INVESTIGATION OF POWER RESONANCES OF A RING LASER WITH A NONLINEARLY ABSORBING CELL

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Submitted July 15, 1970

Zh. Eksp. Teor. Fiz. 60, 117-123 (January, 1970)

We investigate power resonances of traveling waves in a ring laser with a nonlinearly absorbing cell inside the resonator. We show that the power resonances can be much more contrasty and narrower than power resonances due to the Lamb saturation of the absorbing component.

1. INTRODUCTION

ONE of the most effective methods of producing highly stable sources of optical frequency is based on the existence of resonances of the output power of gas lasers with a nonlinearly absorbing molecular gas of low pressure.^[1-3] The output power has a peak at the central frequency of the absorbing gas, connected with the spectral burning-out of the Doppler-broadened line. Such "Lamb" burning of the Doppler line leads to resonance widths that are close in magnitude to the homogeneous line width of the absorbing component. For an He-Ne laser at $\lambda = 3.39 \ \mu$ with CH₄ in the absorbing cell, the observed experimental resonance widths are of the order of 600 kHz;^[2, 3] the relative frequency drift of a laser stabilized against the methane peak amounts to several times 10⁻¹¹ of the carrier frequency.^[3, 4]

In the present paper we investigate the power resonances of a ring laser with a nonlinearly absorbing cell inside the resonator. In the proposed method, the presence of power resonances is connected not only with the "Lamb" burning of the Doppler width, but also the spatial interaction of the waves traveling in opposite directions and having a frequency close to the central frequency of the absorbing gas.^{15, 6, 1}

The power resonances of the traveling waves of a ring laser turn out to be narrower and much more contrasty^[7] than the power resonances due to the "Lamb" burning of the line. All this will apparently make it possible to develop, on the basis of a ring laser, a frequency standard with a reproducibility not lower than several times 10^{-12} .

2. THEORY

Some ring-laser generation regimes for the case of a two-component medium made up of amplifying and absorbing phases were considered in $^{[6]}$. For certain purposes, however, it is more successful to use many-component media with different natural frequencies. (In particular, we present below the results of experimental investigations of a laser with three media, two amplifying and one absorbing.)

For a multicomponent medium, the amplitudes ${\bf E}_i(t)$ of the travelling waves of the field $\langle \mathcal{E}\rangle$ of the laser

$$\mathscr{E}(x, t) = E_1(t)\cos(\omega t + \varphi_1(t) - kx) + E_2(t)\cos(\omega t + \varphi_2(t) + kx)$$
(1)

satisfy, in an approximation cubic in E_i, the following

system of equations:

$$E_{i} + \frac{\Delta \omega}{2} E_{i} = \sum_{m} \frac{\varkappa_{+m}}{2} \left\{ 1 - s_{m}^{2} \alpha_{m} E_{i}^{2} - s_{m}^{2} \beta_{m} E_{j}^{2} \right\} E_{i} + \sum_{n} \frac{\varkappa_{-n}}{2} \left\{ 1 - s_{-n}^{2} \alpha_{-n} E_{i}^{2} - s_{-n}^{2} \beta_{-n} E_{j}^{2} \right\} E_{i},$$
(2)

where $\Delta\omega$ is the resonator line width, κ the gain of the weak field, $s^2 = 2\rho^2/\hbar\gamma^2$, ρ is the matrix element of the dipole moment, and $s^2\alpha$ and $s^2\beta$ are the saturation coefficients of the medium. The plus and minus signs of the coefficients pertain to amplifying and absorbing media, respectively.

Depending on the frequency deviation of the resonator from the amplification and absorption line centers, the system of equations describes the generation of a laser either at two waves of equal intensity (two-wave generation or the standing-wave regime, $E_i = E_j$), or with one travelling wave (single-wave or unidirectional generation, $E_i = 0$, $E_i \neq 0$).

The frequencies corresponding to the boundaries between the foregoing regimes are equal to the roots of the equation

$$\Phi(\omega) = \Phi_+(\omega) - \Phi_-(\omega) = 0,$$

$$\Phi_{+}(\omega) = \sum_{m} \mu_{+m} F_{+m}(\omega), \qquad \Phi_{-}(\omega) = \sum_{n} \mu_{-n} F_{-n}(\omega), \qquad (3)$$

where

$$\mu_{\pm k} = \left| \frac{\varkappa_{\pm k} s_{\pm k}^2}{\varkappa_{\pm i} s_{\pm i}^2} \right|, \quad F_{\pm k}(\omega) = 1 - \left(\frac{\gamma_{\pm k}^0}{k u_{\pm k}} \right)^2 - \left[1 + \left(\frac{\omega - \Omega_{\pm k}}{\gamma_{\pm k}} \right)^2 \right]^{-1}.$$

The condition (3) and the form of the functions $F_{\pm k}(\omega)$ can easily be obtained by taking into account the explicit dependence of the coefficients α and β of the system (2) on the homogeneous γ_k and Doppler ku_k line widths, on the reciprocal lifetime γ_k^0 of the working levels, and on the central transition frequency Ω_k of the k-th component of the laser medium.

When $\Phi(\omega) < 0$, the generation regime of the ring laser is unidirectional, and when $\Phi(\omega) > 0$ the laser generation is with two travelling waves. Figures 1a and 2a show plots of the functions $\Phi_{-}(\omega)$ and the possible form of the function $\Phi_{+}(\omega)$, illustrating the laser generation in the case of a single-component absorbing medium and a single-component (Fig. 1a) or two-component (Fig.2a) amplifying medium. The dependence of the intensity of the travelling waves on the radiation frequency, corresponding to the form of the function $\Phi(\omega)$, is also shown



FIG. 1. Theoretical dependence (a) and oscillogram (b) of the radiation regime in the case of single-component amplifying and absorbing media, c-structure of radiation peak.

FIG. 2. Theoretical dependence (a) and oscillogram (b) of the amplification regimes in the case of single-component absorbing and twocomponent amplifying media.

in Figs. 1a, b, c and 2b. The region Δ of the variation of Ω_{+k} , in which the peak of the radiation can be observed at the central frequency of the absorbing gas, is determined essentially by the form of the curve $\Phi_{+}(\omega)$. In our case, for a two-component active medium, the region Δ of the existence of radiation resonance (Fig. 2a) is much larger than the corresponding region Δ (Fig. 1a) of a single-component medium. With changing partial pressures of the laser active components, changes take place in the line widths and their central frequencies shift (impact broadening and line shift). At the same time, the form of the functions $\Phi_{+}(\omega)$ and $\Phi_{-}(\omega)$ changes in the region of the central frequencies of the amplifying components, within which a narrow power peak can appear at $\omega \simeq \Omega_{-}$ at one of the travelling waves of the laser, and a dip of corresponding width and amplitude can appear at the other wave.

Let us now examine the causes of the narrowing of the peak (dip) in the power of the travelling waves of a ring laser. We shall assume that the central frequency of the absorbing gas Ω_{-1} is close to one of the roots ω_{+1} of the equation $\Phi_{+}(\omega_{+1}) = 0$. For a laser with only active components, the frequencies ω_{+1} separating the qualitatively different radiation regimes (the regimes of unidirectional and two-directional generation), are the bifurcation frequencies. Thus, at generation frequencies $\omega \approx \omega_{+1}$ the active medium is indifferent to the form of the laser field. If, at the same time, the resonator contains an absorbing component with a central frequency Ω_{-1} close to ω_{+1} , then the resultant form of the field (in the vicinity of $\omega \sim \Omega_{-1}$) will be determined only by

the properties of the passive medium. As mentioned above, at a frequency close to the central frequency of the transition, generation of one travelling wave is energywise favored in an active medium, and conversely, generation of two oppositely travelling waves is favored in a passive medium. Therefore, when $\omega_i \approx \omega_{+i}$, for frequencies ω close to Ω_{-1} , a power peak in one of the travelling waves should appear in the laser radiation (and a corresponding dip in the other), corresponding to a transition from the single-wave generation regime to the two-wave regime. Let us estimate the width of the power resonances of the travelling waves. It is natural to assume that the homogeneous widths γ_{+k} of the amplifying components are much larger than the homogeneous width of the absorber line γ_{-1} . Recognizing, in addition, that the characteristic scales of variation of the functions $\Phi_+(\omega)$ and $\Phi_-(\omega)$ are as determined by the quantities γ_{rk} and γ_{-1} , respectively, we find that Eq. (3), subject to the condition $\omega \approx \omega_{+1}$, $\omega \approx \Omega_{-1}$, reduces to the equation

$$\Phi_{-}(\omega) = 1 - \left(\frac{\gamma_{-1}^{0}}{ku_{-1}}\right)^{2} - \left[1 + \left(\frac{\omega - \Omega_{-1}}{\gamma_{-1}}\right)^{2}\right]^{-1} = 0.$$
(4)

The frequencies at which $\Phi_{-}(\omega)$ vanishes thus determine the width $2 \Delta \Omega$ of the laser power resonances:

$$\Delta\Omega \sim \gamma_{-1}(\gamma_{-1}^{0}/ku_{-1}).$$
 (5)

Inasmuch as in an absorbing gas the condition $\gamma_{-1}^0/ku_{-1} \ll 1$ is usually satisfied, the half-width $\Delta\Omega$ of the power peak should be much narrower than the homogeneous width γ_{-1} .

It should be noted that observation of a power peak with a half-width given by (5) involves definite difficulties. Principal among them are the stabilization of the pressure of the amplifying phase of the laser and the elimination of the back reflection of energy from the generator elements.

Stabilization of the pressure of the amplifying phase is equivalent in fact to maintenance of one of the bifurcation frequencies ω_{+1} close to a central frequency of the absorbing gas Ω_{-1} . If at the same time the function $\Phi_+(\omega)$ for $\omega = \Omega_{-1}$ not only vanishes but is also minimal, then the width of the peak does not depend on the sign of the difference $\omega_{+1} - \Omega_{-1}$. On the other hand, if $\Phi_+(\omega)$ does not have a minimum at the point ω_{+1} , then the peak in the radiation should be observed only at a definite sign of the difference $\omega_{+1} - \Omega_{-1}$. In both cases, to obtain narrow resonances it is necessary to let ω_{+1} tend to Ω_{-1} ; the dynamics of the experimentally observed narrowing of the resonances with a change of the difference $\omega_{+1} - \Omega_{-1}$ is illustrated in Fig. 3.

Elimination of back reflections of energy from the generator element excludes the effect of the "mixing" of the travelling waves, which hinders the regime of unidirectional radiation of the ring laser. In the ideal case, when there are no back reflections at all, the power peaks should have a 100% contrast. In the experiment described below, no special measures were taken to eliminate the back reflections. The latter, apparently, was the reason for the 50% contrast in the radiation peaks (Fig. 2). It should be noted that in this case the contrast of the ring-laser power peaks greatly exceeded the contrast of the resonances of a laser with a Fabry-Perot resonator.^[2, 3]







FIG. 3. Narrowing of the power resonances of the travelling waves with changing pressure of the He-Ne mixture.

3. EXPERIMENTAL SETUP

The experiments were performed with an He-Ne laser at $\lambda = 3.39 \,\mu$. The experimental setup is shown in Fig. 4. A laser with a Fabry-Perot interferometer and a laser with a ring resonator were placed on a massive iron plate, which served to protect the laser resonator against external vibrations. The plate was decoupled from mechanical oscillations with the aid of shock absorbers. Both lasers were connected to vacuum and gas-filling systems. The resonators consisted of flat mirrors, making it possible to work at the lowest mode. The perimeter of the ring laser was $\sim 10^2$ cm, and the length of the Fabry-Perot resonator in our experiments was 70 cm. Both lasers operated with continuous flow of gas consisting of a mixture of Ne-He with a partialpressure ratio 1:10 and 1:20. The mixture pressure was measured with an oil manometer. The discharge was excited in the amplifying tube by means of a definite combination of high-frequency and direct currents.

The absorbing cells in both lasers were 37 cm long, had an inside diameter of 2 cm, and were connected to the vacuum and gas-filling systems. The pressure of the methane gas ranged from 10^{-3} to 10^{-1} Torr. One of the mirrors in each laser was fastened to a piezoceramic, to which a voltage from a sawtooth generator was applied. The radiation of each of the waves of the ring laser and of the laser with the Fabry-Perot resonator was extracted through one of the mirrors and was registered with a Ge-Au photoreceiver cooled with liquid nitrogen. The signal from the photoreceiver was fed to the vertical input of an oscilloscope. The horizontal sweep of the oscilloscope was by means of the signal from the generator controlling the piezoceramic. As a result, the oscilloscope screen revealed the dependence of the output laser power on the generation frequency at the standing and travelling waves. Such an experimental setup has made it possible to study simultaneously the power resonances in the Fabry-Perot-resonator laser and in the ring laser.

4. RADIATION REGIMES OF RING LASER FOR SINGLE COMPONENT AMPLIFYING AND ABSORBING MEDIA

The amplifying tube with the Ne²⁰ isotope, having an active length ~25.0 cm, and the absorbing cell with CH₄ were placed in two arms of the ring laser. At a CH₄ pressure ~ 2.5×10^{-2} Torr, the radiation peak was ob-

served in the He-Ne mixture pressure range (partial composition 1:10) from 3.5 to 5.2 Torr. A typical oscillogram of the radiation is shown in Fig. 1b. The contrast of the power peak was of the order of ~20%. The fine structure of the peak is illustrated in Fig. 1c. Figure 1a shows the theoretically obtained dependence of the intensity of the laser travelling waves. Good agreement between the theoretical and experimental results is seen.

5. RING-LASER RADIATION REGIMES FOR A TWO-COMPONENT AMPLIFYING MEDIUM AND A SINGLE-COMPONENT ABSORBING MEDIUM

Two amplifying tubes, one with the Ne²⁰ isotope and operating with continuous flow, and the other with the Ne²² isotope and sealed off at a pressure 1.5 Torr, with the He-Ne mixture partial pressures having a ratio 1:10, were placed in two arms of the resonator. In the third arm there was a CH4 cell with a methane pressure $\sim 2 \times 10^{-2}$ Torr. In view of the fact that the natural frequencies of Ne²⁰ and Ne²² are different (the frequency distance between them is $\sim 63 \text{ MHz}^{[8]}$, the peak in the radiation of the ring laser exists in a wider interval of He-Ne mixture pressures. This is connected mathematically, as indicated above, with the deformation of the function $\Phi_{+}(\omega)$ (see Fig. 2a). In our experiment, the radiation peak was observed at pressures from 1 to 14 Torr. A typical oscillogram of the power peak in one of the travelling waves and of the dip in the other, at the central frequency of CH_4 , is shown in Fig. 2b. It is of interest to point out the high contrast of the resonances: the power of the peak (dip) was 50% of the power of the wave.



FIG. 4. Experimental setup: 1-power supply, 2-vacuum meter, 3generator (f = 50 Hz), 4-Ge-Au receiver, 5-diaphragm, 6-piezoceramic, 7-forevacuum pump, 8-oil manometer, 9-amplifier, 10-reductor, 11diffusion pump, 12-nitrogen trap, A, B, C-flat mirrors.

6. NARROWING OF THE POWER RESONANCES OF THE RING LASER

One amplifying tube, operating with continuous flow of the natural mixture of Ne, and the absorbing cell were placed in two arms of the resonator. The methane pressure in the absorbing cell was $\sim 1.6 \times 10^{-2}$ Torr. The pressure of the He-Ne mixture ranged from 3.2 to 4.7 Torr. The observed narrowing of the power resonances of the travelling waves is shown in Fig. 3. We see that a change of pressure by ~ 1.5 Torr greatly narrows down the width of the resonance. The resonances shown in Figs. 4a, b, c have half-widths ~ 500 , ~90, and ~30 kHz, respectively. The half-width of the resonance of the laser with the Fabry-Perot interferometer and CH₄ absorbing cell, as measured by us, was close to 300 kHz. In this case, a tenfold narrowing of the power peak of the ring laser was observed, compared with the width of the resonance due to the "Lamb" saturation of the methane absorption line. We note that if the ring-laser parameters are optimized, the power resonances of the travelling waves will apparently be narrowed down by one more order of magnitude.

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Translated by J. G. Adashko 16