INSTABILITY OF PLASMA AT FINITE AND HIGH PRESSURE WITH FINITE ELECTRONIC THERMAL CONDUCTIVITY

A. B. MIKHAĬLOVSKIĬ and V. S. TSYPIN

Submitted May 14, 1970

Zh. Eksp. Teor. Fiz. 59, 1694–1701 (November, 1970)

It is shown that an inhomogeneous plasma with a finite ratio of plasma pressure to the magnetic-field pressure is unstable against perturbations whose increment depends on the thermal conductivity of the electrons. Instabilities of this kind can develop even if $\nabla T_0 = 0$ (T_0 is the plasma temperature). They are caused by the density and magnetic-field gradients. An almost force-free high-pressure plasma is likewise unstable if the pressure gradient is directed towards the density gradient. The investigated instabilities can lead to increased particle and heat losses in thermonuclear installations.

1. INTRODUCTION

THE instabilities of an inhomogeneous plasma with finite and large values of the parameter $\beta \equiv 8\pi p_0/B_0^2$, where p_0 is the plasma pressure and $B_0^2/8\pi$ is the pressure of the magnetic field existing in the plasma, are investigated theoretically in this paper. We note beforehand that only a plasma of this type is of interest when it comes to development of thermonuclear reactors with positive yield (see, for example,[1]).

Much less is known at present concerning instabilities of an inhomogeneous plasma with finite and large β than concerning the instabilities of a plasma with $\beta \ll 1$. In particular, the instabilities built up by different dissipative factors in a plasma with $\beta \gtrsim 1$ have hardly ever been analyzed. The only exception is a paper by the present authors[2], where the buildup of oscillations due to ion viscosity was analyzed.

As is well known (see, for example, the review of Galeev et al.[3]), the finite character of the electronic thermal conductivity plays the role of a dissipative factor that causes buildup or damping of oscillations in an inhomogeneous plasma. An important problem of the theory of plasma oscillations is the determination of the conditions under which the finite electronic thermal conductivity gives rise to the buildup of oscillations and hence to instability. Such a problem was considered by Galeev, Oraevskiĭ, and Sagdeev[4] in the approximation $\beta = 0$. It was shown that instability occurs if there exists in the plasma a gradient of the temperature T_0 directed opposite to the gradient of the density n_0, $\eta \equiv \partial \ln T_0/\partial \ln n_0 < 0$.

We show in this paper that at finite values of β, the dissipation connected with the finite character of the electronic thermal conductivity can lead to a buildup of oscillations when $\partial \ln T_0/\partial \ln n_0 \geq 0$, and particularly when $\nabla T_0 = 0$. The corresponding type of instability can produce additional difficulties in the problem of stable containment of a plasma with finite β. We also investigate the instabilities of an almost force-free plasma ($\nabla p_0 \to 0$) of high pressure, $\beta \gg 1$.

2. INITIAL EQUATIONS

Let us assume that the static magnetic field B_0 is directed along the z axis, and the plasma density n_0, temperature T_0, and magnetic field intensity B_0 are not uniform along the x axis. The gradients of these quantities are connected by the equilibrium condition

$$\frac{\partial}{\partial x}(p_{0e} + p_{0i} + B_0^2/8\pi) = 0, \tag{2.1}$$

where $p_{0e} = p_{0i} = n_0 T_0$ is the pressure of the electrons and ions.

We consider deviations from this stationary state, assuming a space-time dependence of the perturbations in the form $\exp[-i\omega t + i\int k_x dx + ik_y y + ik_z z]$. We assume the frequency ω to be small compared with the frequency of the ion-ion collisions ν_i; this makes it possible to use Braginskiĭ's hydrodynamic description of the plasma[5]. We assume the longitudinal phase velocity ω/k_z to be large compared with the ion thermal velocity v_{Ti}. Just as in[4], we can then neglect the longitudinal motion of the ions and assume their perturbed longitudinal velocity V'_{zi} to vanish,

$$V_{zi}' = 0. \tag{2.2}$$

The perturbations of a finite-pressure plasma at finite $\omega/k_z v_{Ti}$ were investigated by Mikhaĭlovskaya[6].

We use the following equations for the field and the plasma.

1. Electrodynamic equations. Departing from[4], we take into account the perturbations of the magnetic field $\mathbf{B}$ and the non-electrostatic character of the perturbations of the electric field $\mathbf{E}$; to this end it is necessary to use the complete system of Maxwell's equations, which, neglecting the displacement currents, takes the form

$$\mathrm{rot}_\perp \mathbf{B} = \frac{4\pi}{c}\mathbf{j}_\perp, \tag{2.3}$$

$$\mathrm{rot}_z \mathbf{B} = \frac{4\pi}{c} e_e n_0 V'_{ze}, \tag{2.4}$$

$$n_e' = n_i', \tag{2.5}$$

$$\mathrm{rot}\mathbf{E}' = \frac{i\omega}{c}\mathbf{B}', \tag{2.6}$$

$$\mathrm{div}\,\mathbf{B} = 0. \tag{2.7}$$

Here n'_e and n'_i are the perturbations of the electron and ion densities, $\mathbf{j}_\perp$ is the perturbation of the density of the electric currents across the static magnetic field, and e_e is the electron charge.

2. Equations for the electronic component. The elec-

trons are described by the equations of continuity, motion, and heat

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{V}_e = 0, \tag{2.8}$$

$$ne_e\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}_e, \mathbf{B} + \mathbf{B}_0]\right) - \nabla p + \mathbf{R}_e = 0, \tag{2.9*}$$

$$\frac{\partial p_e}{\partial t} + (\mathbf{V}_e\nabla)p_e + \gamma p_e \operatorname{div}\mathbf{V}_e + (\gamma - 1)\operatorname{div}\mathbf{q}_e = 0, \tag{2.10}$$

where $\gamma = 5/3$, $\mathbf{R}_e$ is the friction force, and $\mathbf{q}_e$ is the heat flux; these are defined by the relations

$$\begin{gathered} \mathbf{R}_e = -sn_e\nabla T_e - \nu_e m_e V_{ze}\mathbf{e}_z, \\ \mathbf{q}_e = -3{,}16\frac{p_e}{m_e\nu_e}\nabla_{\parallel}T_e + \frac{5}{2}\frac{p_e}{m_e\omega_{Be}}[\mathbf{h}\nabla T_e], \\ \nabla_{\parallel} \equiv \mathbf{h}(\mathbf{h}\nabla), \mathbf{h} = (\mathbf{B} + \mathbf{B}_0)/|\mathbf{B} + \mathbf{B}_0| \approx \mathbf{e}_z + \frac{\mathbf{B}_{\perp}}{B_0}, \\ s = 0.71, \ \omega_{Be} = e_e|\mathbf{B} + \mathbf{B}_0|/m_e c. \end{gathered} \tag{2.11}$$

If we linearize the z-projection of Eq. (2.9) and express B'_x in it in terms of V'_{ze} by means of the following relation, derived from (2.4):

$$B_x' = i\frac{4\pi k_y e_e n_0}{ck_{\perp}^2}V_{ze}', \tag{2.12}$$

then we find that V'_{ze} is proportional to the parameter $\xi(ck_{\perp}/\omega_{pe})^2$:

$$V_{ze}' \sim \xi\left(\frac{ck_{\perp}}{\omega_{pe}}\right)^2 \equiv \frac{c^2k_{\perp}^2 m_e}{4\pi n_0 e^2}\frac{k_z^2 v_{Ti}^2}{\omega^2}\frac{m_i}{m_e} \tag{2.13}$$

Since $k_{\perp} \ll 1/\rho_i$, where ρ_i is the Larmor radius of the ions, it follows that $\xi(ck_{\perp}/\omega_{pe})^2 \gg \xi m_e/m_i\beta$, so that when $\beta \gtrsim \xi m_e/m_i$, this parameter should be regarded as small. We shall neglect small terms of this order. Then the terms with V'_{ze} can be omitted everywhere from the equations (2.8)–(2.11). The magnetic field B'_x, as seen from (2.12) does not contain such a small parameter and must therefore be taken into account—it enters in (2.9)–(2.11). Using (2.9), we can express this field in terms of the perturbations of the electric field E'_y and the perturbations of the density and pressure of the electrons:

$$\begin{gathered} B_x' = ik_zB_0\frac{\overset{*}{\omega}_{Ti}}{\varkappa_T}\{\omega + \overset{*}{\omega}_{pi}(1+s) - s\overset{*}{\omega}_{ni}\}^{-1}\cdot \\ \times\left\{\frac{icE_y'}{B_0}\frac{\varkappa_T}{\omega_{Ti}^*} - \frac{p_e'}{p_{0e}}(1+s) + s\frac{n'}{n_0}\right\} \end{gathered} \tag{2.14}$$

Taking this equation into account, as well as the expression derived from (2.9) for the transverse electron velocity

$$\mathbf{V}_{\perp e} = \frac{1}{m_e\omega_{Be}}\left[\mathbf{h}, \frac{\nabla p_e}{n} - e_e\mathbf{E}\right] \tag{2.15}$$

we transform the linearized equations (2.8) and (2.10) into

$$-i\omega\frac{n'}{n_0} + i\frac{p_e'}{p_{0e}}\omega_{Be}^* + \frac{cE_y'}{B_0}(\varkappa_n - \varkappa_B) + i\frac{B_z'}{B_0}(\omega - \omega_{pe}^*) = 0, \tag{2.16}$$

$$\begin{gathered} -i\frac{p_e'}{p_{0e}}\left(\omega - 2\gamma\omega_{Be}^* + i\Delta\frac{\omega + \omega_{ni}^*}{\tilde{\omega}}\right) - i\frac{n'}{n_0}\left(\gamma\omega_{Be}^* - i\Delta\frac{\omega + \omega_{pi}^*}{\tilde{\omega}}\right) \\ + \frac{cE_y'}{B_0}\left(\varkappa_p - \gamma\varkappa_B + i\Delta\frac{\varkappa_T}{\tilde{\omega}}\right) + i\gamma\frac{B_z'}{B_0}(\omega - 2\omega_{pe}^* + \omega_{ne}^*) = 0. \end{gathered} \tag{2.17}$$

Here

$$\begin{gathered} \Delta = \frac{2}{5}\gamma 3{,}16b\frac{k_z^2 v_{Te}^2}{\nu_e}, \ v_{Te}^2 = \frac{T_0}{m_e}, \ \omega_A^* = \frac{k_yT_0}{m\omega_B}\varkappa_A, \\ \varkappa_A = \frac{\partial \ln A_0}{\partial x}, \quad A_0 = (p_0, n_0, T_0, B_0), \\ \tilde{\omega} = \omega + \omega_{ni}^*(1+s) - s\omega_{ni}^*. \end{gathered}$$

3. Equations for the ionic component. By using the complete system of Maxwell's equations (2.3)–(2.7), we can obtain a closed system of equations without taking into account the continuity condition for the ions. This condition is satisfied automatically, as can be seen by taking the divergence of (2.3) and (2.4) with allowance for (2.8). The equation of ion motion is similar to (2.9), but with account taken of the inertial and viscous terms. We are interested in the motion of the ions only across B_0, since we have assumed above that $V'_{zi} = 0$, see (2.2). For $\omega < \omega_{Bi} \equiv e_iB_0/m_ic$, we obtain approximately from the ion equation of motion a result similar to (2.15)

$$\mathbf{V}_{\perp i} = \frac{1}{m_i\omega_{Bi}}\left[\mathbf{h}, \frac{\nabla p_i}{n} - e_i\mathbf{E}\right]. \tag{2.18}$$

The ion-pressure balance equation is analogous to (2.10), except that the latter contains only the q-component transverse to the magnetic field. Linearization of this equation using (2.18) yields

$$\begin{gathered} -i\frac{p_i'}{p_{0i}}(\omega - 2\gamma\omega_{Bi}^*) - i\frac{n'}{n_0}\gamma\omega_{Bi}^* + \frac{cE_y'}{B_0}(\varkappa_p - \gamma\varkappa_B) \\ + i\gamma\frac{B_z'}{B_0}(\omega - 2\omega_{pi}^* + \omega_{ni}^*) = 0. \end{gathered} \tag{2.19}$$

We now show how to obtain the dispersion equation. By taking the x-projection of (2.3) and substituting in it $j_x = e_en_0(V'_{xe} - V'_{xi})$ with V'_{xe} and V'_{xi} defined by (2.15) and (2.18), we obtain the connection of B'_z with p'_e and p'_i:

$$B_z' = -4\pi(p_i' + p_e')/B_0. \tag{2.20}$$

Once (2.20) is substituted in (2.16), (2.17), and (2.19), all the perturbations turn out to be expressed in terms of E'_y, n', p'_i, and p'_e. If we introduce the new variables X, Y, and Z used in[7],

$$\begin{gathered} X = \frac{p_i' - p_e'}{p_0}, Y = \frac{\omega}{\varkappa_p}\frac{p_i' + p_e'}{p_0} + \frac{icE_y'}{B_0}, \\ Z = \frac{n'}{n_0} - \frac{\varkappa_n}{\varkappa_p}\frac{p_i' + p_e'}{p_0}, \end{gathered} \tag{2.21}$$

then (2.16), (2.17), and (2.19) take the respective forms

$$\begin{gathered} \omega_{Bi}^*X + (\varkappa_B - \varkappa_n)Y - \omega Z = 0, \\ X\left\{\omega + \frac{i\Delta}{2\tilde{\omega}}(\omega + \omega_{ni}^*)\right\} - Yi\frac{\Delta}{2\tilde{\omega}}\varkappa_T \\ + Z\left\{\gamma\omega_{Bi}^* + \frac{i\Delta}{2\tilde{\omega}}(\omega + \omega_{pi}^*)\right\} = 0, \\ X(\omega - 2\gamma\omega_{Bi}^*) + Y(\varkappa_p - \gamma\varkappa_B) + \gamma\omega_{Bi}^*Z = 0. \end{gathered} \tag{2.22}$$

The determinant of this system gives the sought dispersion equation

$$\begin{gathered} \left(1 + \frac{\gamma\beta}{2}\right)\left\{x^2 + \frac{\gamma\beta^2y^2}{4}\left[1 - \frac{2\gamma(1 + {}^1/_2 y\beta)}{y(1 + {}^1/_2\gamma\beta)}\right]\right\} \\ \times\{x + y(1+s) - s\} \\ + i\alpha\{x^2(1 + {}^2/_5\gamma\beta) + x[1 + {}^1/_2\gamma\beta + {}^1/_2 y\gamma\beta(1 + {}^1/_2\beta)] \\ + {}^1/_4\gamma\beta(1 + 2y + \beta y^2 - \gamma^{-1}y^2)\} = 0, \end{gathered} \tag{2.23}$$

where

$$x = \omega/\omega_{ni}^*, \ y = \varkappa_p/\varkappa_n, \ \alpha = \Delta/\omega_{ni}^*.$$

---

*$[\mathbf{V}_e, \mathbf{B} + \mathbf{B}_0] \equiv \mathbf{V}_e \times (\mathbf{B} + \mathbf{B}_0)$.

3. SUMMARY OF SOME PREVIOUSLY-KNOWN RESULTS THAT FOLLOW FROM (2.23)

Equation (2.23) contains the dimensionless parameters β, α and y. The first of them, $\beta \equiv 8\pi p_0 B_0^2$, characterizes the relative pressure of the plasma; the second, $\alpha \equiv (2/3)\, 3.16\, k_z^2 v_{Te}^2 / \nu_e \omega_{ni}^*$ characterizes the longitudinal wavelength of the oscillations; the third, $y \equiv \kappa_p/\kappa_n$, describes the relative pressure gradient. Heretofore, only particular cases of (2.23) were investigated, corresponding to the approximations $\beta = 0$[4]; $\alpha = 0$[7,8]; $\alpha = \infty$, $\beta = \infty$, $y = 0$[9]; $\alpha = \infty$,[6]; $\alpha = 0$, $\beta = 0$[10]. Let us note the main results of these investigations.

1) The approximation of zero-pressure plasma, $\beta = 0$. When $\beta = 0$, Eq. (2.23) reduces to a quadratic equation in the frequency:

$$\omega^2 + \omega[\omega_{ni}^* + (1+s)\omega_{Ti}^* + i\Delta] + i\Delta\omega_{ni}^* = 0. \tag{3.1}$$

Eq. (3.1) has solutions with Im $\omega > 0$, corresponding to instability if

$$\eta < 0.$$

The increment is maximal at

$$k_z v_{Te} \approx (\nu_e \omega_{Te}^*)^{1/2}.$$

It is of the same order as the oscillation frequency:

$$(\mathrm{Im}\,\omega)_{max} \approx \mathrm{Re}\,\omega \approx \omega_{Te}^*.$$

2. Entropy (inertialess) waves in the approximation $\alpha = 0$. When $\alpha = 0$, the equation (2.23), which is cubic in x, breaks up into two equations, one of which is quadratic and takes the form

$$\omega^2 + \gamma\omega_{Bi}^{*2}\left(1 + 2\gamma\frac{\varkappa_B - \varkappa_n}{\varkappa_p - \gamma\varkappa_B}\right) = 0. \tag{3.2}$$

If $\beta \ll 1$, the instability takes place if

$$\eta > 7/3,\ \eta < -1.$$

This instability is aperiodic, Re $\omega = 0$, and its increment is

$$\mathrm{Im}\,\omega \approx \omega_{Bi}^* \approx \beta\omega_{Ti}^*.$$

It is proportional to the parameter β. When $\beta \gg 1$, the plasma is unstable if

$$-4/\beta < y < 0.$$

Just as in the case of $\beta \ll 1$, the instability is aperiodic. Its increment is of the order of

$$\mathrm{Im}\,\omega \approx \omega_{Ti}^*.$$

3. Inertialess waves in the approximation $\alpha = \infty$, $y = 0$, $\beta = \infty$. The approximation $y = 0$ means that there are no pressure or magnetic-field gradients, and the temperature and density gradients have opposite directions and are such that $\kappa_n = -\kappa_T$. This corresponds to the case of the so-called force-free plasma. When $y = 0$, $\alpha = \infty$, $\beta = \infty$, Eq. (2.23) reduces to the quadratic equation

$$(1+\gamma)\omega^2 + 2\gamma\omega\omega_{ni}^* + \gamma\omega_{ni}^{*2} = 0. \tag{3.3}$$

The solutions of this equation are complex. One of the roots corresponds to hydrodynamic instability with an increment on the order of

$$\mathrm{Im}\,\omega \approx \omega_{Ti}^*.$$

4. Inertialess waves in the approximation in which $\alpha \to \infty$ and β and y are finite. In this case we obtain the dispersion equation by equating the coefficient of α in (2.23) to zero. For $\beta \ll 1$, this dispersion equation takes the form

$$\omega^2 - \omega\omega_{ne}^* + \gamma\frac{\beta}{4}\omega_{ne}^{*2}\left[1 + 2\frac{\varkappa_p}{\varkappa_n} - \frac{1}{\gamma}\left(\frac{\varkappa_p}{\varkappa_n}\right)^2\right] = 0. \tag{3.4}$$

The roots of this equation are real. They differ in order of magnitude: One of them,

$$\omega = \omega_{ne}^*,$$

does not depend on β and corresponds to the so-called "drift oscillations" of zero-pressure plasma[10], which have been investigated many times, while the other is proportional to the parameter β:

$$\omega = -\frac{5}{12}\beta\omega_{ni}^*\left[1 + 2\frac{\varkappa_p}{\varkappa_n} - \frac{3}{5}\left(\frac{\varkappa_p}{\varkappa_n}\right)^2\right].$$

For $\beta \gg 1$ and finite $y\beta$, the dispersion equation with the approximation $\alpha \to \infty$ is

$$(1+\gamma)\omega^2 + 2\gamma\omega\omega_{ni}^*(1 + \beta y/2) + \gamma\omega_{ni}^{*2} = 0. \tag{3.5}$$

It is a generalization of (3.3) to the case of finite $y\beta$. It follows from (3.5) that the instability described by (3.3) becomes suppressed if the pressure gradient is small. The instability is possible only if

$$-\frac{2}{\beta}\left(\sqrt{\frac{8}{5}} + 1\right) < \frac{\varkappa_p}{\varkappa_n} < \frac{2}{\beta}\left(\sqrt{\frac{8}{5}} - 1\right).$$

5. Electromagnetic perturbations with $\mathbf{E} \parallel \mathbf{B}_0$ at $\alpha = 0$. If $\alpha = 0$ ($k_z = 0$) Eq. (2.23) describes not only the solutions of type (3.2), corresponding to perturbations with $\mathbf{E} \perp \mathbf{B}_0$, but one other type of perturbation with $\mathbf{E} \parallel \mathbf{B}_0$. Their frequency is given by

$$\omega = \omega_{ne}^* + \omega_{Te}^*(1+s). \tag{3.6}$$

It is independent of the parameter β.

4. INSTABILITY OF A FINITE-PRESSURE PLASMA WITH $\nabla T_0 = 0$

In this section we consider the stability of an important particular case of a plasma with $\nabla n_0 \neq 0$, $\nabla T_0 = 0$, and finite β. (In the approximation $\beta = 0$, see Sec. 3 and and[4], such a plasma is stable.) When $\nabla T_0 = 0$, Eq. (2.23) breaks up into two, one of which, $\omega = \omega_{ne}^*$, describes stable oscillations. The second equation

$$(1 + \gamma\beta/2)\{\omega^2 + \gamma\omega_{Bi}^{*2}[1 - 2\gamma(1+\beta/2)/(1+\gamma\beta/2)]\} + i\Delta[\omega(1 + {}^2/_3\beta) - 2\omega_{Bi}^*(1 + {}^5/_{12}\beta)] = 0 \tag{4.1}$$

has, at not too large β, $0 < \beta < \beta_{lim}$, a solution with Im $\omega > 0$ corresponding to instability. Let us prove this.

In the case $\beta \ll 1$, this equation simplifies to

$$\omega^2 - {}^{35}/_9\,\omega_{Bi}^{*2} + i\Delta(\omega - 2\omega_{Bi}^*) = 0. \tag{4.2}$$

At small k_z we have $\Delta/\omega_{Bi}^* \ll 1$, from which it follows that

$$\mathrm{Re}\,\omega = \pm\sqrt{35}\,\omega_{Bi}^*/3, \quad \mathrm{Im}\,\omega = -{}^1/_2\Delta(1 \pm 6/\sqrt{35}). \tag{4.3}$$

We see that one of the roots has a positive imaginary part, Im $\omega > 0$. The increment is numerically small: Im $\omega \approx \Delta/144$. At large k_z, $\Delta \gg \omega_{Bi}^*$, it follows from

(4.2) that the perturbations that increase in time have a frequency and an increment

$$\mathrm{Re}\,\omega = 2\omega_{Bi}^*, \ \mathrm{Im}\,\omega = \omega_{Bi}^{*2}/9\Delta. \qquad (4.4)$$

The instability described by (4.1) does not develop at too large values of β. By equating to zero the real and imaginary parts of (4.1) at Im $\omega = 0$, we obtain the instability limit

$$\beta = \beta_{\lim} = 1{,}2. \qquad (4.5)$$

When $\beta > \beta_{\lim}$, the plasma is stable.

It follows from the foregoing analysis that a plasma with $\nabla T_0 = 0$ and finite β is unstable. The instability is due to the fact that in a plasma with finite albeit small β there exists a branch of very slow oscillations

$$\omega/k_y \approx \frac{cT_0}{eB_0}\frac{\partial \ln B_0}{\partial x}$$

(magnetic-drift oscillations), which are built up as a result of the negative dissipation due to the finite electronic heat conduction. The longitudinal wave number of this instability is of the order of

$$k_z v_{Te} \approx (\nu_e \omega_{ne}^* \beta)^{1/2},$$

and the maximum increment is numerically small compared with the frequency

$$\mathrm{Im}\omega \approx 0{,}1\mathrm{Re}\omega \approx 0{,}1\beta \frac{k_y cT_0}{eB_0}\frac{\partial \ln n_0}{\partial x}.$$

It is interesting to note that the oscillation modes that build up are essentially connected with temperature perturbations, and the presence of a stationary-temperature gradient is not obligatory. A similar situation was discussed earlier for a different class of instabilities by Moiseev[11].

5. ALMOST FORCE-FREE PLASMA OF HIGH PRESSURE

In the approximation of infinitely large electronic thermal conductivity $\alpha = \infty$, a force-free plasma, $y = 0$, of high pressure, $\beta \gg 1$, is unstable, see Sec. 3, Item 3. If $y \neq 0$, the instability of the approximation $\alpha = \infty$ is suppressed, see Sec. 3, Item 4. A plasma with $y < 0$ is also unstable against perturbations with $\alpha = 0$, but this instability exists only at small values of the parameter $y\beta$, see Sec. 3, Item 2. It is therefore of interest to check on the possible appearance of additional instabilities connected with the finite value of the parameter α, in a plasma that is stable in the approximations $\alpha = \infty$ and $\alpha = 0$.

At finite α and finite $y\beta$, the oscillations of a plasma with $\beta \gg 1$ are described by the following dispersion equation derivable from (2.23):

$$[x^2 - \gamma\beta y(1 + {}^1\!/_4\beta y)](x - s) + {}^1\!/_2 i\alpha\gamma^{-1}[(1+\gamma)x^2 + 2\gamma(1 + \beta y/2)x + \gamma] = 0. \qquad (5.1)$$

Let us examine the solutions of this equation for $y\beta \gg 1$. If $\beta y \gg 1$, two of three roots of (5.1) are large, of the order of βy. They correspond to damped oscillations. The third root is given by

$$x = s\frac{1 + 2i\alpha/\gamma\beta y}{1 + (2\alpha/\gamma\beta y)^2}. \qquad (5.2)$$

When $y > 0$ ($\kappa_p/\kappa_n > 0$), this root corresponds to perturbations that increase in time with an increment

$$\mathrm{Im}\,\omega \approx \frac{13}{5} s \frac{\varkappa_n}{\varkappa_p \beta}\frac{k_z^2 v_{Te}^2}{\nu_e}\frac{1}{1 + ({}^6\!/_5 k_z^2 v_{Te}^2/\nu_e \omega_{Bi}^*)^2}. \qquad (5.3)$$

The increment is maximal at

$$k_z v_{Te} \approx (\nu_e \omega_{Bi}^*)^{1/2}.$$

It is of the order of

$$\mathrm{Im}\,\omega \approx |\omega_{ni}^*|.$$

It is interesting to note that the oscillation mode with frequency (5.2) is determined by the presence of the thermal force in the equations of motion (2.9) and (2.11). The importance of the class of instabilities connected with the thermal force was pointed out earlier by Moiseev[11].

6. CONCLUSION

We have shown that a finite-pressure plasma ($\beta < 1.2$) is unstable even in the absence of a temperature gradient, Sec. 4. This instability is due to the density and magnetic-field gradients. The instability appears as a result of dissipative buildup of magnetic-drift oscillations resulting from the finite thermal conductivity of the electrons. The same type of instability is also observed in a plasma with a positive temperature gradient, $\partial \ln T_0/\partial \ln n_0 > 0$.

The instability of a zero-pressure plasma with negative temperature gradient, $\partial \ln T_0/\partial \ln n_0 < 0$, previously investigated by Galeev et al.[4], exists at all values of β, particularly at $\beta \gg 1$. According to Sec. 5, this instability should play a major role in the transport of an almost force-free plasma of high pressure with $\partial \ln p_0/\partial \ln n_0 > 0$.

[1] A. S. Bishop, State and Prospects of the World Program of Research on Controlled Thermonuclear Fusion. Paper at the Research Council of the National Academy of Sciences of the USA, 11 March 1969. A. S. Bishop, Nuclear Fusion, 10, 85 (1970).

[2] A. B. Mikhaĭlovskiĭ and V. S. Tsypin, Zh. Eksp. Teor. Fiz. 59, 524 (1970) [Sov. Phys.-JETP 32, 287 (1971)].

[3] A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, Atomnaya energiya 15, 451 (1963).

[4] A. A. Galeev, V. N. Oraevskiĭ, and R. Z. Sagdeev, Zh. Eksp. Teor. Fiz. 44, 903 (1963) [Sov. Phys.-JETP 17, 615 (1963)].

[5] S. I. Braginskiĭ, Voprosy teorii plazmy (Problems of Plasma Theory), Gosatomizdat, 1, 183 (1963).

[6] L. V. Mikhaĭlovskaya, Zh. Tekh. Fiz. 40, (1970) [Sov. Phys.-Tech. Fiz. 15, (1970)].

[7] L. V. Mikhaĭlovskaya, ibid. 37, 1974 (1967) [12, 1451 (1968)].

[8] B. B. Kadomtsev, Zh. Eksp. Teor. Fiz. 37, 1096 (1959) [Sov. Phys.-JETP 10, 780 (1960)].

[9] A. B. Mikhaĭlovskiĭ, Dokl. Akad. Nauk SSSR 192, 74 (1970) [Sov. Phys.-Dokl. 15, 471 (1970)].

[10] A. B. Mikhaĭlovskiĭ, Zh. Eksp. Teor. Fiz. 44, 1552 (1963) [Sov. Phys.-JETP 17, 1043 (1963)].

[11] S. S. Moiseev, ZhETF Pis. Red. 4, 81 (1966) [JETP Lett. 4, 55 (1966)].

Translated by J. G. Adashko
193