EXPANSION OF A PLASMA IN VACUUM AND FLOW OF A COLLISIONLESS PLASMA AROUND A PLATE

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The problem of the flow of a supersonic stream of plasma around a plate is reduced to the problem of the expansion of a plasma in a vacuum, and is solved numerically in a one-dimensional formulation with allowance for the Poisson equation. The problem of expansion of a non-isothermal collisionless plasma in vacuum is compared with the self-similar solution[1]. An experimental investigation is made of the structure of the perturbed zone in the wake of a plate moving through a stream of a non-isothermal plasma with a velocity $M = 2$ in the absence of a magnetic field. The density and potential distributions behind the plate are obtained. The results of the experiments and of the calculations are in good agreement.

THE escape of a plasma into a vacuum, and also the associated process of flow of plasma around bodies, are undoubtedly of practical interest and have been recently under intense study. This question was considered in sufficient detail theoretically in[1,2]. In the present paper we report numerical calculations and experiments on the flow of a supersonic collisionless plasma ($T_e \gg T_i$) around a plate. The results of the experiments are compared with numerical calculations, in which, unlike[1,2], the equations of motion of the ions and the Poisson equation are numerically integrated. As follows from the present results, allowance for the Poisson equation gives a picture of escape and flow that is different from the results of[1], where the escape time is short.

In the general case, the solution of our problem is quite complicated, and therefore certain simplifications which follow from physical considerations were made. A Boltzmann distribution was assumed for the electrons, $n_e = n_0 \exp(e\varphi/T_e)$. The ion density was determined as usual, $n_i = \int f(\mathbf{r}, \mathbf{v}, t)d\mathbf{v}$, where f is the ion distribution function determined from the kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{e}{m_i}\frac{\partial \varphi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0. \qquad (1)$$

The two-dimensional stationary problem of flow around a plate that is infinite in the y direction can be reduced to a one-dimensional nonstationary problem of the expansion of plasma in vacuum[1]. This assumption is valid if $d^2\psi/dz^2 \ll d^2\psi/dx^2$ (the z coordinate is directed along the flux and ψ is the dimensionless potential). This condition is expressed for our case by the relation $M^2 \gg \psi$. Here M is the plasma stream velocity in Mach numbers $M = v_z/v_s$, $v_s = \sqrt{T_e/M_i}$. The problem reduces to simultaneous solution of Equation (1) and of the Poisson equation:

$$d^2\varphi / dx^2 = 4\pi e (n_0 \exp(e\varphi/T_e) - n_i). \qquad (2)$$

We used for the solution a method analogous to that described in[3]. In this method, the ions are represented in the form of quasiparticles. The potential was determined not by a continuous function, but by a grid-like function that assumes discrete values at the nodes of the grid. Equation (2) is replaced by a difference equation and the ion density at the node j is determined by

$$n_j = \sum_k [H(x_k - x_j) - H(x_k - x_{j+1})] n_0,$$

where $H(x)$ is the Heaviside function, n_0 is the concentration of the particles in the layer, and x_k is the coordinate of the center of the particle.

Using the results of[4], we introduced in place of Eq. (2) a second equation, the solution of which as $t \to \infty$ is the solution of (2). We used 1872 particles in the calculation. In the cold plasma, the velocities were assumed to be equal to zero, and for a hot plasma ($T_i \neq 0$) a random-number generator produced thermal noise corresponding to the required ion temperature. All the quantities were reduced to dimensionless form:

$$\tau = \omega_{pi} t, \quad \psi = e\varphi / T_e, \quad w = v / v_s.$$

The described method was used to solve the problem of flow of a plasma stream around a plate under conditions that were as close as possible to the experiment described below. The initial distribution of the density is shown in Fig. 5a, $z = 0$ (see below). The symmetry condition for flow around a plate was satisfied by specifying $d\varphi/dx$ equal to zero on the right-hand boundary ($x = 0$ in Fig. 5a; see below), and by taking into account the ions reflected from the boundary. The coordinate z for the points in the wake of the body was determined from the formula $z = M\tau$, where τ is the dimensionless time. In the calculation model, at the initial instant of time, the ions were uniformly distributed in an interval of length equal to 13 Debye lengths. The dimension of the initial distribution of the ions was chosen to be sufficiently large so as to exclude the influence of the left-hand boundary on the results of counting, and the ion density near the point $x = 20$ differed little from the initial value at all instants of time. The width of the region free of ions was chosen from the concrete experimental conditions and equalled 7D in this case ($D = [T_e/(4\pi n_0 e^2)]^{1/2}$).

The results of the calculation for $T_e/T_i = \infty$ and $T_e/T_i = 20$ are given in Fig. 1 (equal-concentration

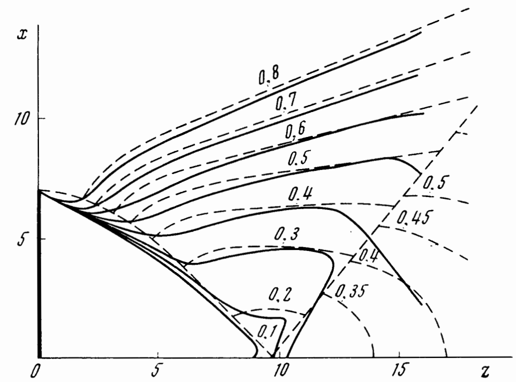

FIG. 1. Surfaces of constant concentration (numbers on the curves) of ions in the case of flow around a plate: dashed curves—$T_e/T_i = \infty$; solid curves—$T_e/T_i = 20$.

lines). The scales of the axes were chosen to be in units of Debye lengths. The plot was constructed for the case M = 2. We see that the distribution of the density in the wake of the body, for a non-isothermal plasma, has a steep leading front, and the presence of thermal scatter of the ions leads to a smearing of the front. The front velocity increases on approaching the axis of the plate, and the amplitude decreases. Starting with a certain distance z, the motion acquires a two-velocity character, corresponding in the calculation model to a reflection of the particles from the right-hand boundary, and under real conditions to the appearance of particles that travel from the opposite edge of the plate.

It is of interest to compare the results of a machine experiment with the self-similar solution obtained for the spreading of a half-space of a plasma into a vacuum or, what is the same (in a certain approximation), for the flow of plasma around a half-plane[1]. Owing to limitations on the capabilities of the electronic computer, we chose the plasma dimensions to be 20D and those of the free space to be 20D. This has made it possible to carry out comparisons down to a concentration level ~ 0.1. The calculation was carried out, just as in the preceding case, for a symmetrical geometry. As can be seen from Fig. 2, the main deviation from the self-similar solution lies in the fact that the motion occurs with a steep leading front. Behind the front, at

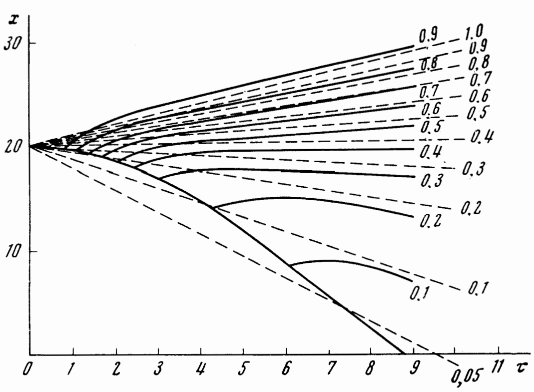

FIG. 2. Surfaces of constant concentration (numbers in the figures) of ions in the case of escape into a vacuum: $T_e/T_i = \infty$; Dashed curves—self-similar solution.

a distance on the order of 2—3D, the solution approaches asymptotically the solution obtained in[1]. The calculation was carried out for the case $T_e/T_i = \infty$.

The experiments were performed with the "Volna" apparatus, a diagram of which is shown in Fig. 3. Gas was injected from the end of a vacuum volume of 80 cm diameter and 150 cm length with the aid of a pulsed injector 1. An incandescent cathode 2 surrounded by a metallic grid was placed in the immediate vicinity of the entry of the gas. The grid was at a positive potential relative to the cathode. The escaping gas (xenon was used in the experiments) was ionized by a stream of accelerated electrons from the cathode. The profile of the escaping gas had a front ~ 10 cm and a velocity $\approx 4 \times 10^4$ cm/sec. Owing to the large pressure drop of the gas along the axis of the volume (the residual pressure in the volume was 10^{-6} Torr, the pressure in the injection region was $\sim 10^4$ Torr), ionization takes place in the region occupied by the gas. The plasma produced in this manner flows into a region of the volume where there is no gas. The parameters of the plasma, measured in the region where the experiments were performed, were $n_e \approx (1-3) \times 10^7$ cm^{-3} and $T_e \approx 5-8$ eV. Within a time of 1 msec from the instant of gas injection, the plasma flow assumes a quasistationary character, and for 100—150 microseconds the potential and the velocity of the plasma remained constant.

In the experiments, we investigated flow around a metallic plate 4, of width 2R = 5.4 cm (14 Debye radii) and length l = 40 cm, placed at a distance ~ 120 cm from the cathode, in a place where the plasma stream velocity changed little with distance. The large ratio 1/R has made it possible to exclude the influence of the ends. In all the experiments, the plate had a floating potential. The measurement of the plasma density distribution in the wake of the plate was carried out with the aid of a flat probe 5 of 5 mm diameter, working in the saturation ion-current regime. The plasma potential was measured by the incandescent-probe method. Owing to the relatively small resistance of the plasma-probe junction and owing to the large resistance of the output divider, the probe potential was close to the plasma potential. To ensure the necessary time resolution, the filament supply circuit of the probe 6 was disconnected during the measurement time. The method of securing the plate and of the probes has made it possible to move the plate in the direction perpendicular to the plasma stream, and the probes in a direction along the stream, by the same token ensuring the possibility of measuring the profile of the density behind the plate in the xz plane. The electron temperature and density were measured with a Lang-

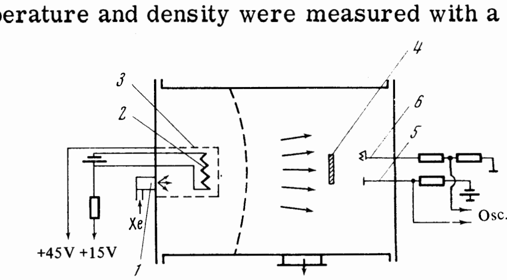

FIG. 3. Diagram of experimental setup.

muir probe with a logarithm-determining element. The transverse temperature of the ions (along the x axis) was determined with a grid energy analyzer and amounted to 0.3–0.5 eV. The probe was mounted in such a way that the grid and the analyzing collector were parallel to the plasma stream.

Figure 4 shows the experimental dependence of the plasma velocity in Mach numbers on the coordinate z. The M(z) dependence was obtained by measuring the total energy of the plasma ions with the energy analyzer and measuring the plasma potential at the same point with the incandescent probe. In addition, the velocity of the plasma stream at several points was measured by determining the velocity of the ion-acoustic wave of small amplitude. As seen from Fig. 4, in that region of the volume where the experiments were performed the plasma flow velocity changed little and reached ~ 2M.

Figure 5 shows successive profiles of the ion density behind the plate (in steps of $\Delta z = 4$ D), normalized to the unperturbed plasma density, for different distances from the plate. The x axis shows the distances in units of Debye lengths. The dashed and the solid lines represent here the calculated curves obtained by numerical methods for the cases $T_e/T_i = \infty$ and $T_e/T_i = 20$, respectively. The points denote the experimental data. It is seen from Fig. 5a that at small values of z ($z < 8$), the plasma moves, just as in the calculation, with a rather steep front, slightly smeared out as a result of thermal motion of the ions. On the boundary between the two streams traveling from different edges of the plates, an ion-density jump is produced (Fig. 5c). At large values of z, the jump spreads out gradually and the density distribution assumes a smooth character (Fig. 5d).

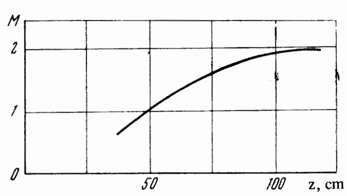

FIG. 4. Experimental dependence of the plasma velocity (in Mach numbers) on the distance to the source.

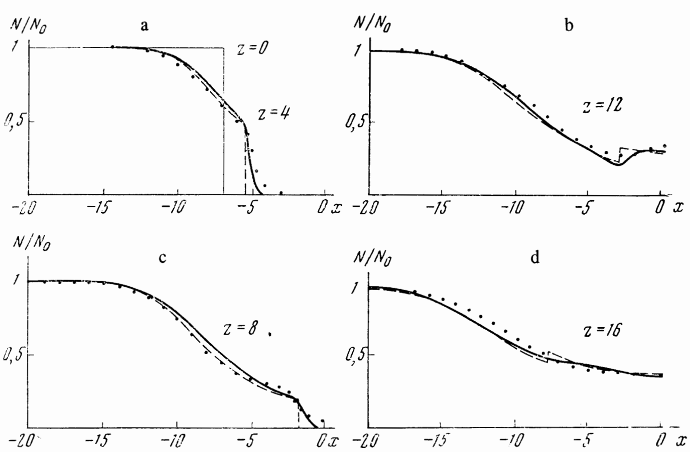

FIG. 5. Concentration of ions behind a plate with R = 7D as a function of x for different z. Points–experiment; dashed lines ($T_e/T_i = \infty$) and solid lines ($T_e/T_i = 20$)–calculation.

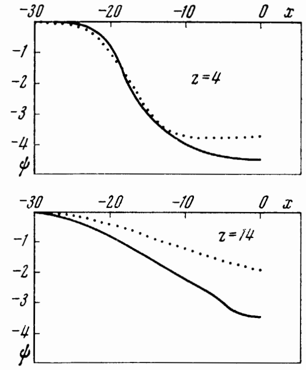

FIG. 6. Potential of plasma behind a plate with R = 20D as a function of x for different z. Points–experiment; solid curve–calculation ($T_e/T_i = \infty$).

An experiment was performed on flow around a plate with dimension 2R = 40D. The profiles of the potentials at distances z = 4 and z = 14 in the wake of the body are shown in Fig. 6. The figure also shows for comparison the calculated curves obtained for the case $T_e/T_i = \infty$. The essential difference between the calculation and the experiment can be attributed to violation of the condition $M^2 \gg \psi$, that is, to the fact that the one-dimensional solution is not involved in this case.

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