INDUCTIVE AND DEFORMATION ABSORPTION OF SOUND IN CONDUCTORS

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The interaction of conduction electrons with acoustic waves in anisotropic conductors is investigated starting from the general principles of quantum mechanics. A microscopic expression for the deformation-potential tensor λ_{ik} is obtained. A theory of collisionless sound absorption (of the Landau-damping type) in conductors is constructed. Absorption both in the absence and in the presence of a magnetic field is investigated. At relatively low frequencies ω, the coefficient of absorption of transverse sound in the absence of a magnetic field increases in proportion to the frequency, reaches a maximum, begins to decrease like ω^{-3}, and finally again increases in proportion to ω after passing through a minimum. In calculating the absorption coefficient in the high-frequency region it is necessary to take into account the Stewart-Tolman effect. The absorption coefficient of transverse sound in a strong magnetic field (when the Larmor radius is smaller than the length of the acoustic wave) does not depend on the magnetic field and is proportional to ω. It is shown that the contribution made by inductive effects to the coefficient of collisionless absorption of sound should in general be smaller than or of the order of the contribution of the deformation interaction.

1. INTRODUCTION

IN the study of the absorption of sound in conductors one can distinguish between two limiting cases. The case of frequent collisions of conduction electrons ($ql \ll 1$, where $\mathbf{q}$ is the wave vector of the sound and l the mean free path of the electrons) was first considered by Akhiezer[1]. The opposite case of collisionless absorption (of the Landau-damping type) was investigated in a number of papers[2-6]. The purpose of the present paper is to study the distinguishing features of collisionless absorption of transverse sound in conductors both in the absence and in the presence of a constant magnetic field $\mathbf{H}$.

In the absence of a magnetic field, the absorption of transverse sound by conduction electrons is due to two mechanisms. The first is the interaction of the electrons with the periodic field of the crystal lattice, and the second is the interaction of the electrons with the solenoidal electrical field accompanying the transverse sound wave in the conductor. We have investigated the frequency dependence of the absorption coefficient. At sufficiently low frequencies, the absorption coefficient is proportional to the acoustic frequency ω, and then, after reaching a maximum, it begins to decrease like ω^{-3}, finally passing through a minimum and again increasing in proportion to ω[1)]. In this latter region, it became necessary to take the Stewart-Tolman effect into account.

We then investigated the limiting value to which the sound absorption coefficient tends when the potential V_0 of the self-consistent periodic field in which the conduction electrons move tends to zero. The point is that in many metals the potential of the periodic field (more accurately, its pseudopotential) is small compared with the Fermi energy ζ_0, i.e., the conduction electrons are almost free[7] (see also[8]); this makes it urgent to investigate the sound absorption coefficient as $V_0 \to 0$.

To this end, we had to start with the construction of a microscopic theory of sound absorption, starting from the basic principles of quantum mechanics, and then change over to the classical description. It was possible to trace the transition to the limit as $V_0 \to 0$ by analyzing the corresponding microscopic expressions describing the interaction between the electrons and the sound.

In Secs. 4 and 5 we consider collisionless absorption of sound in a magnetic field. One of us had previously introduced[9] the concept of two sound-absorption mechanisms in this case—deformation and inductive. The investigation made in Sec. 4 makes it possible to conclude that the inductive interaction of the electrons with the sound can be separated by different means. One of them, employed in[9], corresponds in essence to a changeover to a coordinate system that moves together with the lattice. In this system, the inductive field is given by $\dot{\mathbf{u}} \times \mathbf{H}/c$. It is necessary to add to the interaction connected with this field also the interaction proportional to the tensor of the deformation potential λ_{ik} introduced in[1,3], and the Stewart-Tolman interaction (incidentally, as will become clear in Sec. 5, its role in a strong magnetic field is negligibly small).

Another method is to change over to coordinate systems moving with that additional velocity $\Delta\mathbf{v}$ which is acquired by the electrons dragged by the lattice. As shown by Holstein[10] (see also[11]), the conduction electrons are not completely dragged by the lattice in the absence of collisions. If, for example, the lattice executes periodic motion with velocity $\mathbf{u}$, then in the simplest case when the spectrum is isotropic the electrons acquire an additional velocity

$$\Delta\mathbf{v} = (1 - m_0/m)\dot{\mathbf{u}}, \tag{1.1}$$

where m_0 is the mass of the free electron and m is the effective mass. In this method, the inductive field is

[1)] The first two sections of the frequency dependence were investigated earlier (see [2,3,6,16]).

$$\frac{1}{c}\left(1-\frac{m_0}{m}\right)[\mathbf{uH}]. \qquad (1.2)^*$$

It is necessary to add to the interaction connected with this field also the deformation interaction, which this time is described not by the tensor λ_{ik}, but by the tensor L_{ik} introduced by formula (2.24) of Sec. 2. This method, incidentally, is convenient because it makes it possible to trace directly the transition to the limit as $V_0 \to 0$, for in this case the quantities $1 - m_0/m$ and L_{ik} also tend to zero (unlike λ_{ik}).

Finally, if the electron dynamics is described in the laboratory frame, no explicit inductive term appears at all.

The most important fact in the collisionless case is that, generally speaking, it is meaningless to consider inductive absorption in itself, separately from the deformation absorption, since the induction effects can only be either smaller than the deformation effects or of the same order of magnitude[2]. This is verified both by a general analysis based on the kinetic equation (Sec. 4) and by concrete calculations for an isotropic electron spectrum (Sec. 5).

2. DERIVATION OF KINETIC EQUATION FOR THE ELECTRONS IN THE FIELD OF AN ACOUSTIC WAVE

Let us consider a conduction electron moving in the self-consistent field of the other electrons and ions of the lattice. Let the lattice-displacement vector $\mathbf{u}$ depend on the coordinates and on the time. We assume the deformation to be small, so that

$$|\partial u_i/\partial r_k| \ll 1. \qquad (2.1)$$

The Hamiltonian of the electron is

$$\mathcal{H} = \frac{\hat{\mathbf{p}}^2}{2m_0} + V_0(\mathbf{r}-\mathbf{u}) + V'(\mathbf{r}-\mathbf{u}) + V(\mathbf{r}-\mathbf{u}). \qquad (2.2)$$

Here $V_0(\mathbf{r})$ is the periodic potential of the undeformed lattice, $V(\mathbf{r})$ the potential of the impurity atoms, and $V'(\mathbf{r})$ the change of the periodic potential as a result of the deformation. The latter, obviously, is of the form

$$V'(\mathbf{r}) = V_{ik}' u_{ik}, \qquad (2.3)$$

where u_{ik} is the deformation tensor.

We shall find it convenient to carry out a canonical transformation corresponding to a transition to a coordinate frame moving together with the lattice. Let us consider this transformation first for the simplest case when the displacement $\mathbf{u}$ is independent of the coordinates and is a function of the time only. We choose the canonical-transformation operator in the form

$$\hat{T} = \exp\left(-\mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right) = \exp\left(-\frac{i}{\hbar}\dot{\mathbf{u}}\hat{\mathbf{p}}\right). \qquad (2.4)$$

The transformed energy operator is

$$\mathcal{H}' = \hat{T}^{-1}\mathcal{H}\hat{T} + i\hbar\hat{T}^{-1}\frac{\partial \hat{T}}{\partial t} = \frac{\hat{\mathbf{p}}^2}{2m_0} + V_0(\mathbf{r}) + V(\mathbf{r}) - \dot{\mathbf{u}}\hat{\mathbf{p}}. \qquad (2.5)$$

The coordinate is simultaneously transformed in the following manner: $\mathbf{r} \to \mathbf{r} + \mathbf{u}$. Expression (2.5), apart from a constant term, coincides with the Hamiltonian of an electron moving in an electric field with a vector potential

$$\mathbf{A}^{ST} = (c/e)m_0\dot{\mathbf{u}}, \qquad (2.6)$$

where e is the electron charge. This means that in the co-moving system the electron behaves as if it were moving in an external Stewart-Tolman electric field equal to

$$\mathbf{E}^{ST} = -\frac{1}{c}\frac{\partial \mathbf{A}^{ST}}{\partial t} = -\frac{m_0}{e}\ddot{\mathbf{u}}. \qquad (2.7)$$

From this we obtain in the approximation linear in the field $\mathbf{E}^{ST}$ the following expression for the current density:

$$j_i' = \sigma_{ik}(\omega)E_k^{ST}, \qquad (2.8)$$

where $\sigma_{ik}(\omega)$ is the electric-conductivity tensor.

In the general case, when the displacement vector $\mathbf{u}$ depends on the coordinates (say, for a monochromatic acoustic wave), the canonical-transformation operator is conveniently chosen in the form

$$\hat{T} = \exp\left[-\frac{i}{2\hbar}(\dot{\mathbf{u}}\hat{\mathbf{p}} + \hat{\mathbf{p}}\dot{\mathbf{u}})\right]. \qquad (2.9)$$

Using the condition (2.1) and putting

$$\hbar q \ll p, \qquad (2.10)$$

where p is the characteristic value of the electron quasimomentum[3], we obtain the following expression for the transformed operator:

$$\mathcal{H}' = \frac{\hat{\mathbf{p}}^2}{2m_0} + V_0(\mathbf{r}) + V'(\mathbf{r}) + V(\mathbf{r}) - \frac{1}{m_0}u_{ik}\hat{p}_i\hat{p}_k - \dot{\mathbf{u}}\hat{\mathbf{p}}. \qquad (2.11)$$

Let us consider the equation for the single-particle electron density matrix with the energy operator (2.11). When conditions (2.1) and (2.10) are satisfied, it is easy to verify that the equation for the density-matrix elements that are diagonal in the band index reduces to an equation for the classical distribution function

$$\frac{\partial F'}{\partial t} + \frac{\partial \varepsilon'}{\partial \mathbf{p}'}\frac{\partial F'}{\partial \mathbf{r}'} - \frac{\partial \varepsilon'}{\partial \mathbf{r}'}\frac{\partial F'}{\partial \mathbf{p}'} + \hat{S}'F' = 0. \qquad (2.12)$$

We shall not present here the derivation of the classical kinetic equation from the quantum-mechanical equation for the density matrix, since the procedure for such a derivation is described in[13]. In (2.12) we have primed the symbols for the classical coordinate $\mathbf{r}'$, for the momentum $\mathbf{p}'$ canonically conjugate to it (which in this case coincides with the electron quasimomentum), and the other quantities, emphasizing thereby that they pertain to the co-moving coordinate system. The quantity ϵ' in (2.12) is the classical Hamiltonian of the electron

[2] Strictly speaking, we obtained the corresponding proof for the case when the contribution of the solenoidal fields accompanying the acoustic wave to the absorption coefficient can be neglected. But if these fields are appreciable, then we have the special case of helicon-acoustic resonance wherein, as shown by Skobov and Kaner [12], the inductive interaction predominates.

*$[\mathbf{uH}] \equiv \mathbf{u} \times \mathbf{H}$.

[3] We call attention to the difference in the notation for the momentum operator $\hat{\mathbf{p}} = -i\hbar\partial/\partial\mathbf{r}$ and for the electron quasimomentum $\mathbf{p}$.

and is obtained by averaging (2.11) (after subtracting the impurity potential) and is equal to[4)]

$$\varepsilon'(\mathbf{p}', \mathbf{r}') = \varepsilon_0(\mathbf{p}') + \lambda_{ik}(\mathbf{p}')u_{ik} - m_0\dot{u}_i v_i, \tag{2.13}$$

where $\epsilon_0(\mathbf{p}')$ is the unperturbed value of the electron energy and

$$\lambda_{ik}(\mathbf{p}') = \left\langle n\mathbf{p}' \left| \frac{\hbar^2}{m_0}\frac{\partial^2}{\partial r_i \partial r_k} + V_{ik}' \right| n\mathbf{p}' \right\rangle \tag{2.14}$$

is a diagonal matrix element on the Bloch wave functions of the electron with band index n (corresponding in this case to the conduction band) and quasimomentum $\mathbf{p}'$. When $\mathbf{p}' = 0$, expression (2.14) goes over into that obtained earlier in[14].

The collisional term $S'F'$ is obtained by the standard method (see, e.g.,[15]) and has, in the Born approximation, the usual form

$$\hat{S}'F' = \frac{2\pi}{\hbar}\sum_{\mathbf{p}_1'} |V_{\mathbf{p}'\mathbf{p}_1'}|^2 [F'(\mathbf{p}', \mathbf{r}') - F'(\mathbf{p}_1', \mathbf{r}')]\delta[\varepsilon'(\mathbf{p}', \mathbf{r}') - \varepsilon'(\mathbf{p}_1', \mathbf{r}')], \tag{2.15}$$

i.e., it is made to vanish by the equilibrium function $F_0(\epsilon')$, where F_0 is the Fermi function.

In the derivation of the kinetic equation (2.12) we have set the macroscopic electric fields E equal to zero. In the presence of these fields, there appears in the left-hand side of (2.12) an additional term

$$e\mathbf{E}\partial F'/\partial \mathbf{p}'. \tag{2.16}$$

What is the role of the density-matrix elements that are not diagonal in the band number? It is easy to verify that they make no contribution to the sound absorption. This statement is valid so long as

$$\hbar\omega \ll \varepsilon_g, \tag{2.17}$$

where ω is the frequency of the ultrasound and ϵ_g is the width of the forbidden band. At the same time, contributions to the macroscopic current density $\mathbf{j}'$ in the co-moving system of coordinates are made by both the diagonal density-matrix elements and those not diagonal in the band number. Their summary contribution can be represented in the following classical form[5)]

$$\mathbf{j}' = e\int F'(\mathbf{p}', \mathbf{r}')\frac{\partial \varepsilon'}{\partial \mathbf{p}'}d\tau_{\mathbf{p}'}, \qquad d\tau_{\mathbf{p}'} = \frac{2d^3p'}{(2\pi\hbar)^3}. \tag{2.18}$$

Thus, with respect to perturbations satisfying the conditions (2.1), (2.10) and (2.17), the conduction electrons behave like classical particles with a dispersion law (2.13).

We now calculate with the aid of (2.12) the coefficient of transverse-sound absorption in a metal for the limiting case $\omega \gg \nu$ (ν is the frequency of the electron collisions). We confine ourselves in this section to an analysis of the simplest case, when the electric fields produced upon propagation of the sound can be neglected. We seek a solution of (2.12) in the form

$$F'(\mathbf{p}', \mathbf{r}') = F_0(\varepsilon') + f'(\mathbf{p}', \mathbf{r}'). \tag{2.19}$$

As shown in Appendix 1, the absorption coefficient describing the spatial damping of the sound intensity in the metal is expressed in terms of the function f' as follows:

$$\Gamma = \frac{1}{2wU_s}\mathrm{Re}\int \varepsilon'^* f' d\tau_{\mathbf{p}'}, \tag{2.20}$$

where $U_S = \rho|\dot{\mathbf{u}}|^2/2$ is the energy density of the sound wave, ρ the crystal density, and w the speed of sound. Substituting (2.19) in (2.12), we obtain the following equation for f':

$$(-i\omega + i\mathbf{qv} + \nu)f' = -(\lambda_{ik}\dot{u}_{ik} - m_0 v_i \dot{u}_i)\frac{\partial F_0}{\partial \varepsilon'}. \tag{2.21}$$

Substituting the solution of this equation in (2.20), we obtain as $\nu \to 0$

$$\Gamma = -\frac{\pi}{2wU_s}\int |\lambda_{ik}\dot{u}_{ik} - m_0 v_i \dot{u}_i|^2 \frac{\partial F_0}{\partial \varepsilon'}\delta(\omega - \mathbf{qv})d\tau_{\mathbf{p}'}. \tag{2.22}$$

Since the integrand contains $\delta(\omega - \mathbf{q}\cdot\mathbf{v})$, we can replace $-m_0\dot{v}_i\dot{u}_i$ by $m_0 v_i v_k \dot{u}_{jk}$, thus obtaining

$$\Gamma = -\frac{\pi}{2wU_s}\int |L_{ik}\dot{u}_{ik}|^2 \frac{\partial F_0}{\partial \varepsilon'}\delta(\omega - \mathbf{qv})d\tau_{\mathbf{p}'}, \tag{2.23}$$

where

$$L_{ik} = \lambda_{ik} + m_0 v_i v_k. \tag{2.24}$$

As can be seen from the microscopic expression (2.14), the quantity L_{ik} vanishes when $V_0 \to 0$; and with it the absorption coefficient (2.23) also tends to zero. As shown in[13], the physical meaning of the tensor $L_{ik}(\mathbf{p})$ is that when multiplied by the deformation tensor u_{ik} it describes the average energy increment of an electron with quasimomentum $\mathbf{p}$ in the case when the crystal is uniformly deformed.

We see, however, that in the case of transverse sound it is necessary to take into account the Stewart-Tolman effect for such a limiting transition, whereas the Stewart-Tolman effect plays no role in the absorption of longitudinal sound in metals (see Appendix 2).

Let us obtain an explicit expression for the absorption coefficient of transverse sound in the case of an isotropic electron spectrum, when

$$L_{ik} = C(\mathbf{p})\frac{p_i p_k}{m} + C_1(\mathbf{p})\frac{p^2}{m}\delta_{ik}. \tag{2.25}$$

Integrating (2.23), we obtain

$$\Gamma = \frac{3\pi}{4}C^2(\mathbf{p}_F)\frac{n_0 m\omega}{\rho v_F}, \tag{2.26}$$

where n_0 is the concentration of the conduction electrons; $\mathbf{p}_F$ and $\mathbf{v}_F$ are the momentum and the Fermi velocity. This expression turns out to be smaller than the absorption coefficient of longitudinal sound, calculation in[2-6], by a factor $(w/v_F)^2$.

In concluding this section we note that (at least in the case of closed Fermi surfaces) it is possible to change over from the classical variables $\mathbf{p}'$ and $\mathbf{r}'$, in terms of which the kinetic equation (2.12) has been written, to other variables corresponding to the laboratory frame by using a canonical transformation[6)] with a generating function

$$\Phi(\mathbf{r}', \mathbf{p}, t) = \mathbf{p}(\mathbf{r}' + \mathbf{u}). \tag{2.27}$$

[4)] We note that expression (2.13) appeared in the papers of Akhiezer et al. [3] and of Kontorovich [16].

[5)] We do not present the derivation of this expression. The idea of such a derivation is described in detail in [13].

[6)] The idea of such a transformation was advanced by L. D. Landau.

This transformation takes the form

$$p_i' = \frac{\partial \Phi}{\partial r_i'} = p_i + p_k \frac{\partial u_k}{\partial r_i'}, \qquad r_i = \frac{\partial \Phi}{\partial p_i} = r_i' + u_i; \tag{2.28}$$

$$\varepsilon(\mathbf{p}, \mathbf{r}) = \varepsilon'(\mathbf{p}', \mathbf{r}') + \frac{\partial \Phi}{\partial t} = \varepsilon_0(\mathbf{p}) + (\mathbf{p} - m_0\mathbf{v})\mathbf{u} + (\lambda_{ik} + v_i p_k)\frac{\partial u_k}{\partial r_i}. \tag{2.29}$$

For free electrons $\mathbf{p} = m_0\mathbf{v}$, and the second and third terms in the right-hand side of (2.29) vanish, as they should. The kinetic equation for the function $F(\mathbf{p}, \mathbf{r}) = F'(\mathbf{p}', \mathbf{r}')$ in the laboratory frame is of the form

$$\frac{\partial F}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{r}} - \frac{\partial \varepsilon}{\partial \mathbf{r}}\frac{\partial F}{\partial \mathbf{p}} + e\mathbf{E}\frac{\partial F}{\partial \mathbf{p}} + \frac{2\pi}{\hbar}\sum_{\mathbf{p}_1} |V_{\mathbf{p}\mathbf{p}_1}|^2 (F_{\mathbf{p}} - F_{\mathbf{p}_1})\delta(\varepsilon_{\mathbf{p}} - \dot{\mathbf{p}}\mathbf{u} - \varepsilon_{\mathbf{p}_1} + \mathbf{p}_1\dot{\mathbf{u}}) = 0. \tag{2.30}$$

(We have included here the term with the electric field.) This equation is contained in this form in the paper of Kontorovich[16]. The electron current density $\mathbf{j}^e$ satisfies the equation

$$\mathbf{j}^e = e\int \frac{\partial \varepsilon}{\partial \mathbf{p}} F\, d\tau_{\mathbf{p}}. \tag{2.31}$$

We note that the functions $F'(\mathbf{p}', \mathbf{r}')$ and $F(\mathbf{p}', \mathbf{r}')$, with identical arguments, are connected by the simple relation

$$F'(\mathbf{p}', \mathbf{r}') = F(\mathbf{p}', \mathbf{r}') - p_i' \frac{\partial u_i}{\partial r_k'}\frac{\partial F_0}{\partial p_k'}. \tag{2.32}$$

This means that by substituting (2.32) in (2.12) and in the expression

$$\mathbf{j}^e = n_0 e\dot{\mathbf{u}} + \mathbf{j}' \tag{2.33}$$

for the density of the electron current in the laboratory frame we can change over to (2.30) and (2.31), as can be readily verified directly.

Finally, we note one more canonical transformation, which makes it possible to write the kinetic equation in a form where the quantity $L_{ik} = \lambda_{ik} + m_0 v_i v_k$ appears explicitly. We choose the generating function of the canonical transformation in the form

$$\tilde{\Phi} = \tilde{\mathbf{p}}\mathbf{r}' + m_0\mathbf{v}\mathbf{u}. \tag{2.34}$$

We have

$$p_i' = \frac{\partial \tilde{\Phi}}{\partial r_i'} = \tilde{p}_i + m_0 v_k \frac{\partial u_k}{\partial r_i'}, \qquad \tilde{r}_i = \frac{\partial \tilde{\Phi}}{\partial \tilde{p}_i} = r_i' + m_0 \frac{\partial v_k}{\partial \tilde{p}_i} u_k; \tag{2.35}$$

$$\varepsilon(\tilde{\mathbf{p}}, \tilde{\mathbf{r}}) = \varepsilon'(\mathbf{p}', \mathbf{r}') + \frac{\partial \tilde{\Phi}}{\partial t} = \varepsilon_0(\tilde{\mathbf{p}}) + (\lambda_{ik} + m_0 v_i v_k)\frac{\partial u_k}{\partial \tilde{r}_i}. \tag{2.36}$$

Such a transformation effects the transition to a coordinate system moving with a velocity

$$\tag{2.37}$$

which in the general case depends on $\mathbf{p}$. We obtain for the distribution function $\tilde{F}(\tilde{\mathbf{p}}, \tilde{\mathbf{r}}) = F'(\mathbf{p}', \mathbf{r}')$ the equation

$$\frac{\partial \tilde{F}}{\partial t} + \frac{\partial \tilde{\varepsilon}}{\partial \tilde{\mathbf{p}}}\frac{\partial \tilde{F}}{\partial \tilde{\mathbf{r}}} - \frac{\partial \tilde{\varepsilon}}{\partial \tilde{\mathbf{r}}}\frac{\partial \tilde{F}}{\partial \tilde{\mathbf{p}}} + \frac{2\pi}{\hbar}\sum_{\tilde{\mathbf{p}}_1} |V_{\tilde{\mathbf{p}}\tilde{\mathbf{p}}_1}|^2 (\tilde{F}_{\tilde{\mathbf{p}}} - \tilde{F}_{\tilde{\mathbf{p}}_1}) \times \delta(\tilde{\varepsilon}_{\tilde{\mathbf{p}}} - m_0\mathbf{v}\dot{\mathbf{u}} - \tilde{\varepsilon}_{\tilde{\mathbf{p}}_1} + m_0\mathbf{v}\dot{\mathbf{u}}) = 0, \tag{2.38}$$

and the current density in the laboratory frame is

$$j_i^e = e\int \frac{\partial \tilde{\varepsilon}}{\partial \tilde{p}_i}\tilde{F} d\tau_{\mathbf{p}} + e\dot{u}_j \int d\tau_{\tilde{\mathbf{p}}}\left(\delta_{ij} - m_0 \frac{\partial^2 \varepsilon_0}{\partial \tilde{p}_i \partial \tilde{p}_j}\right) F_{0\tilde{\mathbf{p}}}. \tag{2.39}$$

From (2.12) and (2.33) we can go over directly to (2.38) and (2.39) with the aid of the transformation

$$F'(\mathbf{p}', \mathbf{r}') = F(\mathbf{p}', \mathbf{r}') - m_0 v_i \frac{\partial u_i}{\partial r_k'}\frac{\partial F_0}{\partial p_k'}. \tag{2.40}$$

3. CALCULATION OF THE ABSORPTION COEFFICIENT OF TRANSVERSE SOUND WITH ALLOWANCE FOR THE SOLENOIDAL FIELDS

The propagation of transverse sound in a conductor is accompanied by transverse electric currents $\mathbf{j}$. These currents, in turn, induce transverse electric fields. In the preceding section we disregarded the contribution of these fields to the sound absorption. In the present section we shall take this contribution into account for the case of an isotropic conduction-electron dispersion. The electric field $\mathbf{E}$ is determined from Maxwell's equations[7]

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}. \tag{3.1}$$

Since $\mathbf{j} \sim \exp(i\mathbf{q}\cdot\mathbf{r} - i\omega t)$, we therefore have

$$q^2\mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{j}. \tag{3.2}$$

To determine the current density, we use the kinetic equation (2.30). In the case of an isotropic dispersion, its solution is best sought in the form

$$F(\mathbf{p}, \mathbf{r}) = F_0(\varepsilon) - \dot{\mathbf{p}}\mathbf{u}\left[C + \left(1 - \frac{m_0}{m}\right)\right]\frac{\partial F_0}{\partial \varepsilon} + f_1, \tag{3.3}$$

where $m^{-1} = (\partial^2\epsilon/\partial p^2)_{p_F}$, and we obtain for the function f_1 as $\nu \to 0$ the following equation:

$$(-i\omega + i\mathbf{q}\mathbf{v} + \nu)f_1 + e\mathbf{E}^{\text{eff}}\frac{\partial F_0}{\partial \mathbf{p}} = 0, \tag{3.4}$$

where the effective electric field $\mathbf{E}_{\text{eff}}$ is

$$\mathbf{E}^{\text{eff}} = \mathbf{E} - C\frac{m}{e}\ddot{\mathbf{u}}. \tag{3.5}$$

The total current density $\mathbf{j}$, which is equal to the sum of the electron current $\mathbf{j}^e$ calculated from (2.31) and the lattice current $-n_0 e\dot{\mathbf{u}}$, is

$$\mathbf{j} = \left(C - \frac{m_0}{m}\right)n_0 e\dot{\mathbf{u}} + \sigma\mathbf{E}^{\text{eff}}. \tag{3.6}$$

Let the sound propagate along the ζ axis, and let it be polarized along the ξ axis. Then σ in (3.6) is the component $\sigma_{\xi\xi}(\omega, \mathbf{q})$, equal to

$$\sigma_{\xi\xi} = -e^2\int \frac{v_\xi^2}{-i\omega + iqv_\zeta + \nu}\frac{\partial F_0}{\partial \varepsilon} d\tau_{\mathbf{p}}. \tag{3.7}$$

Calculating this integral with allowance for the inequality $\omega \ll qv_F$, we obtain

$$\sigma = 3\pi n_0 e^2/4qp_F. \tag{3.8}$$

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[7] It is actually necessary to add to Maxwell's equations also the condition for the electroneutrality of the metal, $\operatorname{div}\mathbf{j} = 0$, but this condition is satisfied automatically in the case of transverse sound. To explain the connection with the results of [3] and [16], we note that in our case

$$\overline{\lambda_{ik}(\mathbf{p})u_{ik}} = \overline{L_{ik}(\mathbf{p})u_{ik}} = 0,$$

where the bar denotes averaging over the Fermi surface.

Since (3.4) has the same form as the equation for the correction that must be introduced in the equilibrium distribution function to allow for the field E^{eff}, the expression for the sound absorption coefficient can obviously be represented in the form of the ratio of the Joule loss in this field to the energy flux density of the sound wave wU_S:

$$\Gamma = \frac{1}{2wU_s} \operatorname{Re}\sigma |E^{\text{eff}}|^2. \tag{3.9}$$

Substituting in (3.2) the expression for the current density (3.6), we obtain

$$eE^{\text{eff}} = -m\ddot{u}\left[C + \left(C - \frac{m_0}{m}\right)\frac{\omega_p^2}{c^2q^2}\right]\left(1 - \frac{4\pi i\omega\sigma}{c^2q^2}\right)^{-1}, \tag{3.10}$$

where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the Langmuir frequency.

Substituting (3.10) in (3.9), we obtain the transverse-sound absorption coefficient

$$\Gamma = \frac{4n_0\omega p_F}{3\pi\rho w^2}\left(C - \frac{m_0}{m} + C\frac{\omega^2}{\omega_p^2}\frac{c^2}{w^2}\right)^2\left(1 + \frac{16\omega^4 v_F^2 c^4}{9\pi^2\omega_p^4 w^6}\right)^{-1}. \tag{3.11}$$

This expression depends on two dimensionless parameters. The first of them

$$c^2q^2/4\pi\sigma\omega = 4\omega^2 v_F c^2/3\pi\omega_p^2 w^3 \tag{3.12}$$

is the square of the ratio of the depth of the skin layer (in the case of the anomalous skin effect, characterized by expression (3.8) for the conductivity) to the length of the sound wave. This parameter assumes a value on the order of unity at a frequency

$$\omega_1 \approx (\omega_p w/c)\sqrt{w/v_F},$$

which amounts to approximately 10^9 sec^{-1} for typical metals. The second parameter is

$$(\omega c/\omega_p w)^2. \tag{3.13}$$

It assumes a value on the order of unity at a frequency $\omega_2 \approx \omega_p w/c$ which lies for typical metals in the interval between 10^{10} and 10^{11} sec^{-1}.

When $\omega \ll \omega_1$, the absorption of sound is due exclusively to the solenoidal fields produced by the propagation of the sound. The transverse-sound absorption coefficient is proportional to ω and is equal in order of magnitude to the longitudinal-sound absorption coefficient. By measuring the absorption we can determine the difference $C - m_0/m$. For free electrons $C = 0$ and $m_0/m = 1$, so that when $\omega \ll \omega_1$, formula (3.11) goes over into the expression obtained by Pippard[2] (see also Blount's paper[6], which contains more general assumptions concerning the electron spectrum).

When $\omega_2 \gg \omega \gg \omega_1$, the collisionless absorption is determined as before by the solenoidal field. However, the field intensity decreases with increasing frequency, and with it also the absorption coefficient, which in this region is proportional to ω^{-3}.

Finally, when $\omega \gg \omega_2$, the pure deformational collisionless sound absorption comes into play. The absorption coefficient, which in this frequency interval is again proportional to ω, satisfies formula (2.26) obtained in the preceding section.

Such a frequency dependence of the collisionless absorption of transverse sound was in essence established earlier by Blount[6]. Our result, however, differs from Blount's in the coefficient preceding ω in the region of the high-frequency linear section. The difference reduces to the fact that (in our notation) this coefficient is proportional in our case to C^2 as against $(C - m_0/m)^2$ in Blount's case; the difference is due to allowance for the Stewart-Tolman effect. We recall that for free electrons, $C^2 = 0$ whereas $(C - m_0/m)^2 = 1$. In the low-frequency linear section, the sound absorption coefficient is proportional to $(C - m_0/m)^2$. For free electrons this means, physically, that they interact only with the macroscopic fields produced by the lattice current $-n_0 e\mathbf{u}$.

It should be noted that the last (high-frequency) section of the $\Gamma(\omega)$ plot (3.11) is difficult to observe, since it is necessary to satisfy very stringent conditions in order to be able to neglect the influence of the collisions on the sound absorption in this region. The corresponding inequality is of the form $\omega/\nu \gg v_F/w$ and is obtained by comparing our results with the corresponding formulas of Pippard and Blount.

4. INTERACTION OF ELECTRONS WITH SOUND IN A MAGNETIC FIELD

The quantum-mechanical part of the problem in the presence of a magnetic field $\mathbf{H}$ is not much more complicated than the corresponding problem at $\mathbf{H} = 0$. We therefore confine ourselves to a discussion of the differences, avoiding duplication of cumbersome derivations.

The Hamiltonian of the electron in a magnetic field is obtained from (2.2) by making the substitution

$$\hat{\mathbf{p}} \to \hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\mathbf{r}) \tag{4.1}$$

($\mathbf{A}$ is the vector potential of the external constant magnetic field $\mathbf{H}$).

A similar substitution must be made in the operator $\hat{T}$ (2.9) of the quantum-mechanical canonical transformation. As a result of the canonical transformation of the initial Hamiltonian, we obtain expression (2.11), but with the substitution

$$\hat{\mathbf{p}} \to \hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\mathbf{r}) + \frac{e}{c}[\mathbf{uH}]. \tag{4.2}$$

The last term in this expression is the result of the non-commutativity of the different components of the operator $\mathbf{p} - e\mathbf{A}/c$. This term can be regarded as an increment $\Delta\mathbf{A}$ to the vector potential in the new coordinate frame. It describes the inductive field produced in this coordinate frame:

$$\mathbf{E}^{(i)} = -\frac{1}{c}\frac{\partial \Delta\mathbf{A}}{\partial t} = \frac{1}{c}[\mathbf{uH}]. \tag{4.3}$$

We confine ourselves further to a classical magnetic field. To change over to the classical treatment it is necessary to make a substitution of the type (4.2) in the classical Hamiltonian in the co-moving coordinate system, i.e., it is necessary to replace the canonical momentum $\mathbf{p}'$ by the combination

$$\mathbf{P}' - \frac{e}{c}\mathbf{A}(\mathbf{r}') + \frac{e}{c}[\mathbf{uH}]. \tag{4.4}$$

It is easy to verify that this leads to the appearance of the following additional terms in the left-hand side of the kinetic equation (2.12):

$$\frac{e}{c}[\mathbf{uH}]\frac{\partial F'}{\partial \mathbf{p}'} + \frac{e}{c}\left[\frac{\partial \varepsilon'}{\partial \mathbf{p}'}\mathbf{H}\right]\frac{\partial F'}{\partial \mathbf{p}'}. \tag{4.5}$$

Here $\mathbf{p}'$, as in the preceding section, denotes the quasimomentum of the electron, which no longer coincides with the canonical momentum $\mathbf{P}'$ if a magnetic field is present. Expression (2.18) for the current density remains the same.

The presence of an inductive term—the first term in (4.5)—in the kinetic equation was in fact the reason why one of us[9] introduced the concept of two different mechanisms of collisionless sound absorption—inductive and deformation. This distinction between the absorption mechanisms was subsequently used in a number of papers. It is therefore useful to discuss the extent to which such a distinction is unique and is meaningful in general.

We start with the remark that the classical variables can be subjected to a canonical transformation similar to (2.28) corresponding to a transition to the laboratory frame. As a result we obtain a Hamiltonian in the form (2.29), in which the momentum $\mathbf{p}$ should be replaced by $\mathbf{P} - e\mathbf{A}/c$, where $\mathbf{P}$ is the canonical momentum. The kinetic equation in terms of the variables $\mathbf{p} = \mathbf{P} - e\mathbf{A}/c$ and $\mathbf{r}$ is

$$\frac{\partial F}{\partial t}+\frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{r}}-\frac{\partial \varepsilon}{\partial \mathbf{r}}\frac{\partial F}{\partial \mathbf{p}}+e\mathbf{E}\frac{\partial F}{\partial \mathbf{p}}+\frac{e}{c}\left[\frac{\partial \varepsilon}{\partial \mathbf{p}}\mathbf{H}\right]\frac{\partial F}{\partial \mathbf{p}} +\frac{2\pi}{\hbar}\sum_{\mathbf{p}_1}|V_{\mathbf{pp}_1}|^2(F_{\mathbf{p}}-F_{\mathbf{p}_1})\delta(\varepsilon_{\mathbf{p}}-\mathbf{pu}-\varepsilon_{\mathbf{p}_1}+\mathbf{p}_1\mathbf{u})=0 \tag{4.6}$$

This equation, which does not contain the inductive term explicitly, coincides with the corresponding equation of Kontorovich[16]. We can go over from (2.12) with the additional terms (4.5) directly to (4.6) by making the substitution (2.32).

Finally, it would also be possible to carry out a canonical transformation of the type (2.35), signifying a transition to a coordinate system moving together with the electrons dragged by the lattice (which is equivalent to the substitution (2.40)). In the left side of the kinetic equation (2.38) there appear terms of the form

$$\frac{c}{e}[\Delta\mathbf{v}\mathbf{H}]\frac{\partial \tilde{F}}{\partial \tilde{\mathbf{p}}}+\frac{e}{c}\left[\frac{\partial \tilde{\varepsilon}}{\partial \tilde{\mathbf{p}}}\mathbf{H}\right]\frac{\partial \tilde{F}}{\partial \tilde{\mathbf{p}}}. \tag{4.7}$$

We thus verify that the concrete form of the inductive term in the kinetic equation depends on the choice of the coordinate system. In particular, the inductive term does not appear explicitly at all in an equation of the form (4.6). This means, in particular, that when collisionless sound absorption (of the Landau-damping type) is considered, it is meaningless to consider the inductive absorption separately from the deformation absorption, since the inductive effects cannot exceed the deformation effects in order of magnitude[8]. This is illustrated by the example in the next section.

5. CALCULATION OF THE ABSORPTION COEFFICIENT OF TRANSVERSE SOUND IN A METAL IN A MAGNETIC FIELD

Let us calculate the coefficient of collisionless absorption of transverse sound in a strong magnetic field, for an isotropic quadratic electron spectrum $\epsilon_0(\mathbf{p}) = p^2/2m$. We assume that the magnetic field $\mathbf{H}$ is directed along the z axis, the wave vector $\mathbf{q}$ of the sound lies in the yz plane, and the sound polarization makes an angle χ with this plane.

We seek the solution of the kinetic equation (4.6) in the form (3.3). We obtain the following equation for the function f_1 as $\nu \to 0$:

$$(-i\omega + i\mathbf{q}\mathbf{v})f_1+\frac{e}{c}[\mathbf{v}\mathbf{H}]\frac{\partial f_1}{\partial \mathbf{p}}+e\mathbf{E}^{\text{eff}}\frac{\partial F_0}{\partial \mathbf{p}}=0. \tag{5.1}$$

It coincides with the equation for the non-equilibrium addition to the electron distribution function in an external electric field

$$\mathbf{E}^{\text{eff}}=\mathbf{E}+\mathbf{E}^{(1)}, \tag{5.2}$$

$$\mathbf{E}^{(1)}=\left(C+1-\frac{m_0}{m}\right)\frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]-C\frac{m}{e}\ddot{\mathbf{u}}. \tag{5.3}$$

By analogy with (3.9), we obtain for the absorption coefficient

$$\Gamma=\frac{1}{2wU_s}\operatorname{Re}\sum_{ik}\sigma_{ik}E_i^{\text{eff}*}E_k^{\text{eff}}. \tag{5.4}$$

The electric field $\mathbf{E}$ is determined from Maxwell's equations (3.1), which reduce to

$$\operatorname{rot}\operatorname{rot}\mathbf{E}=\frac{4\pi i\omega}{c^2}\mathbf{j}, \tag{5.5}$$

and the electroneutrality condition, which can be written in the form

$$\operatorname{div}\mathbf{j}=0. \tag{5.6}$$

Here $\mathbf{j}$ is the total current density (3.6).

It is then convenient to calculate the effective field $\mathbf{E}^{\text{eff}}$ directly. We choose a coordinate system ξ, η, ζ, with the ξ axis directed along x, the ζ axis along $\mathbf{q}$, and the η axis in the plane of z and $\mathbf{q}$. Solving Eqs. (5.5) and (5.6), we obtain

$$\begin{aligned}E_\xi^{\text{eff}}=&\left[\left(q^2E_\xi^{(1)}+\frac{4\pi i\omega}{c^2}j_\xi^{(1)}\right)\left(q^2-\frac{4\pi i\omega}{c^2}s_{\eta\eta}\right)\right.\\&\left.+\frac{4\pi i\omega}{c^2}s_{\xi\eta}\left(q^2E_\eta^{(1)}+\frac{4\pi i\omega}{c^2}j_\eta^{(1)}\right)\right]\left[\left(q^2-\frac{4\pi i\omega}{c^2}s_{\xi\xi}\right)\right.\\&\left.\times\left(q^2-\frac{4\pi i\omega}{c^2}s_{\eta\eta}\right)+\left(\frac{4\pi\omega}{c^2}\right)^2s_{\xi\eta}s_{\eta\xi}\right]^{-1},\end{aligned} \tag{5.7}$$

where

$$s_{\beta\beta'}=\sigma_{\beta\beta'}-\sigma_{\beta\zeta}\sigma_{\zeta\beta'}/\sigma_{\zeta\zeta},\qquad \mathbf{j}^{(1)}=(C-m_0/m)n_0e\mathbf{u},$$

and β and β' run through the values of ξ and η. The expression for E_η^{eff} is obtained from (5.7) by making the substitution $\xi \rightleftarrows \eta$, and

$$E_\zeta^{\text{eff}}=-\frac{\sigma_{\zeta\xi}}{\sigma_{\zeta\zeta}}E_\xi^{\text{eff}}-\frac{\sigma_{\zeta\eta}}{\sigma_{\zeta\zeta}}E_\eta^{\text{eff}}. \tag{5.8}$$

To interpret (5.4), we use the general expressions for the components of the tensor $\sigma_{ik}(\omega, \mathbf{q}, \mathbf{H})$, given in the review of Kaner and Skobov ([17], p. 634, formula (122)). As a result we find that when $|e|H/mc > \omega + |q_z|v_F$ the value of the absorption coefficient Γ is

$$\begin{aligned}\Gamma=\frac{1}{\rho w|\dot{u}|^2}\frac{3\pi n_0e^2}{2m}\int_{-1}^{1}&|-i\sqrt{1-\mu^2}J_1(\varkappa)E_x^{\text{eff}}\\&+\mu J_0(\varkappa)E_z^{\text{eff}}|^2\delta(\omega-q_zv_F\mu)\,d\mu,\end{aligned} \tag{5.9}$$

[8] One of us [9] has stated that situations where the inductive absorption predominates are possible. We wish to note that this statement is not valid for cases when the solenoidal fields can be neglected.

where

$$\varkappa = -q_y v_F \sqrt{1-\mu^2}(mc/eH), \tag{5.10}$$

$J_0(\kappa)$ and $J_1(\kappa)$ are Bessel functions.

Let us consider further the case of strong magnetic fields, when

$$qR \ll 1, \tag{5.11}$$

where $R = p_F c/|e|H$ is the Larmor radius of the electron. We can confine ourselves here to the lowest terms in the expansion of the Bessel function in the argument κ. Furthermore, we assume that the contribution of the solenoidal fields to the sound absorption can be neglected. This is permissible if

$$4\pi|\sigma_{\xi\eta}|\omega/c^2q^2 = 4\pi n_0|e|\omega/Hcq^2|\cos\theta| \ll 1. \tag{5.12}$$

The condition (5.12) is satisfied at the frequencies

$$\omega \gg 4\pi n_0|e|w^2/Hc|\cos\theta|. \tag{5.13}$$

(Here θ is the angle between the vectors **q** and **H**.) Then

$$E_\xi^{\text{eff}} = E_\xi^{(t)},\ E_\eta^{\text{eff}} = E_\eta^{(t)}. \tag{5.14}$$

Determining E_ζ^{eff} from formula (5.8) and calculating the integral in (5.9), we get

$$\Gamma = \frac{3\pi n_0 m}{8\rho w} q v_F \left(C + 1 - \frac{m_0}{m}\right)^2 \cos^2\chi \sin^2\theta|\cos\theta|. \tag{5.15}$$

We have neglected here small terms of the order w/v_F. The Stewart-Tolman effect, as can be readily verified, makes no contribution to the absorption in this approximation.

The quantity $C + 1 - m_0/m$ in (5.15) has the meaning of a dimensionless constant for the coupling of the electrons with the transverse sound. At **H** = 0 this role was played by the quantity C (see formula (2.26)). If it is desired to separate the inductive effects, it is most natural to relate their contribution to the difference between these constants. This corresponds to an expression for the inductive field in the form

$$\frac{1}{c}\left(1 - \frac{m_0}{m}\right)[\dot{\mathbf{u}}\mathbf{H}].$$

Incidentally, this concrete example likewise shows that the contribution from the inductive effects does not exceed, in order of magnitude, the contribution from the deformation effects, and must be taken into account together with the latter.

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APPENDIX 1

DERIVATION OF AN EXPRESSION FOR THE SOUND ABSORPTION COEFFICIENT

Let us consider an acoustic wave packet occupying a bounded region of space, and let us calculate the acoustic energy dissipated per unit time as a result of the interaction with the conduction electrons. As is well known, this energy is equal to TS, where

$$S = -\int d^3r' \int F_{p'} \ln F_{p'}\, d\tau_{p'} \tag{A.1.1}$$

is the electron entropy and T is the temperature. Putting $F' = F_0(\epsilon') + f'$, we can write the expression for $T\dot{S}$ in the form

$$T\dot{S} = \int d^3r' \int \left(\varepsilon'_{p'} - \zeta_0 + \frac{f_{p'}}{\partial F_0/\partial\varepsilon'}\right)\dot{F}_{p'}\, d\tau_{p'}. \tag{A.1.2}$$

We first transform the integral of $(\epsilon'_{p'} - \zeta_0)\dot{F}'_{p'}$. To this end, we express $\dot{F}'_{p'}$ with the aid of the kinetic equation (2.12). After straightforward but rather cumbersome transformations, it reduces to the form

$$\int \mathbf{j}'\mathbf{E}\, d^3r. \tag{A.1.3}$$

In calculating the integral of $f'_{p'}\dot{F}'_{p'}(\partial F_0/\partial\epsilon')^{-1}$, we should substitute $(\partial F_0/\partial\epsilon)\dot{\epsilon}'$ for $\dot{F}'_{p'}$. Adding this integral to the quantity (A.1.3), averaging over the period of the sound, and dividing by the sound energy $\frac{1}{2}\int\rho|\dot{u}|^2 d^3r$ and by the speed of sound w, we obtain the following final expression for the absorption coefficient:

$$\Gamma = \frac{1}{2wU_s}\operatorname{Re}\left(\mathbf{j}'^*\mathbf{E} + \int \varepsilon'^* f_{p'}\, d\tau_{p'}\right). \tag{A.1.4}$$

APPENDIX 2

Let us investigate the role of the Stewart-Tolman effects in collisionless absorption of longitudinal sound. Let the sound propagate along the symmetry axis of the crystal, so that there are no transverse electric fields. The longitudinal electric field **E** differs from zero in this case, and it is therefore necessary to add in the left-hand side of (2.12) the term $e\mathbf{E}\partial F'/\partial\mathbf{p}'$. To facilitate comparison with the results of earlier investigations, we replace λ_{ik} in (2.12) by $\Lambda_{ik} = \lambda_{ik} - \bar{\lambda}_{ik}$, where the bar denotes averaging over the Fermi surface. This substitution makes it possible to separate the part of the macroscopic field corresponding to the static deformation. The field **E** is determined from the neutrality condition

$$\int F'(\mathbf{p}')\, d\tau_{p'} = 0. \tag{A.2.1}$$

We seek a solution in the form $F' = F_0(\epsilon') + f'$ and obtain

$$f' = -\frac{\partial F_0}{\partial\varepsilon'}\frac{\Lambda_{ik}\dot{u}_{ik} + e\mathbf{E}_1\mathbf{v}}{i(-\omega + \mathbf{qv}) + \nu}, \tag{A.2.2}$$

where $\mathbf{E}_1 = \mathbf{E} - (m_0/e)\ddot{\mathbf{u}}$, and $\nu \to 0$. From (A.2.1) we obtain the following expression with which to determine the field $\mathbf{E}_1$:

$$\int \frac{\partial F_0}{\partial\varepsilon'}\frac{\Lambda_{ik}\dot{u}_{ik} + e\mathbf{E}_1\mathbf{v}}{-i\omega + i\mathbf{qv} + \nu}\, d\tau_{p'} = 0. \tag{A.2.3}$$

From this, with allowance for the condition $\operatorname{div}\mathbf{j}' = 0$ and formula (2.20), we obtain for the absorption coefficient

$$\Gamma = -\frac{\pi}{2wU_s}\int \frac{\partial F_0}{\partial\varepsilon'}|\Lambda_{ik}\dot{u}_{ik} + e\mathbf{E}_1\mathbf{v}|^2\,\delta(\omega - \mathbf{qv})\, d\tau_{p'}. \tag{A.2.4}$$

This expression coincides with that obtained in[3]. Actually, the longitudinal field $\mathbf{E}_1$ is determined from Eq. (A.2.3), which does not contain terms proportional to m_0. This means that allowance for the Stewart-Tolman effect does not change the absorption coefficient of the longitudinal sound. The reason lies in the fact that the longitudinal Stewart-Tolman field is cancelled out by part of the macroscopic field.

On going over to the limit of free electrons, when

$$\Lambda_{ik}(\mathbf{p}') \to -\frac{p_i' p_k'}{m_0} + \frac{p_F^2}{3m_0}\delta_{ik}, \qquad (A.2.5)$$

we are left in (A.2.4)) with only the contribution from the macroscopic electric field $\mathbf{E} + \bar{\lambda}_{ik}\partial u_{ik}/\partial \mathbf{r}$, the interaction with which is indeed the cause of sound absorption in this case.

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192