}. \quad (10)$$

The bar indicates averaging of the period of rotation of the electron. It follows from Eq. (10) that the condition for the effectiveness of the interaction of the electrons with sound (2) corresponds to the formation of a resonance denominator  $[v - i\omega + ik\bar{v}_z \sin \varphi]^{-1}$ . The electrons with the largest drift velocity  $v_{z \max}$ , belonging to the vicinity of the reference points on the Fermi surface  $p_z = p_0$  ( $p_0 = p_{z \max} = |p_{z \min}|$ ), determine the inclination effect.<sup>[3]</sup> Therefore, replacing the functions  $\bar{\Lambda}_{\zeta\zeta}^2(\mathbf{p}_z)$  and  $\bar{v}_z$  by their values at  $p_z = p_0$ , and separating the real and imaginary parts, we obtain

$$\text{Re } \Phi = \sum_{\alpha} \omega A^{(\alpha)} \frac{s}{\bar{v}_{z \max}^{(\alpha)} \sin \varphi} Y^{(\alpha)}, \quad (11)$$

$$\Delta\omega = \text{Im } \Phi = \sum_{\alpha} \omega A^{(\alpha)} \frac{s}{\bar{v}_{z \max}^{(\alpha)} \sin \varphi} Z^{(\alpha)}, \quad (12)$$

$$A = \frac{2\pi \Lambda_{\zeta\zeta}^2(p_0) m p_0}{s^2 \rho (2\pi\hbar)^3}, \quad (13)$$

$$Y = \text{arctg} \left[ \omega \tau_0 \left( \frac{\bar{v}_{z \max} \sin \varphi}{s} - 1 \right) \right] + \text{arctg} \left[ \omega \tau_0 \left( \frac{\bar{v}_{z \max} \sin \varphi}{s} + 1 \right) \right], \quad (14)$$

$$Z = \frac{1}{2} \ln \frac{1 + \omega^2 \tau_0^2 (1 + \bar{v}_{z \max} \sin \varphi / s)^2}{1 + \omega^2 \tau_0^2 (1 - \bar{v}_{z \max} \sin \varphi / s)^2} \quad \tau_0 = \tau(p_0). \quad (15)$$

We shall give the value of  $\Delta\omega$  for  $\varphi = 0$ , which will be used below for an estimate of the constants of the deformation potential:

$$\Delta\omega(\varphi = 0) = \sum_{\alpha} \omega A^{(\alpha)} \frac{2\omega^2 \tau_0^2}{1 + \omega^2 \tau_0^2} \quad (16)$$

Figure 1 shows the curves of the angular dependence of the sound velocity  $\Delta s/sA = \text{Im } \Phi/\omega A$  for one group of carriers for different values of the parameter  $\omega\tau_0$ . As is seen from the drawing, the inclination effect for the sound velocity is observed for  $\omega\tau_0 > 1$ . For  $\omega\tau_0 \approx 1$ , the curve is characterized by a smooth decay for increasing values of the angle  $\varphi$ .

The function  $\Phi(\varphi)$  (8) can be computed exactly for a metal with a spherical Fermi surface. The deformation potential tensor  $\Lambda_{ik}(\mathbf{p})$  for this case will have the form

$$\Lambda_{ik}(\mathbf{p}) = \Lambda \left( \frac{p_i p_k}{p_0^2} - \frac{1}{3} \delta_{ik} \right). \quad (17)$$

After elementary integration, we get

$$\text{Re } \Phi = \omega B \left\{ \frac{1}{a} C_1 Y + \frac{1}{a} \frac{1}{\omega\tau} C_2 Z + \frac{1}{a^2 \omega\tau} C_3 \right\}, \quad (18)$$

$$\text{Im } \Phi = \omega B \left\{ \frac{1}{a} C_2 Z - \frac{1}{a\omega\tau} C_2 Y + \frac{1}{a^2} C_4 \right\}; \quad (19)$$

$$B = \frac{2\pi \Lambda^2 m p_0}{s^2 \rho (2\pi\hbar)^3}, \quad a = \frac{v_0 \sin \varphi}{s},$$

$$C_1 = \frac{1}{36} - \frac{1}{6a^2} \left( 1 - \frac{1}{\omega^2 \tau^2} \right) + \frac{1}{4a^4} \left( 1 - \frac{6}{\omega^2 \tau^2} + \frac{1}{\omega^4 \tau^4} \right),$$

$$C_2 = \frac{1}{3a^2} - \frac{1}{a^4} \left( 1 - \frac{1}{\omega^2 \tau^2} \right), \quad C_3 = -\frac{1}{6} + \frac{1}{2a^2} \left( 3 - \frac{1}{\omega^2 \tau^2} \right),$$

$$C_4 = \frac{1}{6} - \frac{1}{2a^2} \left( 1 - \frac{3}{\omega^2 \tau^2} \right). \quad (20)$$

In the case of large values of the parameter  $\omega\tau$  ( $\omega\tau \gg 1$ ) and  $\varphi \approx \varphi_{\text{cr}}$ , Eqs. (18)–(20) are materially simplified. The quantity  $\Lambda^2 C_1$  is identical, with accuracy to terms of order  $(\omega\tau)^{-2}$ , with the value of the component  $\Lambda_{\zeta\zeta}^2(p_0)$  at the reference points of the Fermi surface, and can be used in Eqs. (11) and (12) for the functions  $\text{Re } \Phi$  and  $\text{Im } \Phi$ . For  $\omega\tau \lesssim 2$ , the shapes of the sound velocity dispersion curves computed by Eqs. (12) and (19) are identical. This was to have been expected, since the inclination effect is subject only to the fulfillment of the condition for the effectiveness of the interaction of the electrons with the sound—the phase relation (2).

## 2. EXPERIMENTAL METHOD

Figure 2 shows the arrangement for the investigation of the dispersion of the sound velocity in an inclined magnetic field. The specimen, with  $\text{LiNbO}_3$  ultrasonic transducers attached to it, is placed in a cryostat which is filled with liquid helium. The transducers with the specimen are connected in the positive

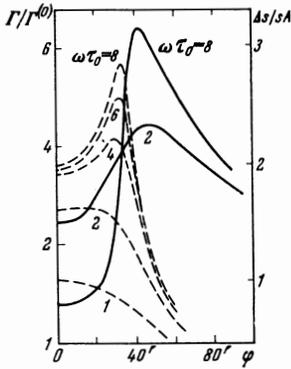


FIG. 1

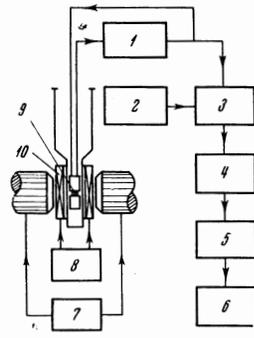


FIG. 2

FIG. 1. Curves of the angular dependence of the sound velocity  $\Delta s/sA$  and absorption  $\Gamma/\Gamma^{(0)}$  for different values of  $\omega\tau_0$  for the parameters  $v_{z \max} = 2 \times 10^7$  cm/sec,  $s = 2.02 \times 10^5$  cm/sec (dashed lines— $\Delta s/sA$ , solid lines— $\Gamma/\Gamma^{(0)}$ ).

FIG. 2. Block diagram of the apparatus for the study of the sound velocity dispersion in an inclined magnetic field: 1—amplifier, 2—reference generator, 3—mixer, 4—low-frequency amplifier, 5—frequency meter, 6—recorder, 7—power supply for magnet, 8—sweep for magnetic field of solenoid, 9—sample, 10—solenoid.

feedback loop of an amplifier. The amplifier consisted of a resonance system with central frequency 165 MHz and bandwidth  $\Delta f = 10$  MHz; its amplification factor could be varied from 0 to 23 dB. To achieve a definite amplification factor exceeding the total damping of the system specimen-transducers, the amplifier was excited at a frequency determined by the band pass of the amplifier, the bandwidths of the transducers, and the frequency characteristic of the specimen. In addition, the signal was fed into a mixer, to which was also applied the signal of a reference generator, which had a frequency stability no worse than  $10^{-6}$  sec $^{-1}$ . The difference frequency after the mixer was fed to a broadband low-frequency amplifier and amplified to the value necessary for registering by a frequency meter. A signal of constant voltage, proportional to the frequency, was recorded by the frequency meter and fed to a recorder through an appropriate circuit. The inclination of the magnetic field was produced by means of superposition of two mutually perpendicular magnetic fields: the field of the electromagnet  $H'$  = const and the field of the solenoid  $H''$ . Upon change in the field  $H''$ , which is produced by means of an electronic rheostat, the resultant vector  $H$  is inclined at the desired angle  $\varphi$ . Since the width of the dispersion curve in angular units is small ( $\varphi_{\max} \approx 2^\circ$ ), the absolute change in the value of the magnetic field can be neglected if the condition of a large magnetic field (1) is satisfied. For an exact determination of the characteristic points on the dispersion curve, a recording was made on both sides of the position  $\mathbf{k} \cdot \mathbf{H} = 0$ , i.e., the resulting vector can depart from the perpendicular direction relative to  $\mathbf{k}$  by an angle  $\varphi$  and by  $-\varphi$ . Such a recording of the curves does not require an exact initial orientation  $\mathbf{k} \cdot \mathbf{H} = 0$ .

For the study of the dispersion, we used Bi samples with resistance ratio  $R_{300}/R_{4.2} = 330$ .<sup>1)</sup> The samples

<sup>1)</sup>We take this opportunity to thank B. N. Aleksandrov for kindly providing the bismuth for our experiments.

were cut from a single crystal and had the shape of disks of diameter 7–8 mm and thickness  $d = 1.2$  mm. The temperature during the course of the measurements was determined from the saturated vapor pressure of the helium.

To determine the dependence of the frequency of oscillation of the system sample–amplifier, it was necessary to estimate the effect of the change in the absorption and sound velocity. In the given case, the sample was an acoustic resonator of the Fabry-Perot type. Its frequency characteristic is shown in Fig. 3. If we denote the initial amplitude of the wave by  $u_0 = \bar{u}_1$ , then the complex amplitude of the signal and its modulus will be equal:

$$\begin{aligned} \bar{u} &= u_0 \sum_{n=1}^{\infty} \exp[-(n-1)2\Gamma d + (n-1)ikd] \\ &= u_0 [1 - \exp(-2\Gamma d + ikd)]^{-1}, \quad u = |\bar{u}| = u_0 [1 - 2e^{-\Gamma d} \cos \varphi + e^{-2\Gamma d}]^{-1/2}, \quad \varphi = 2kd, \quad k = \omega/s. \end{aligned} \quad (21)$$

Assuming the sound absorption to be small ( $2\Gamma d < 1$ ) and expanding Eq. (21) in a power series in  $2\Gamma d$ , we get  $u = A[(2\Gamma s)^2 + (\omega - \omega_k)^2]^{-1/2}$ , where  $\varphi \sim (\omega - \omega_k)$ ,  $\omega_k$  is the characteristic frequency of vibration of the sample lying inside the complete resonance curve of the amplifier and transducers. The resonance frequency of the system is found from the condition  $\partial u / \partial \omega = 0$ :

$$\omega_p = k_n s - \Gamma s^2 \frac{\partial \Gamma}{\partial \omega}, \quad k_n = \frac{\omega}{s} = \frac{\pi n}{d}; \quad n = 1, 2, \dots$$

We estimate the frequency shift of the acoustic resonator for a change in the absorption and sound velocity:

$$\Delta \omega_{p\Gamma} = - \left[ s^2 \frac{\partial \Gamma}{\partial \omega} + \Gamma s^2 \frac{\partial}{\partial \Gamma} \left( \frac{\partial \Gamma}{\partial \omega} \right) \right] \Delta \Gamma, \quad (22)$$

$$\Delta \omega_{ps} = \left[ k_n - 2\Gamma s \frac{\partial \Gamma}{\partial \omega} \right] \Delta s = \frac{\omega_p}{s} \Delta s - \Gamma s \frac{\partial \Gamma}{\partial \omega} \Delta s. \quad (23)$$

Substituting the corresponding values of the quantities in Eqs. (22), (23), it can be established that  $\Delta \omega_{p\Gamma} / \Delta \omega_{ps} \ll 1$ , i.e., the frequency shift of the acoustic resonator is completely determined by the change in the sound velocity if the inequality  $2\Gamma d < 1$  is satisfied. It must be noted that a similar method is justified when it is necessary to measure the small change in the sound velocity in materials with a small absorption coefficient. To the inadequacies of this method, we must add the impossibility of shifting the frequency of the system over a wide range.

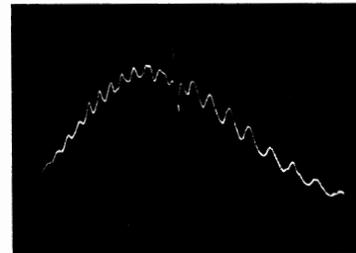


FIG. 3. Frequency characteristic of the sample (photographed with a measuring screen X1-19). The marker corresponds to a frequency of 160 MHz.

## 3. DISCUSSION OF RESULTS

Figure 4 shows a comparison of the theoretical curve with the experimental points. The experimental recording refers to the case in which the sound is propagated along the trigonal axis  $C_3$  and the vector  $H$  (for  $\varphi = 0$ ) lies in the plane of the binary and bisector axes  $C_1 \cdot C_2$  with angle of inclination  $\beta \approx 6^\circ$  relative to the  $C_2$  axis. The drift velocity of the electrons from the vicinity of the reference points  $\bar{v}_{Z \max} = 2 \times 10^7$  cm/sec, the sound velocity along the  $C_3$  is equal to  $2.02 \times 10^5$  cm/sec. The parameter  $\omega\tau_0 \sim 1$  at a frequency  $f = 165$  MHz and  $T = 1.4^\circ\text{K}$ . The theoretical curve is constructed by Eq. (12) for the values of  $\bar{v}_{Z \max}$ ,  $s$  and  $\omega\tau_0$  given above. As is seen from the drawing, there is excellent agreement between theory and experiment.

It follows from the theory that the deviation effect (the growth of the sound velocity with increase in the angle  $\varphi$ ) is observed for  $\omega\tau_0 > 1$ . For comparison, the absorption curves are plotted in Fig. 1, in addition to  $\Delta s/sA$ . The maxima of the dispersion curves are located at the same values of the angles  $\varphi_{CR}$  as the inflection points on the absorption curves. From the position of the maxima, one can calculate the velocities of the electrons from the neighborhood of the reference points  $v_{Z \max}$ , using Eq. (4). The measurement of the quantity  $\Delta s$  at  $\varphi = 0$  makes it possible to estimate the constants of the deformation potential  $|\Lambda_{\zeta\zeta}(p_0)|$  (see (16)).

For  $\omega\tau_0 \approx 1$ , as was observed previously, the dispersion curve is a smoothly decreasing one for increase in the angle  $\varphi$ . The values of  $\Delta s/s$  at  $\varphi = 0$  were measured in our experiments for the purpose of determining the constants of the deformation potential. For longitudinal sound, the measurements were made in the directions:

$$\mathbf{k} \parallel C_3, \vec{\chi} \parallel H, C_2 \approx 6^\circ; \mathbf{k} \parallel C_2, H \parallel C_3.$$

In the first case,  $\Delta s/s = 1.06 \times 10^{-4}$ ,

$$|\bar{\Lambda}_{33}(p_0)| = (1.9 \pm 0.2) \text{ eV} \quad (24)$$

In the second case  $\Delta s/s = 1.36 \times 10^{-4}$

$$|\bar{\Lambda}_{22}(p_0)| = (3.7 \pm 0.2) \text{ eV} \quad (25)$$

(the values of the cyclotron masses are taken from<sup>[9]</sup> and the values of  $p_0$  from<sup>[10]</sup>).

In the specified directions, the sound interacts effectively with the electrons; therefore, we can say nothing about the corresponding components of the deformation potential for holes. In the direction  $\mathbf{k} \parallel C_1$ , it was not possible to obtain a satisfactory resolution of the absorption and dispersion. The other components of the tensor  $\Lambda_{ik}$  can be measured in a similar way by using transverse sound.

It is of interest to compare the values of  $|\bar{\Lambda}_{22}(p_0)|$  and  $|\bar{\Lambda}_{33}(p_0)|$ , obtained by us, with the results of other authors. For example, the components  $\Lambda_{ik}$ , computed

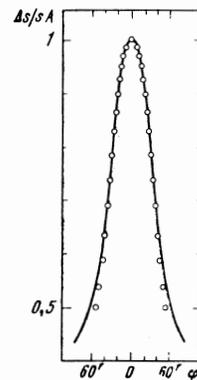


FIG. 4. Comparison of the theoretical curve and experimental points for the sound velocity dispersion  $\Delta s(\varphi)/sA$  for  $\omega\tau_0 \approx 1$ . The solid curve represents theory, the circles, experimental points.

from the anisotropy of giant quantum oscillations in the sound absorption, are given in<sup>[11]</sup>. It is known that the giant quantum oscillations (GQO) are due to electrons belonging to the extremal cross sections of the Fermi surface (in the case of convex surface—the central cross section with  $p_z = 0$ ). The components of the deformation potential for the corresponding electron groups are equal to

$$|\Lambda_{22}| = (5.9 \pm 0.8) \text{ eV}, \quad |\Lambda_{33}| = (1.71 \pm 0.36) \text{ eV}.$$

from the data of<sup>[11]</sup>.

In conclusion, it is our pleasant duty to thank É. A. Kaner for useful discussions and V. I. Beletskii for help in the measurements.

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