# ELECTROMAGNETIC FIELD SPIKES AND THE SIZE EFFECT IN CADMIUM, DUE TO A CHAIN OF ELECTRON TRAJECTORIES

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We investigate the excitation of spikes of an electromagnetic field by a chain of electron trajectories at radio frequencies in cadmium. The theoretical dependences of the transfer coefficient of the field along the links of the chain on the angle of inclination of the magnetic field to the surface of the metal and on the number of the size-effect line are obtained. In the main, the experimental results confirm the theoretical relations. It is shown that the difference between the Fermi surface and a sphere does not change the results qualitatively, but leads to a decrease of the transfer coefficient and to its anisotropy; the latter is due to nonresonant electrons.

## 1. INTRODUCTION

As is well known, the interaction of the conduction electrons of a metal with an external electromagnetic field leads under ordinary conditions to the occurrence of the skin effect, wherein the field penetrates to a slight depth $\delta$ (skin layer) into the metal. However, at low temperatures and in a magnetic field when the mean free path of the electron $l$ and the radius of the electron trajectory R become large ($l \gg R$, $R \gg \delta$), the electromagnetic field can be carried by the electrons to a considerable distance from the surface of the metal. Becoming accelerated in the skin layer and acquiring a velocity increment, the electrons produce a current not only in the skin layer, but at any place in the metal where their velocity is $v_z = 0$ (the z axis is parallel to the normal n to the surface of the metal). In particular, in the case of motion on a circular trajectory, the current will occur at a depth equal to $2R = D$. If the number of electrons having a trajectory of equal diameter is not small, then an electromagnetic field spike (EFS) will be produced in the interior of the metal, and will in fact duplicate the skin layer. The electrons moving along the trajectories passing through the new skin layer can again produce an EFS at a distance 2D, etc. Thus, a system of periodic spikes produced by a chain of trajectories will arise in the metal.

The field amplitude in the EFS depends on a number of factors. First, the condition requiring that the number of electrons producing the EFS be large, and consequently that the electrons have a diameter spread $\Delta D \sim \delta$, is equivalent to requiring that D have an extremal value. It is perfectly obvious that the number of such resonant electrons depends on the sharpness of this extremum. In addition, if we disregard the unrealistic case of one cylindrical Fermi surface, then different numbers of electrons will take part in the production of the skin current and of the EFS, namely, the skin current is formed by all the electrons, and the EFS by the resonant electrons only.[1)] This difference can increase appreciably in the case of complicated Fermi surfaces, particularly non-convex ones, when the contribution of the nonresonant electrons to the conductivity is large. The relation between the role of the resonant and nonresonant electrons can change if the magnetic field is inclined to the surface of the metal[1] or if the resonant electrons are accelerated more under conditions of cyclotron resonance.[2]

In a magnetic field parallel to the surface of the metal, all the electrons return many times to the skin layer. As the field becomes inclined ($\delta/l \ll \varphi \ll \delta/R$), the nonresonant electrons, drifting along **H**, begin to leave the skin layer after passing through it only once. Their contribution to the skin layer will thus decrease, and this will lead to an increase of the field in the EFS. Such a process will continue until resonant electrons also begin to leave. Therefore a further increase of the inclination will lead to a decrease of the transfer coefficient of the field. Thus, the maximum of the field is reached in the EFS when the relative contribution of the resonant electrons to the conductivity becomes maximal. In the case of a spherical Fermi surface, inclination of the field makes it possible to realize conditions under which practically the same group of electrons takes part in both the conductivity and in the production of the EFS, and the transfer coefficient becomes close to unity.[1] On the other hand, if the Fermi surface is complicated and has several stationary cross sections with $v_H = 0$, then the number of nonresonant electrons will always be larger than the number of resonant ones, and the field in the spikes will be relatively weak (here, of course, the electrons of other stationary cross sections will produce their own systems of EFS).

A theoretical analysis of the conditions for the formation of the EFS for a spherical Fermi surface is given in [1]. However, the obtained formulas are asymptotic approximations and are not quite convenient for a detailed comparison with experiment.

The EFS has been investigated experimentally using the radio-frequency size effect (RSE) in a number of metals (tin,[3] bismuth,[4] rubidium[5]). Although these investigations have shown that the decay of the EFS changes with inclination of the magnetic field and depends on the crystallographic directions, they were

[1)]The EFS effect is essentially a geometric resonance, which arises upon interaction of electrons with the spatial harmonics of the field in the metal having wave numbers that are multiples of the characteristic dimension of the electron trajectory, in the particular case, multiples of the extremal diameter of the trajectories $D_0$.

not systematic in character. The purpose of the present paper is to fill this gap.

## 2. THEORY

According to Maxwell's equations, the electric field in a metal can be described by the formula

$$E_\alpha(z) = -2\pi^{-1}E_\beta'(0)\,T_{\alpha\beta}(z), \qquad (1)$$

where $E'_\beta(0) = \partial E_\beta/\partial z|_{z=0}$; $\alpha, \beta = x, y$, the z axis is directed along the inner normal to the surface of the metal, and

$$T_{\alpha\beta}(z)=\int\limits_0^\infty \frac{\cos qz\,dq}{q^2\delta_{\alpha\beta}-4\pi i\omega c^{-2}\sigma_{\alpha\beta}(\omega, q)}, \qquad (2)$$

where $\mathbf{q}$ is the wave vector of the electromagnetic field, $\delta_{\alpha\beta}$ is a unit tensor, and the high-frequency conductivity tensor $\sigma_{\alpha\beta}$ is determined from the kinetic equation.

Henceforth, assuming $\mathbf{H} \perp \mathbf{E}$ and $\mathbf{E} = E_x$, we shall be interested only in the component $\sigma_{xx}(\omega, q)$ which is most most important for the excitation of EFS by a chain of trajectories. According to [1], $\sigma_{xx}(q)$ is given for $qR \gg 1$ and low frequencies ($\omega \ll \nu, \Omega$) by the following asymptotic expression in the case of a spherical Fermi surface:

$$\sigma_{xx}(q) = \sigma_M(q) - \sigma_{os}(q), \qquad (3)$$

where

$$\sigma_M(q)=\frac{3\sigma_0}{2\pi ql}\sum_{n=-\infty}^{\infty}\int\limits_{-1}^{1}\frac{d\mu\sqrt{1-\mu^2}}{\gamma_0+in+iqR_0\mu\sin\varphi}$$
$$=\frac{3\sigma_0}{2ql}\sum_{n=-\infty}^{\infty}\frac{\gamma_0+in}{(qR_0\sin\varphi)^2}\left[1-\sqrt{1+\left(\frac{qR_0\sin\varphi}{\gamma_0+in}\right)^2}\right] \qquad (4)$$

is the monotonic part of the conductivity and

$$\sigma_{0e}(q)=\frac{3\sigma_0}{2\pi ql}\sum_{n=-\infty}^{\infty}\int\limits_{-1}^{1}\frac{d\mu\sqrt{1-\mu^2}\sin(qD_0\sqrt{1-\mu^2}-n\pi)}{\gamma_0+in+iqR_0\mu\sin\varphi}$$
$$=\overset{0}{\sigma}_{os}\cos(qD_0+\Theta) \qquad (5)$$

the oscillating part.

Here $\sigma_0 = Ne^2/m^*\nu$ is the static conductivity, $\nu$ the collision frequency, $\gamma_0 = \nu/\Omega = R_0/l$, $\Omega = eH/m^*c$ the cyclotron frequency $l = v/\nu$ the free path of the electron, $D_0 = 2R_0$, $R_0 = cp/eH$ the radius of the electron trajectory, p the radius of the Fermi sphere (momentum), $\mu = p_H/p$, and $p_H$ is the projection of $\mathbf{p}$ on the direction of $\mathbf{H}$.

The amplitude of the conduction oscillations $\sigma^0_{os}$ can be represented in the form

$$\overset{0}{\sigma}_{os}=\frac{3}{\sqrt{2}}\frac{\sigma_0}{q^2lR_0\sin\varphi}F(w) \qquad (6)$$

where

$$F(w) = \{[S(w^2)-{}^1/_2]^2+[C(w^2)-{}^1/_2]^2\}^{1/2}, \qquad (7)$$

and

$$\Theta=\frac{\pi}{4}+w^2+\operatorname{arctg}\frac{C(w^2)-{}^1/_2}{S(w^2)-{}^1/_2}, \qquad (8)$$

$$w=\gamma_0\,/\,(qR_0)^{1/2}\sin\varphi; \qquad (9)$$

$S(w^2)$ and $C(w^2)$ are Fresnel integrals.

The oscillating character of the conductivity, as shown by Kaner,[1] leads to the occurrence of EFS in the interior of the metal. If the quantity $\xi = \frac{1}{2}\sigma^0_{os}/\sigma_M$, which can be called the transfer coefficient along the links of the chain, is smaller than $\frac{1}{2}$, than the field in the next spike decreases like a power of the parameter $\xi$, and the field distribution in the m-th spike (i.e., near $t \equiv z/D_0 = m$) can be described by the function

$$T(t)=J\frac{\exp(i\pi m/2)}{M}\xi^m\int\limits_0^\infty\frac{y^\alpha\cos[M(t-m)y-m\Theta]dy}{(y^{\alpha+2}-i)^{m+1}}, \qquad (10)$$

where J is a certain constant that depends weakly on the magnetic field, $M = D_0/\delta_{eff}$, and $\alpha$ and the effective skin-layer depth $\delta_{eff} = c(4\pi\omega\sigma_M)^{-1/2}$ depend on the inclination of the field and are listed in the table for different angle intervals. It is assumed here that q in $\xi$ (i.e., in $\sigma_M$ and $\sigma^0_{os}$) is equal to $\delta^{-1}_{eff}$. Since the transfer coefficient $\xi$ is determined by the ratio of the oscillating to the monotonic part of the conductivity, its value at different inclination angles of the magnetic field will depend on the behavior of $\sigma_M$ and $\sigma_{os}$.

The inclination of the magnetic field leads to a decrease of both parts of the conductivity. However, the characteristic angle intervals that determine these changes turn out to be different. For the monotonic part, the characteristic angles are $\delta/l$ and $\delta/R$, at which none of the electrons, drifting along $\mathbf{H}$, has time to leave the skin layer within the free-path time and within the period of revolution in the magnetic field, respectively. The oscillating part is characterized by the angle $(R\delta)^{1/2}/l$, at which resonant electrons do not leave the skin layer within the free-path time. This means that the relative contributions to the conductivity from the resonant and nonresonant electrons will depend significantly at different inclination angles on the ratio of $\delta/R$ and $(R\delta)^{1/2}/l$, which in turn varies with the magnetic field. In the limiting case when $\delta/R \gg (R\delta)^{1/2}/l$, it is relatively easy to obtain from (4) and (6) the asymptotic values of $\sigma_M$, $\sigma^0_{os}$, and $\xi$ far from the characteristic angles (see the table). However, when these angles are of the same order of magnitude, as is usually the case in the experiment, the asymptotic approximations become incorrect. Nonetheless, the dependences of $\sigma_M$, $\sigma^0_{os}$ and $\xi$ on $\varphi$ and $D_0$ can be calculated ap-

| $\varphi$ | $\sigma_M$ | $\overset{\circ}{\sigma}_{os}$ | $\xi$ | $\delta^*_{eff}$ | $\alpha$ |
|---|---|---|---|---|---|
| $<\frac{\delta}{l}$ | $\frac{3}{4}\frac{\sigma_0}{qR_0}$ | $\frac{3}{2}\frac{\sigma_0}{qR_0}(\pi qR_0)^{-1/2}$ | $\left(\frac{\delta_{eff}}{\pi R_0}\right)^{1/2}$ | $\delta_0\left(\frac{\pi R_0}{l}\right)^{1/3}$ | 1 |
| $\frac{\delta}{l}\ll\sin\varphi\ll\frac{(R_0\delta)^{1/2}}{l}$ | $\frac{3}{2}\frac{\sigma_0}{q^2R_0l\sin\varphi}$ | $\frac{3}{2}\frac{\sigma_0}{qR_0}(\pi qR_0)^{-1/2}$ | $\frac{l\sin\varphi}{2(\pi R_0\delta_{eff})^{1/2}}$ | $\delta_0\left(\frac{\pi R_0\sin\varphi}{2\delta_0}\right)^{1/4}$ | 2 |
| $\frac{(R_0\delta)^{1/2}}{l}\ll\sin\varphi\ll\frac{\delta}{R_0}$ | $\frac{3}{2}\frac{\sigma_0}{q^2R_0l\sin\varphi}$ | $\frac{3}{2}\frac{\sigma_0}{q^2R_0l\sin\varphi}$ | $\frac{1}{2}$ | $\delta_0\left(\frac{\pi R_0\sin\varphi}{2\delta_0}\right)^{1/4}$ | 2 |
| $\sin\varphi\gg\frac{\delta}{R_0}$ | $\sigma_a^{**}$ | $\frac{3}{2}\frac{\sigma_0}{q^2R_0l\sin\varphi}$ | $\frac{\delta_{eff}}{\pi R_0\sin\varphi}$ | $\delta_0$ | 1 |

* $\delta_0=(c^2l/3\pi^2\omega\sigma_0)^{1/3}$.

** $\sigma_a={}^3/_4\pi\sigma_0/ql$.

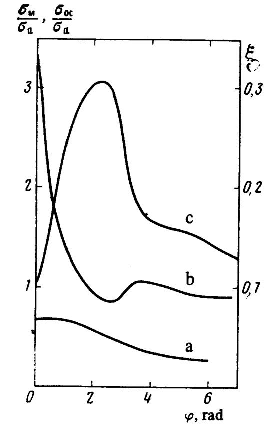


FIG. 1. Plots of the oscillating $\sigma_{os}^0$ and monotonic parts of the conductivity (a and b) and of the transfer coefficient ξ (c) against the angle of inclination of **H** to the surface of the metal at $\gamma_0 = 0.1$ and $l = 0.95$ mm.

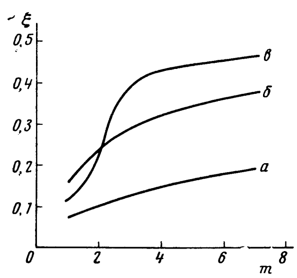


FIG. 2. Plot of the transfer coefficient ξ against the magnetic field, characterized by the quantity $m = d/D_0$, where $d = 3.8 \times 10^{-2}$ cm, at $\varphi = 0$ (a), 50′ (b), and 3°10′ (c).

proximately for concrete conditions by using formulas (4) and (6). Figure 1 shows plots of the indicated quantities under the condition[2)] that $\gamma_0 = 0.1$, $R_0 = d/4 = 9.5 \times 10^{-3}$ cm. We see that ξ has a maximum equal to 0.307 in the angle interval

$$(\delta R_0)^{1/2}/l \approx 1°20' < \varphi < \delta / R_0 \approx 3°.$$

An increase of the magnetic field, besides directly increasing ξ, intensifies the inequality between these angles, which also leads to a growth of the transfer coefficient, which tends to ½ in the limit. This is clearly seen from Fig. 2, which shows plots of ξ against $m = d/D_0$ at different inclination angles φ. (It should be borne in mind that in order to satisfy the conditions of the anomalous skin effect it is necessary to have $m \ll D_0/\delta$.)

So far we have assumed that the metal is semi-infinite and is in a constant magnetic field of given intensity. The EFS in this case change the impedance somewhat and cannot appear directly. To observe them experimentally it is necessary to monitor the transparency of a plane-parallel plate to the magnetic field; this transparency appears if the next spike emerges on the opposite side of the plate, i.e., under the condition $d = mD_0$, where d is the thickness of the plate. In addition, one measures in the experiment not the field itself, but its derivative $dT/dH = (dT/dt)(dt/dH)$.

Differentiating (10), we obtain an expression that determines the RSE lines:

$$\frac{dT}{dH} = J_0 \xi^m \exp\frac{i\pi m}{2} \cdot \int_0^\infty \frac{y^{\alpha+1}}{(y^{\alpha+2} - i)^{m+1}} \sin\left(\frac{d - mD_0}{\delta_{\mathrm{eff}}} y - m\Theta\right) dy. \qquad (11)$$

2)The values of $\gamma_0$ and d were obtained in the experiment described below.

For the line intensity, assuming the sine function in (11) equal to unity, we obtain the expression

$$J(m) = J_0 \xi^m m^{-1} (-1)^m / (\alpha + 2), \qquad (12)$$

where $\xi = \xi(\varphi, m)$. As ξ approaches ½, the validity of (12) is violated. In this case the EFS, as shown in [6], will be attenuated more slowly, but we were unable to obtain an exact expression for the decrease of the RSE lines. However, as shown by experiment, formula (12) can be used in first approximation even at relatively large ξ.

Let us see now how the nonquadratic character of the dispersion law can become manifest. The deviation of the Fermi surface from a sphere, or the presence of other surfaces, should change quantitatively both the monotonic and the oscillating parts of the conductivity. For most metals, these changes (with the exception of the case of a cylindrical Fermi surface), will be such that $\sigma_M(q)$ increases and $\sigma_{os}^0(q)$ decreases. In addition, since the principal role is played by the rate of change of $\sigma_M(q)$ and $\sigma_{os}^0(q)$ as functions of φ, which is determined by the dependence of the electron drift velocity along the field on μ, the characteristic angle intervals can also change somewhat. However, the general character of the change of both parts of the conductivity should remain the same. Quantitative changes are very difficult to account for, since they can depend strongly on the direction of **H** relative to the crystallographic axes and, in particular, on the angle φ. In other words, one should expect the transfer coefficient to decrease in metals with a complicated Fermi surface, and the crystallographic anisotropy of $\sigma_M$ and $\sigma_{os}^0$ should become superimposed on the angular dependence of the transfer coefficient.

## 3. EXPERIMENT

The experiment was performed with apparatus of the nuclear spectrometer type. The autodyne generator operated at 3.15 MHz. The magnetic-field modulation frequency was 39 Hz. Bilateral excitation was used, wherein the sample was placed in the inductance coil of the tank circuit of the autodyne, so that currents were induced in the sample on both sides. The electromagnetic fields, passing through the sample as a result of anomalous penetration, also produced currents on both sides, thereby changing the impedance of the plate and producing a reaction of the generator.

In the experiment we measured the change of the generation amplitude a with changing field, $da/dH \sim dR/dH \sim dT/dH$. The sample was in the form of a disk of thickness $d = 0.38$ mm, diameter 11 mm, and was grown in a dismountable quartz mold from initial cadmium with a ratio of resistivities at room and helium temperatures $\rho(293°K)/\rho(4.2°K) = 150\,000$. The flat surface of the sample coincided within 1–2° with the $(1\bar{2}10)$ plane. The crystallographic directions were monitored by x-ray diffraction. The direction of polarization of the high-frequency field **E** was perpendicular to the direction of **H**, which was directed in the plane of the sample along the $[10\bar{1}0]$ axis, with accuracy ± 3°. To obtain arbitrary inclinations of **H** to the surface of the sample, the latter, together with the autodyne coil, was mounted ver-

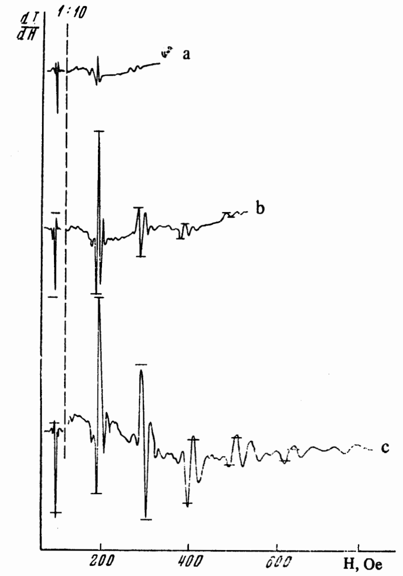


FIG. 3. RSE lines at different angles of inclination of **H** to the surface of the sample: a—$\varphi = 0°$, b—$\varphi = 50'$, c—$\varphi = 3°10'$. The horizontal strokes denote the calculated intensities of the corresponding lines. The first line is reduced in scale by a factor of 10 compared with the others.

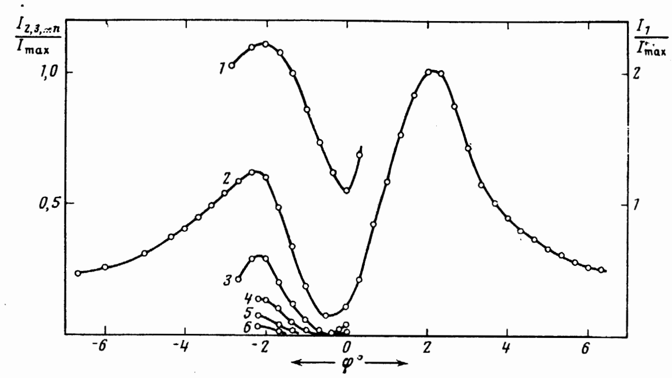


FIG. 4. Experimental dependence of the relative intensity of the RSE lines with different numbers on the angle of inclination of the magnetic field to the surface of the sample. $I_{max}$ corresponds to the maximum of the second harmonic at positive values of $\varphi$. The numbers at the curves correspond to the numbers of the RSE lines.

tically, and the magnet was rotated in a horizontal plane. The angles were read with accuracy $\sim 1'$.

A study of the character of the EFS in the metal following penetration along a chain of trajectories was carried out for the lenticular Fermi surface of Cd in the third zone. The "lens" produces intense RSE lines and has a simple convex form, which makes it possible to use the formulas obtained for a sphere in the comparison with the theory.

Experiment has shown that in a magnetic field strictly parallel to the surface of the metal, the EFS attenuates extremely rapidly, so that one can hope to register only two lines of the size effect (see Fig. 3a). However, inclination of the field by only $10'$–$20'$ gives an appreciable increase of the EFS amplitude, making it possible to observe four and more RSE lines. Figures 3b and 3c show corresponding plots of the RSE lines at $\varphi = 50'$ and $\varphi = 3°10'$, demonstrating the appreciable increase of the EFS amplitude and of the transfer coefficient $\xi$ with inclination of the field.

Figure 4 shows the change of the relative intensity of the size-effect lines with different numbers m as a function of the angle of inclination of the magnetic field to the sample surface.

From an examination of the curve for $m = 2$ we see that it is not symmetrical, either with respect to the height of the maxima or with respect to their placement relative to the minimum with respect to $\varphi$.

We note that the ratio of the intensities at the maxima, as well as the ratio of their positions relative to the minimum in $\varphi$, amounts to $\sim 1.6$. The behavior of lines with numbers $m > 2$ corresponds qualitatively to the behavior of the line with $m = 2$. The minima of the amplitudes of the lines coincide for $m = 2, 3, 4, \ldots$ etc. The line with $m = 4$ appears upon inclination by $10'$ from the position of the minimum of the line with $m \geq 2$. An inclination by $30'$ leads to the appearance of the line with $m = 5$. Lines up to $m = 7$ are observed at an inclination on the order of $1$–$2°$. Further increase of the inclination angle of **H**, say to $7°$, while resulting in a general lowering of the line intensity, does not lead to a decrease of their number, although the observation of the lines is made difficult because of the appearance of RSE lines from other sections of the Fermi surface near the "lens" line. The minimum of the line with $m = 1$ is shifted relative to the others by $30'$, thus introducing a certain uncertainty in the determination of the point $\varphi = 0$.

It should be noted that a change in the direction of **H** in the plane $(1\bar{2}10)$ affects the shift of the minima of the first and succeeding RSE lines relative to one another; this shift may reach $1°$, and the minima of the second and succeeding lines move the farthest. The minimum of the first RSE was therefore chosen to be $\varphi = 0$ in the reduction of the experimental data.

## 3. DISCUSSION

Cadmium, which was used in the experimental study of the EFS, has a relatively complicated Fermi surface. In addition to the electrons of the "lens," a large group of electrons belonging to a Fermi surface of the "monster" type takes part in the production of the skin layer. The trajectories of the electrons of the "monster" have a very complicated form, which depends to a considerable degree on the direction of the magnetic field relative to the crystallographic axes. A particularly strong dependence should be observed when **H** is directed near the twofold axis,[7] as was the case in the experiment. At the same time, for a chosen experimental geometry, the characteristics of the resonant electrons of the "lens" were close to those that should obtain at the Fermi sphere.

Presumably, therefore, $\sigma_M$ is anisotropic and somewhat larger in magnitude than for the Fermi sphere, and $\sigma_{0s}^{0}$ is isotropic. This circumstance will decrease the value of $\xi$ and strongly distort the theoretically-predicted angular dependence, as is indeed observed in the experiment. The unequal magnitudes of the maxima of the intensity of the lines (i.e., $\xi^m$), their different angular positions relative to the minimum and the equality of the ratios of these quantities (1.6) that are observed in the experiment when the direction of the

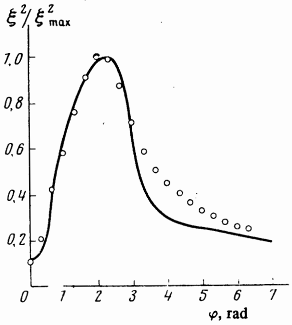


FIG. 5. Theoretical plot of $\xi^2/\xi^2_{max}$ for $\gamma_0 = 0.1$, $d = 3.8 \times 10^{-2}$ cm, m = 2 against the angle of inclination of the magnetic field to the surface of the sample. The points denote the experimentally observed relative intensities of the RSE lines for m = 2.

normal to the surface does not coincide exactly with a rational direction, all favor the foregoing assumption.

This point of view is also supported by the fact that the maxima and minima of the line intensities change when the inclination of the field direction to the sample plane is altered slightly, although the characteristics of the electrons of the "lens" remain practically constant in this case. In spite of this, however, the general character of the angular dependence of the RSE line intensity agrees well with the theory.

In the angle interval $\varphi < \delta/l$, which amounted to $\sim 15'$ in the present experiments, the EFS in the interior of the metal attenuate very strongly and the intensity of the RSE lines is practically independent of $\varphi$.[3)] Further inclination of **H**, which according to the theory leads to the growth of $\xi$, produces an increase in the intensity of the RSE lines, especially those with large numbers, as is demonstrated in Fig. 3.

Figure 5 shows a theoretical plot of $\xi^2/\xi_{max}$ for $\gamma_0 = 0.1$, $d = 3.8 \times 10^{-2}$ cm, and $m = 2$, together with the experimental values of $I_2/I_{max}$ taken from Fig. 4 for positive values of $\varphi$.

We see that the agreement between the experimental points and the theoretical curve is much better than could be expected in view of the statements made above concerning $\sigma_M$. It should be noted that the line intensity, according to (12), varies with $\varphi$ in a somewhat different manner than $\xi^m$, since the coefficient $\alpha$ also depends on $\varphi$. If this circumstance is taken into account, then the agreement on the right-hand side of the plot turns out to be even better.

The agreement between the theoretical curve and experiment for negative values of $\varphi$ is somewhat worse, this apparently being connected with the greater role played by the nonresonant electrons in this region of the inclination angles. In particular, the much smaller value of $I_{max}$ indicates that here $\sigma_M$ is larger by 1.26 times than at positive $\varphi$.

A convincing verification of the validity of the theory could be the experimental dependence of the transfer coefficient $\xi$ on the line number m. However, as seen from Fig. 2, $\xi(m)$ cannot be expressed mathematically in simple fashion for arbitrary $\varphi$, all the more since the contribution of the nonresonant electrons of the "monster" is not known exactly. In addition, it should be borne in mind that $\sigma_M$ can vary with the number.

[3)]The electron mean free path $l = 1$ mm was determined from the temperature dependence of $\delta_{eff}$ [8].

This is connected with the fact that at a fixed plate thickness, certain nonresonant electron trajectories will be "cut off" (i.e., will pass through the skin layer only once), making a small contribution to the monotonic part of the conductivity.

An increase of the magnetic field can change such trajectories into closed ones, thereby increasing their role. In other words, the contribution made to the conductivity by nonresonant electrons of other sections of the Fermi surface in the plate will be the same as in a semi-infinite metal, but only in a sufficiently strong magnetic field, when all the trajectories lie inside the plate. This effect should become more strongly pronounced at small inclination angles.

In spite of the indicated complications, the experimentally observed decrease of line amplitude with increasing number can be compared with that calculated in accordance with formula (12) by using the theoretical values of the transfer coefficient $\xi$, shown in Fig. 2, and and by choosing $I_0/(\alpha/2)$ such as to make the line intensities coincide, say, for $m = 2$. The horizontal strokes in Fig. 3 denote the expected calculated amplitudes at the corresponding angle of inclination.

We see that there is convincing agreement with theory, especially for $\varphi = 50'$. Some excess of the observed amplitudes over the calculated ones for $m > 3$ at $\varphi = 3°10'$ is connected with the fact that $\xi$ approaches strongly the value ½ ($\xi \geq 0.42$). This follows naturally from the limiting estimates of the behavior of $\xi$.[6] However, the observed insignificant discrepancy gives grounds for assuming that the obtained expressions for $I_m$ and $\xi$ are perfectly satisfactory for arbitrary $\xi$.

It should be borne in mind that not all estimates should be based on the line with $m = 1$, for in the case of two-sided excitation of the field in the sample the formation of this line is determined not only by the anomalous penetration of the field into the metal, which is considered in this paper, but also by the size-governed "cutoff" effect.

In light of the results, we must raise the question of the role of the electrons that are specularly reflected from the surface of the metal. It is known that specularly reflected electrons, moving along the surface, can lead to a growth of the monotonic part of the conductivity, causing, for example, a sharp decrease of the cyclotron resonance.[9] The conductivity produced by these electrons should not change strongly with inclination of the magnetic field. This in turn means that the transfer coefficient should be much smaller and its angular dependence should differ strongly from that obtained in the present paper. It can therefore be assumed that the specular fraction is insignificant, at least in cadmium.

Thus, a theoretical and experimental study of the EFS along a chain of trajectories leads to a number of conclusions concerning the characteristics of this phenomenon.

In a magnetic field **H** strictly parallel to the surface of the metal, the attenuation of the EFS is maximal and extremely strong. The attenuation of the field in the EFS is characterized by a certain parameter $\xi$, which can be called the transfer coefficient along the links of the chain of trajectories. The value of $\xi$ is equal to half the ratio of the oscillating part of the conductivity to the monotonic part.

The field in the m-th spike is proportional to $\xi^m$. The theoretical formulas that determine the conductivity of the metal in an inclined magnetic field for diffuse reflection of the electrons from the boundary and the distribution of the electromagnetic field in the interior of the metal, also describe correctly the main features of the EFS effect, such as the angular dependence of $\xi$, the dependence of $\xi$ on the magnetic field at different inclination angles, and consequently the law governing the decrease of the spikes under the RSE. The best agreement between theory and experiment is obtained for $\xi < 1/2$.

The difference between the Fermi surface and the sphere does not lead to qualitative changes of the results. At the same time, an increase of the monotonic part of the conductivity is possible, leading to a decrease of $\xi$. The change of $\xi$ is very anisotropic and in the general case difficult to take into account.

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