THE DENSITY OF STATES FOR EXCITATIONS IN THIN SUPERCONDUCTING FILMS IN A MAGNETIC FIELD

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An expression is obtained for the density of states of the excitations in thin pure films in a magnetic field satisfying the condition $eHd^2 \ll 1$ (d is the thickness of the film). The effect of impurities on the excitation spectrum of thin current-carrying films is investigated for specular reflection of the electrons by the boundary.

1. INTRODUCTION

THE tunneling and high-frequency characteristics of thin films are determined by the shape of the excitation spectrum. In this connection an investigation of the excitation spectrum of thin films is of great interest. The excitation spectrum has been studied by different authors both for specular and for diffuse reflection of the electrons from the walls. The general shape of the excitation spectrum for specular reflection was obtained in [1]. It was shown that in this case the excitations are characterized by quasimomenta, where in sufficiently strong magnetic fields the gap in the excitation spectrum vanishes for certain directions of the quasimomentum. A strong change in the excitation spectrum appears even in weak fields $evHd \sim \Delta$ (v denotes the velocity of the electrons at the Fermi surface, and d is the film's thickness) and is associated with electrons which are moving at small angles to the surface of the film. In fields $evHd \sim \Delta$ the ordering parameter Δ does not depend on the coordinates. The spatial dependence $\Delta(\mathbf{r})$ appears only in fields $eHd^2 \sim 1$, i.e., in fields of the order of the critical field as $T \to 0$. Below an exact solution of the system of equations for the Green's function of thin pure films in a magnetic field satisfying the condition $eHd^2 \ll 1$ is obtained under the assumption $\Delta = \text{const}$, and the density of states is found in different limiting cases. The reflection of the electrons at the boundary of the sample was assumed to be specular.

The density of states and the gap in the excitation spectrum substantially depend on the mean free path of the electrons. The problem of a thin current-carrying film in the absence of an external magnetic field is considered in order to qualitatively investigate the effect of impurities on the excitation spectrum. An expression is obtained for the gap in the excitation spectrum and for the density of states near threshold.

2. DENSITY OF STATES OF THE EXCITATIONS IN A MAGNETIC FIELD

In the quasiclassical approximation the system of equations for the Green's function at coincident points

$$\hat{G}_{\mathbf{p}}(\mathbf{r}) = \frac{i}{\pi}\int \hat{G}_{\mathbf{p}}(\mathbf{r}, \xi)\, d\xi$$

has the form[2]

$$\left(\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)\hat{G}_{\mathbf{p}}(\mathbf{r}) + \hat{\omega}\hat{G}_{\mathbf{p}}(\mathbf{r}) - \hat{G}_{\mathbf{p}}(\mathbf{r})\hat{\omega} = 0,$$

$$\hat{\omega} = \omega\tau_z - ie(\mathbf{vA})\tau_z - i\hat{\Delta} + in\Sigma_{\mathbf{pp}}(\mathbf{r}), \tag{1}$$

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix}, \quad \Sigma_{\mathbf{pp'}} = \chi_{\mathbf{pp'}} - \frac{i\vartheta}{4}\int \chi_{\mathbf{pp}_1}\hat{G}_{\mathbf{p}_1}(\mathbf{r})\Sigma_{\mathbf{p}_1\mathbf{p'}}\, d\Omega_{\mathbf{p}_1},$$

$$\hat{G}^2_{\mathbf{p}}(\mathbf{r}) = 1, \quad \mathrm{Sp}\hat{G}_{\mathbf{p}}(\mathbf{r}) = 0, \tag{1'}$$

where v is the velocity of the electrons at the Fermi surface, $\theta = mp_0/2\pi^2$ is the density of states at the Fermi surface, n is the concentration of impurities, and $\chi_{\mathbf{pp'}}$ is related to the scattering amplitude by the relation

$$f_{\mathbf{pp'}} = \chi_{\mathbf{pp'}} - \frac{i\vartheta}{4}\int \chi_{\mathbf{pp}_1} f_{\mathbf{p}_1\mathbf{p'}}\, d\Omega_{\mathbf{p}_1}. \tag{2}$$

We shall confine our investigation to a thin, pure film of thickness $d \ll \xi_0, \delta$ (δ is the penetration depth, ξ_0 is the coherence length of the superconductor) in a magnetic field satisfying the condition

$$eHd^2 \ll 1. \tag{3}$$

We choose a coordinate system with the y-axis along the magnetic field and with the z-axis across the film. $z = \pm d/2$ at the film boundaries. For $d \ll \delta$ one can neglect the magnetic field of the superconducting currents and choose the vector potential $\mathbf{A}$ in the form

$$A_x = Hz, \quad A_y = A_z = 0. \tag{4}$$

The ordering parameter Δ is real in such a gauge for the vector potential $\mathbf{A}$, and by virtue of condition (3) it does not depend on the coordinates. In the absence of impurities and for $\Delta = \text{const}$ the system (1) can be solved exactly, and the expression for the Green's function has the form

$$\hat{G}_{\mathbf{p}}(\mathbf{r}) = f_1\tau_z + 2^{-1/2}f_2(\tau_y + i\tau_x) + 2^{-1/2}f_3(\tau_y - i\tau_x), \tag{5}$$

where

$$f_1 = B_1(\Phi\Phi^* + {}^1/_2\nu^2t^2\Phi_+\Phi_+^*) - B_2\nu t e^{t^2/4}\Phi^*\Phi_+^* + B_3\nu t e^{-t^2/4}\Phi\Phi_+,$$

$$f_2 = -B_1\nu t\Phi^*\Phi_+ + B_2 e^{t^2/4}(\Phi^*)^2 - {}^1/_2 t^2\nu^2 e^{-t^2/4}\Phi_+^2 B_3,$$

$$f_3 = B_1\nu t\Phi\Phi_+^* - {}^1/_2 B_2\nu^2t^2 e^{t^2/4}(\Phi_+^*)^2 + B_3 e^{-t^2/4}\Phi^2; \tag{6}$$

$$t = \frac{\sqrt{2}(1+i)}{x}\left(\frac{\omega}{v} - ieHz(1-x^2)^{1/2}\cos\varphi\right)$$

$$\times\left(\left|\frac{eH(1-x^2)^{1/2}\cos\varphi}{x}\right|\right)^{-1/2}\exp\left[-\frac{i\pi}{4}(2 - \operatorname{sign} x - \operatorname{sign}\cos\varphi)\right],$$

$$\nu = \frac{(1+i)\Delta}{2evH(1-x^2)^{1/2}\cos\varphi}\left(\left|\frac{eH(1-x^2)^{1/2}\cos\varphi}{x}\right|\right)^{1/2}.$$

$$\times \exp\left[\frac{i\pi}{4}(2-\operatorname{sign}x-\operatorname{sign}\cos\varphi)\right], \quad x=\cos\theta, \quad (7)$$

$\Phi \equiv \Phi(\nu^2/2, 1/2, t^2/4)$, and $\Phi_+ \equiv \Phi(1+\nu^2/2, 3/2, t^2/4)$ are confluent hypergeometric functions. The * sign indicates the replacement of t^2 by $-t^2$ and ν^2 by $-\nu^2$. The coefficients B_i are determined from the boundary conditions and satisfy the normalization condition (1′). From formulas (5) and (6) and from the normalization condition (1′) it follows that

$$B_1^2 + 2B_2B_3 = 1. \quad (8)$$

We shall confine our examination to the case of specular reflection of the electrons from the walls. In this case the boundary conditions for the Green's function have the form

$$\hat{G}(z=\pm d/2, x) = \hat{G}(z=\pm d/2, -x). \quad (9)$$

From formulas (8) and (9) we obtain the following expression for the coefficients B_i:

$$B_1^2 = \left\{1 + 2\frac{(\gamma_1\alpha_2-\gamma_2\alpha_1)(\gamma_1\beta_2-\gamma_2\beta_1)}{(\alpha_1\beta_2-\alpha_2\beta_1)^2}\right\}^{-1} \quad (10)$$

$$B_2 = B_1\frac{\gamma_1\beta_2-\gamma_2\beta_1}{\alpha_1\beta_2-\alpha_2\beta_1}, \quad B_3 = B_1\frac{\gamma_1\alpha_2-\gamma_2\alpha_1}{\alpha_1\beta_2-\alpha_2\beta_1},$$

where

$$\alpha_{1,2} = \exp(t_{1,2}^2/4)\,[(\Phi^*)^2 + {}^1/_2\,\nu^2 t_{1,2}^2(\Phi_+^*)^2], \quad \beta_{1,2} = \alpha_{1,2}^*,$$

$$\gamma_{1,2} = \nu t_{1,2}(\Phi\Phi_+^* + \Phi^*\Phi_+), \quad t_{1,2} = t(z=\mp d/2). \quad (11)$$

Formulas (5)–(11) completely determine the Green's function of a pure film in an arbitrary magnetic field. The density of states is expressed in terms of the Green's function by the formula

$$\rho = (2d)^{-1}\rho_0 \operatorname{Im}\{(i/4\pi)\operatorname{Sp}\tau_z\int_{-d/2}^{d/2} dz \int \hat{G}_p(-i\omega)\,d\Omega_p\}, \quad (12)$$

where ρ_0 is the density of states in the normal metal, and $\hat{G}_p(-i\omega)$ is the analytic continuation of the function $\hat{G}(\omega)$ with the values $\omega = \pi T(2n+1)$ on the imaginary axis. From formulas (5), (6), and (12) we obtain the following expression for the density of states:

$$\rho = \frac{2}{\pi d}\rho_0 \operatorname{Re}\int_0^1 dx \int_0^{\pi/2} d\varphi \int_{-d/2}^{d/2} dz\,\{B_1(\Phi\Phi^* + {}^1/_2\,\nu^2 t^2 \Phi_+\Phi_+^*)$$
$$+ \nu t(B_3 e^{-t^2/4}\Phi\Phi_+ - B_2 e^{t^2/4}\Phi^*\Phi_+^*)\}. \quad (13)$$

First of all let us find the gap in the excitation spectrum. From formulas (10) and (13) it follows that only the angular regions for which the real part of B_1 does not vanish give a contribution to the density of states, and the gap in the excitation spectrum is determined from the condition $B_1 = \infty$, which has the form

$$(\alpha_1\beta_2-\alpha_2\beta_1)^2 + 2(\gamma_1\alpha_2-\gamma_2\alpha_1)(\gamma_1\beta_2-\gamma_2\beta_1) = 0. \quad (14)$$

For $|t_{1,2}| < 2\sqrt{2}\,|\nu|$ Eq. (14) does not have any solutions, and in this region B_1 is pure imaginary. The left-hand side of Eq. (14) first vanishes for $|t_1| = 2\sqrt{2}\,|\nu|$, $x = 0$, $\varphi = 0$. From here we obtain the following expression for the gap ϵ in the excitation spectrum:

$$\varepsilon = \Delta - evHd/2. \quad (15)$$

Thus, the gap in the excitation spectrum of a superconductor in the presence of a magnetic field is decreased by an amount of the order of the interaction energy of the electrons with the magnetic field, and it vanishes for $evHd = 2\Delta$. We also note that threshold excitations appear due to the electrons traveling parallel to the film's boundary.

Formula (13) gives a general expression for the density of states associated with an arbitrary magnetic field. However, this expression is very complicated; therefore we shall investigate it in different special cases.

1) $|\omega - \Delta| \ll \Delta$. In this case one can show that angles for which $|t|, |\nu| \ll 1$ give the major contribution to the density of states. Expanding expressions (10) and (13) in powers of t and ν and carrying out the angular integrations, we obtain

$$\rho = \frac{1}{\pi}\left(\frac{2\Delta}{|\omega-\Delta|}\right)^{1/2}\rho_0$$
$$\times\begin{cases}(1+z^2)^{-1/2}\ln(z+(1+z^2)^{1/2}), & \omega<\Delta,\\ (z^2-1)^{-1/2}\ln(z+(z^2-1)^{1/2}), & \Delta<\omega<\Delta(1+(eHd^2)^2/60),\\ (1-z^2)^{-1/2}\operatorname{arctg}(z^{-2}-1)^{1/2}, & \Delta(1+(eHd^2)^2/60)<\omega,\end{cases} \quad (16)$$

where $z = eHd^2\Delta^{1/2}(60|\omega-\Delta|)^{-1/2}$. In the region of weak fields, $evHd < \Delta$, the condition for the applicability of formula (16) turns out to be more stringent and has the form

$$|\omega-\Delta| \ll \Delta(evHd/\Delta)^4.$$

This is associated with the fact that expression (16) is not applicable near threshold, where angles for which $|t|, |\nu| \gtrsim 1$ give the major contribution to the density of states.

2) $evHd \gg \Delta$. In this case angles for which $x \sim eHd^2$ give the major contribution to the density of states. In this angular region $|\nu| \ll 1$, which makes it possible to expand all of the expressions determining the density of states in powers of ν. After simple transformations we obtain the following result from formulas (10) and (13):

$$\rho = \frac{eHd^2}{2\pi}\rho_0 \operatorname{Re}\int_0^{\infty}[x^2+(eHd^2/4)^2]^{-1}\left[1-\frac{\pi\Delta^2}{2\omega^2 x}\chi^2(x)\right]^{-1/2}dx, \quad (17)$$

where $\chi(x) = \cos x\,C(\sqrt{x}) + \sin x\,S(\sqrt{x})$, S(x) and C(x) are Fresnel integrals.[3] Expression (17) for the density of states can be obtained from the results of article [1].

Let us consider expression (17) in two limiting cases.

a) $\omega \ll \Delta$. In this case only the neighborhoods of the zeros of the function $\chi(x)$ give a contribution to the density of states. Expanding the integrand near the zeros of $\chi(x)$ and carrying out the integration, we obtain

$$\rho = (2\pi)^{-1/2}\rho_0\frac{\omega}{\Delta}eHd^2\sum_n x_n^{-3/2}|\chi'(x_n)|^{-1}, \quad (18)$$

where $\chi(x_n) = 0$. Carrying out the numerical summation in formula (18) we obtain

$$\rho = 0{,}34\rho_0\frac{\omega}{\Delta}eHd^2. \quad (19)$$

b) $|\omega - \Delta| \ll \Delta$. In this case the basic contribution to the density of states arises from the angular interval for which $x \ll 1$. Expanding the integrand in powers of x and performing the integration, we obtain an expression for the density of states which agrees with formula (16).

3. THE DENSITY OF STATES NEAR THRESHOLD ($\omega - \Delta + evHd/2 \ll evHd, \Delta$)

An investigation of the general expression (13) for the density of states near the edge of the spectrum turns out to be very complicated. In this case it is convenient to use the system of equations for the Green's function $\hat{G}_p(\mathbf{r}, \xi)$ which depends on the energy variable ξ:[2]

$$\left[-\xi + i\left(\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right) + \omega\tau_z + e(\mathbf{v}\mathbf{A})\tau_z + \hat{\Delta}\right]\hat{G}_p(\mathbf{r}, \xi) = 1. \quad (20)$$

One can show that the solution of Eq. (20) for a thin film with specular reflection at the boundaries is equivalent to finding the solution of this equation in an infinite space with a periodic potential $\mathbf{A}$ whose period is 2d.[4] As is well-known, a band structure of the energy levels arises in the presence of a periodic field. Near the edge of the spectrum ($\omega - \Delta + evHd/2 \ll evHd, \Delta$) excitations appear due to electrons moving at small angles to the film's surface. In this case the potential barriers between neighboring wells are high, which enables us, with an exponential degree of accuracy, to neglect the presence of neighboring wells and to consider the motion in a single well with the potential

$$A_x = H(d/2 - |z|), \quad A_y = A_z = 0. \quad (21)$$

It is easy to solve the system (20) with the potential (21), and we obtain the following expression for the Green's function $G_p(\mathbf{r}, \xi)$:

$$G_p(\mathbf{r}, \xi) = \frac{\alpha(\xi + \omega + ev_xHd/2)\exp(-i\xi z/v_z)}{v_z^2[V(-\beta/\alpha)(V'(-\beta/\alpha))^* + V^*(-\beta/\alpha)V'(-\beta/\alpha)]}$$

$$\times\left\{\varphi(z)\int_{-\infty}^{z}\varphi^*(-y)\exp\left(\frac{i\xi y}{v_z}\right)dy + \varphi^*(-z)\int_{z}^{\infty}\varphi(y)\exp\left(\frac{i\xi y}{v_z}\right)dy\right\}, \quad (22)$$

where

$$\varphi(z) = \begin{cases} V\left(\dfrac{z-\beta}{\alpha}\right), & z > 0, \\ C_1V^*\left(-\dfrac{z+\beta}{\alpha}\right) + C_2W^*\left(-\dfrac{z+\beta}{\alpha}\right), & z < 0, \end{cases}$$

$$V(y) = 2\int_0^\infty dt\cos\left(yt + \frac{t^3}{3}\right),$$

$$W(y) = i\left[\exp\left(\frac{2\pi i}{3}\right)V\left(y\exp\left(\frac{2\pi i}{3}\right)\right) - \exp\left(-\frac{2\pi i}{3}\right)V\left(y\exp\left(-\frac{2\pi i}{3}\right)\right)\right], \quad (23)$$

$$C_1 = (4\pi)^{-1}[V(-\beta/\alpha)(W'(-\beta/\alpha))^* + V'(-\beta/\alpha)W^*(-\beta/\alpha)],$$
$$C_2 = -(4\pi)^{-1}[V(-\beta/\alpha)(V'(-\beta/\alpha))^* + V'(-\beta/\alpha)V^*(-\beta/\alpha)],$$
$$\alpha = v_z^{2/3}\left[2ev_xH'\left(\omega + \frac{ev_xHd}{2}\right)\right]^{-1/3},$$
$$\beta = \frac{(\omega + ev_xHd/2)^2 - \Delta^2 + iev_xv_zH}{2ev_xH(\omega + ev_xHd/2)}.$$

To within the factor $2^{-1}\pi^{-1/2}$, the function V(y) coincides with the Airy function, which is tabulated in [5]. The complex conjugation sign in these formulas does not apply to the infinitesimal imaginary part of the frequency.

Integrating the Green's function over ξ and substituting it into formula (13), we obtain the following expression for the density of states:

$$\rho = -\frac{1}{\pi d}\rho_0$$
$$\times\operatorname{Im}\int d\Omega_p\frac{\alpha(\omega + ev_xHd/2)}{|v_z|[V(-\beta/\alpha)(V'(-\beta/\alpha))^* + V^*(-\beta/\alpha)V'(-\beta/\alpha)]}$$
$$\times\operatorname{Re}\int_0^\infty dz\,V\left(\frac{z-\beta}{\alpha}\right)\varphi^*(-z). \quad (24)$$

The imaginary part in formula (24) arises only because of going around the zeros of the denominator. Calculating the imaginary part in formula (24), we find

$$\rho = \frac{2\sqrt{2}}{5}\rho_0\frac{\Delta d}{v}\operatorname{Re}\left\{\left(\frac{\omega - \varepsilon}{evHd/2}\right)^{5/2} + \left(\frac{-\omega - \varepsilon}{evHd/2}\right)^{5/2}\right\}\sum_n(x_n^{-3} + y_n^{-3}), \quad (25)$$

where $\epsilon = \Delta - evHd/2$, and x_n and y_n are the zeros of the Airy function and its first derivative. Carrying out the numerical summation in formula (25), we obtain

$$\rho = 0{,}61\rho_0\frac{\Delta d}{v}\operatorname{Re}\left\{\left(\frac{\omega - \varepsilon}{evHd/2}\right)^{5/2} + \left(\frac{-\omega - \varepsilon}{evHd/2}\right)^{5/2}\right\}. \quad (26)$$

Formula (25) for the density of states is valid along the entire edge of the spectrum with the exception of the exponentially narrow region of small frequencies

$$\frac{\omega}{\Delta} \sim \left(1 - \frac{2\Delta}{evHd}\right)\exp\left\{-\frac{\pi^2}{8\sqrt{2}}\left(1 - \frac{2\Delta}{evHd}\right)^{-3/2}\right\}.$$

This is associated with the fact that the method used to derive formula (25) is not applicable when the energy of the excitations becomes of the order of the width of the band. Formula (25) is also invalid immediately on the boundary of the spectrum, where small angles for which $(\mathbf{v}\cdot\partial/\partial\mathbf{r}) \sim \partial^2/\partial\mathbf{r}^2$ are important. In this angular interval the quasiclassical approximation, which was used to derive formula (20), is not applicable. A simple estimate leads to the following restriction on the range of validity of formula (25):

$$\omega - \varepsilon \gg \Delta(v^2e^2H^2/p_0\Delta^2)^{2/5}.$$

4. THE EFFECT OF IMPURITIES ON THE GAP IN THE EXCITATION SPECTRUM

The expressions obtained above for the density of states and for the gap in the excitation spectrum are valid only in the case of a pure film. In the presence of impurities both the gap in the excitation spectrum and the density of states are modified. In order to qualitatively investigate the effect of impurities on the excitation spectrum, let us consider the simpler problem of a thin film carrying current in the absence of any external magnetic field. In this case the vector potential $\mathbf{A}$ is directed along the film and does not depend on the coordinates. In what follows we shall assume that the impurity scattering is isotropic, and the reflection at the boundaries of the sample is specular. In this case the Green's function does not depend on the coordinates and has the form

$$\hat{G}_p(\mathbf{r}) = \alpha(\mathbf{p})\tau_z + \beta(\mathbf{p})\tau_y, \quad (27)$$

where the coefficients $\alpha(\mathbf{p})$ and $\beta(\mathbf{p})$ only depend on the angles of the vector $\mathbf{p}$. Substituting expression (27) for the Green's function into the system of equations (1), we obtain

$$\alpha = \frac{i}{2evA}[(a^2 + (b - ievA)^2)^{1/2} - (a^2 + (b + ievA)^2)^{1/2}],$$
$$\beta = \frac{ia}{2evA}\ln\frac{b - ievA + (a^2 + (b - ievA)^2)^{1/2}}{b + ievA + (a^2 + (b + ievA)^2)^{1/2}}, \quad (28)$$

where

$$\alpha = (4\pi)^{-1}\int\alpha(\mathbf{p})d\Omega_p, \quad \beta = (4\pi)^{-1}\int\beta(\mathbf{p})d\Omega_p,$$
$$a = \Delta + \frac{\pi\vartheta n\chi^2\beta}{1 + (\pi\vartheta\chi)^2(\alpha^2 + \beta^2)}, \quad b = \omega + \frac{\pi\vartheta n\chi^2\alpha}{1 + (\pi\vartheta\chi)^2(\alpha^2 + \beta^2)}.$$

For isotropic scattering χ is related to the total cross section by the formula

$$\sigma = m^2\chi^2 / \pi[1 + (\pi\vartheta\chi)^2].$$

The order parameter Δ and the density of states are expressed in terms of the coefficients α and β:

$$\Delta = \frac{|\lambda| mp_0}{2\pi} T \sum_{\omega} \beta, \quad \rho = \rho_0 \operatorname{Re} \alpha(-i\omega). \tag{29}$$

From formulas (27)–(29) it follows that for $n = 0$ the gap in the excitation spectrum and the density of states are given by the expressions

$$\varepsilon = \Delta - evA,$$

$$\rho = \rho_0 \operatorname{Re} \left\{ \frac{[(evA + \omega)^2 - \Delta^2]^{1/2} - [(evA - \omega)^2 - \Delta^2]^{1/2}}{2evA} \right\}. \tag{30}$$

Let us consider the effect of impurities on the excitation spectrum. At the edge of the spectrum the coefficients α and β, regarded as functions of ω, have branch points and their derivatives with respect to ω tend to infinity

$$\partial\alpha / \partial\omega|_{\omega=\varepsilon} = \infty. \tag{31}$$

Equation (31) together with the system (28) determines the gap in the excitation spectrum. Let us find the gap in two limiting cases.

1) $\tau\Delta \ll 1$. In this case one can write the system of equations (28) in the form

$$\alpha^2 + \beta^2 = 1, \quad \alpha\Delta - \beta\omega = \alpha\beta\Gamma, \quad \Gamma = {}^2/_3 (evA)^2\tau. \tag{32}$$

From formulas (29), (31), and (32) it follows that the gap in the excitation spectrum and the density of states near threshold are given by the expressions

$$\varepsilon = \Delta \left(1 - \left(\frac{\Gamma}{\Delta}\right)^{2/3}\right)^{3/2}, \quad \rho = \rho_0 \left(\frac{2(\omega - \varepsilon)}{3\Gamma}\right)^{1/2} \left(1 - \left(\frac{\Gamma}{\Delta}\right)^{2/3}\right)^{-1/4} \left(\frac{\Delta}{\Gamma}\right)^{1/6}. \tag{33}$$

2) $\Delta\tau \gg 1$. Confining our attention to an examination of the Born approximation, we find that the gap in the excitation spectrum is given by the expression

$$\varepsilon = \begin{cases} \Delta(1 - (\Gamma/\Delta)^{2/3})^{3/2}, & evA\Delta\tau^2 \ll 1, \\ \Delta - evA + (2\tau)^{-1}(\beta - i\alpha), & evA\Delta\tau^2 \gg 1, \quad (\Delta - evA)\tau \gg 1, \end{cases} \tag{34}$$

where

$$\alpha = -i\left(\frac{\Delta}{evA} - 1\right)^{1/2},$$

$$\beta = -\frac{i\Delta}{2evA} \ln\left[1 - \frac{2evA}{\Delta} + 2i\left(\frac{evA}{\Delta}\right)^{1/2}\left(1 - \frac{evA}{\Delta}\right)^{1/2}\right].$$

From formula (34) it follows that in the region $evA\Delta\tau^2 \ll 1$ the gap in the excitation spectrum has the same form as in the case of a very dirty superconductor ($\tau\Delta \ll 1$). In the region $evA\Delta\tau^2 \gg 1$, $(\Delta - evA)\tau \gg 1$ the density of states near threshold has the form

$$\rho = 2^{-1/2}\rho_0 \frac{[\Delta(\omega - \varepsilon)]^{1/2}}{evA}. \tag{35}$$

In the limit $\tau \to \infty$ formula (35) goes over into expression (30). In the region $\tau\Delta \gg 1$ the gap in the excitation spectrum tends to zero for $evA = \Delta + (\pi/4\tau)$.

5. CONCLUSION

An expression has been obtained for the gap in the excitation spectrum of a thin, pure film for specular reflection of the electrons from the walls. This result is also valid for diffuse reflection at the boundaries since the threshold excitations arise due to electrons which are traveling parallel to the surface of the film; such electrons are not sensitive to the form of the boundary conditions. Also an expression has been obtained for the density of states near the edge of the spectrum. With specular reflection from the walls, there is a logarithmic singularity in the density of states for $\omega = \Delta$. Diffuse scattering at the boundary leads to a smoothing of this singularity, and the appearance in the density of states of a finite maximum at $\omega = \Delta$.[6] In the case of strong magnetic fields, $evHd \gg \Delta$, the density of states for specular reflection in the frequency range $\omega < \Delta$ is proportional to $\rho_0 eHd^2$ in contrast to the case of diffuse reflection, where the density of states in strong fields is proportional to $\rho_0 d/\xi_0$.[7]

The presence of impurities leads to a displacement of the edge of the spectrum toward the region of larger energy values. The corresponding formulas in the case $\tau\Delta \ll 1$ are obtained without making any assumptions about the Born nature of the scattering of electrons by impurities. The case of small impurity concentrations ($\tau\Delta \gg 1$) is considered in the Born approximation. In this case, in the region of fields $evA\Delta\tau^2 \gg 1$ the displacement of the edge of the spectrum is proportional to τ^{-1}. In the region of fields $evA\Delta\tau^2 \ll 1$ the same formulas are valid as in the case of a very dirty superconductor, $\tau\Delta \ll 1$.

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