EXCITATION AND AMPLIFICATION OF GALVANOMAGNETIC AND GALVANOMAGNETIC-ACOUSTIC WAVES

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It is shown that when transverse electromagnetic waves are excited in a conductor, excitation of coupled acoustic and electromagnetic waves is possible. Propagation of an electromagnetic wave through a current-carrying crystal is investigated. The magnetic field dependence of the electric field exciting the oscillations is found to be non-monotonic.

1. INTRODUCTION

A theory of the microwave radiation from crystals following passage of sufficiently strong currents was constructed in[1]. It was shown that in the presence of a current in the medium (or a current and an external magnetic field), transverse, almost purely magnetic waves are connected with the lattice vibrations. Therefore, upon excitation of transverse waves, the simultaneous excitation of coupled waves of the field and the sound is possible. The amplitude of the acoustic vibrations increases sharply at certain values of the drift velocity, i.e., the excitation of the sound has a quasi-resonant character.

If an electromagnetic wave is incident on a crystal along which a current flows, then there are three possibilities depending on the current density j or on the current density and the magnetic field H (in the case of a strong magnetic field perpendicular to the current). For a wave frequency higher than some critical value, the presence of a current or current and magnetic field has no effect on the damping of the wave. However, the critical frequency itself is proportional to j^2 or jH.

For much lower frequencies, the incident wave is much less damped than in the absence of the current. Finally, for frequencies less than some definite frequency (proportional to j^2 or jH), amplification of the wave is possible. In the case of a strong magnetic field parallel to the current, only the first two possibilities exist and amplification of the wave is not possible.

Upon increase in the magnetic field perpendicular to the current, the electric field necessary for excitation of the oscillations decreases, reaching a minimum; thereafter it again increases. This is due to the fact that the particle concentration is nonuniformly distributed in the Hall direction and the current density is higher in the vicinity of the surface of the conductor for the same electric field. On the other hand, excitation of waves is impossible for a certain range of the magnetic field.

2. COUPLED GALVANOMAGNETIC-ACOUSTIC WAVES

Upon excitation of transverse galvanomagnetic waves, coupled galvanomagnetic-acoustic oscillations are generated. Their production is not connected with the ordinary[2] electro-acoustic instability, for when $kR < 1$ the transverse galvanomagnetic waves are excited at drift velocities v much greater than the sound velocity s, and under these conditions excitation of sound is virtually impossible. The transverse galvanomagnetic waves can excite acoustic oscillations if the frequencies of the two coincide, i.e., $(\mathbf{k}\cdot\mathbf{v}) = ks$. Since $v \gg s$, this is possible only under the condition that the wave vector is almost perpendicular to the drift velocity.

The set of equations describing the coupled oscillations of the crystalline lattice and the field have the form

$$\partial \mathbf{H}'/\partial t = -c \operatorname{rot} \mathbf{E}', \tag{1}$$

$$\mathbf{E}' = \eta \mathbf{j}' + \eta_1[\mathbf{j}'\mathbf{H}] + \eta_1[\mathbf{j}\mathbf{H}'] + \eta_2\mathbf{H}'(\mathbf{j}\mathbf{H}) + \eta_2\mathbf{H}(\mathbf{j}'\mathbf{H}) + \eta_2\mathbf{H}(\mathbf{j}\mathbf{H}') + \left[\frac{\partial \mathbf{u}}{\partial t}, \mathbf{H}\right] \Big/ c, \tag{2*}$$

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} + \nu \frac{\partial}{\partial t}\mathbf{u} - s^2\mathbf{u} = \frac{1}{\rho c}\{[\mathbf{j}\mathbf{H}'] + [\mathbf{j}'\mathbf{H}]\} \tag{3}$$

(ρ is the density, u the displacement, ν the kinematic viscosity). Here

$$\eta = \frac{\sigma}{\sigma^2 + (\sigma_1 H)^2}, \quad \eta_1 = -\frac{\sigma_1}{\sigma^2 + (\sigma_1 H)^2}$$

$$\eta_2 = \frac{(\sigma\sigma_2 - \sigma_1^2)}{[\sigma^2 + (\sigma_1 H)^2][\sigma + \sigma_2 H^2]}, \quad \eta - \eta_2 H^2 = \eta_0,$$

where σ, σ_1, and σ_2 are the ordinary, Hall, and focusing conductivities; $\eta_0 = \sigma_0^{-1}$, and σ_0 is the conductivity in the absence of a magnetic field. For all mechanisms of scattering of the current carriers, we have in practice $\eta\eta_2 \geq \eta_1^2$. The force due to the deformation interaction with electrons is absent, since it is proportional to the gradient of the oscillations of the concentration and is equal to zero for transverse oscillations.

Let us first consider the case $H_0 = 0$ (where H_0 is the external magnetic field).

Substituting (2) in (1), we obtain the result that the effect of the term containing u is unimportant if uω $u\omega \cdot 4\pi ne/H'ck < 1$. We shall see below that this condition is always satisfied. (We note that, if we use for H′ the result obtained in[1], $H'^2 = (c/\mu_-)^2(j - j_{cr})/j_{cr}$,

*[jH′] ≡ j × H′.

where j_{cr} is the current necessary for the excitation of galvanomagnetic waves, then the resultant condition is violated only for a flux density of the acoustic energy that is larger than $\rho c^2 s(ck/\sigma)(j - j_{cr})/j_{cr}$, which cannot be obtained even for small supercriticality.) We therefore neglect the term $c^{-1}[\partial u/\partial t \times H]$ in (2). Then the conditions for the excitation of magnetic waves are found in[1]. For solution of Eq. (3), we limit ourselves to the case of a free lateral surface of the cylinder along which the current density j flows. On this surface,

$$(\sigma_{ik} + T_{ik})n_k = 0 \quad \text{for } r = R \tag{4}$$

(n_k is the unit vector normal to the surface, σ_{ik} and T_{ik} are the elastic and Maxwell stress tensors). We seek a solution of (3) in the form $u = u^0 f(r) \exp[ikz \pm i\varphi - i\omega t]$. Taking it into account that $H'_z \ll H'_{r,\varphi}$ from[1], and also the fact that, with accuracy up to $(kR)^2 \ll 1$, the quantities $H'_{r,\varphi}$ depend on r, and that $v^2 \gg s^2$, we find

$$\begin{aligned} u_r &= C_r J_1\left\{kr\left[\frac{v^2}{s^2}\left(1+\frac{i\omega\nu}{s^2}\right)^{-1} - 1\right]^{1/2}\right\} + \frac{j_0 H_\varphi'}{\rho c\omega^2}, \\ u_\varphi &= C_\varphi J_1\left\{kr\left[\frac{v^2}{s^2}\left(1+\frac{i\omega\nu}{s^2}\right)^{-1} - 1\right]^{1/2}\right\} - \frac{j_0 H_r'}{\rho c\omega^2}, \\ u_z &= C_z J_1\left\{kr\left[\frac{v^2}{s^2}\left(1+\frac{i\omega\nu}{s^2}\right)^{-1} - 1\right]^{1/2}\right\} + \frac{H_j \operatorname{rot}_z H'}{4\pi\rho\omega^2}. \end{aligned} \tag{5}$$

Here J_1 is a Bessel function of first order, v the drift velocity, and H_j the field of the current.

Substituting (5) in (4), we get a system for the determination of C_r, C_φ, and C_z. For their solution we must consider two cases: $kRv/s \lesssim 1$ and $kRv/s \gg 1$. In the first case, the frequency of the galvanomagnetic waves does not coincide with the frequency of the acoustic waves and the amplitude is small. We limit ourselves to the second case. Let

$$kRv/s = \pi(p + 3/4), \quad p = 1, 2, \ldots. \tag{6}$$

The denominators of the expressions for C_r, C_φ, and C_z decrease in the ratio

$$\frac{\nu\omega}{s^2}\frac{1}{\pi^2(p+3/4)^2} \ll 1.$$

The numerators of these expressions decrease more weakly; consequently, at $kRv/s = \pi(p + 3/4)$ there is a significant increase in the sound amplitude, due to the coincidence of the galvanomagnetic and acoustic oscillation frequencies. In this case,

$$u_z^0 = \frac{H_j H'_{0\varphi}}{N}\left(kR\frac{v}{s}\right)^{1/2} l_{ph}(1+\gamma)$$

(here γ is Poisson's ratio, N Young's modulus, l_{ph} the damping length for sound under scattering by phonons, and H'_0 the amplitude of the oscillations of the magnetic field). The values u_r^0 and u_φ^0 are smaller by the factors kR and $\nu\omega/s^2$, respectively. The ratio of the acoustic energy density to the energy density of the oscillatory magnetic field is

$$\frac{8\pi\rho u^2\omega^2}{H'^2} = 8\pi^4\left(p+\frac{3}{4}\right)^3\left(\frac{v}{s}\right)^2\frac{(nel_{ph})^2}{\rho c^2} \tag{7}$$

for $v/s \approx 30$, $n \approx 3\times10^{15}$ cm^{-3}, $l_{ph} \approx 10$ cm and p = 1, the ratio amounts to 10^{-2}.

The ratio of the flux density of acoustic energy to the Joule losses per unit length of the crystal (the efficiency of the sound generator) is of interest. This quantity is equal to

$$s\left(\frac{c}{\mu_-}\right)^2\frac{\sigma_0}{l_{ph}j_{cr}^2}\frac{j-j_{cr}}{j_{cr}}\left(kR\frac{v}{s}\right)^3\left(\frac{v}{s}\right)^2\frac{(nel_{ph})^2}{\rho c^2} \sim n\mu^2E^3$$

and can amount to 10^{-2}.

Let us estimate the discarded term in (2). We have

$$\frac{4\pi neu\omega}{ckH'} \approx \frac{nel_{ph}H}{\rho s^2}\frac{v}{c}\left(kR\frac{v}{s}\right)^{1/2},$$

which is less than unity up to $n \approx 10^{19}$ cm^{-3}, and neglect of the term $c^{-1}[\partial u/\partial t, H]$ in (2) is valid. Calculation shows $|\operatorname{curl} u|/|\operatorname{div} u| \approx v/s \gg 1$, i.e., transverse sound is excited.

The presence of a strong external magnetic field does not affect the condition for sound excitation, but leads to an increase in amplitude by the factor $\mu_- H_0/c$ (if $c/\sqrt{\mu_-\mu_+} < H_0 < c/\mu_+$; $\mu_\mp$ is the mobility). The efficiency here can increase by one order of magnitude.

We note that the simultaneous excitation of transverse galvanomagnetic and sound waves was observed in a number of experiments (see the review[3]).

3. AMPLIFICATION OF AN ELECTROMAGNETIC WAVE IN A CRYSTAL ALONG WHICH A CURRENT IS FLOWING

If a current of density j flows along the axis of a conductor of cylindrical shape (length L and radius $R \ll L$), then, as was shown in[1], transverse waves can be produced in the conductor, with frequency ω and wave vector k (along the axis of the cylinder) which, for $kR \gg 1$ satisfy the dispersion relation

$$k^2 - \frac{i\eta_1}{2c\eta}(\mathbf{kj}) - \left(\frac{2\pi j}{c}\right)^2\frac{\eta_2}{\eta} - \frac{4\pi i\omega}{\eta c^2} = 0. \tag{8}$$

Solving (8) for k, we find that the presence of a constant current affects the value of the wave vector only for frequencies ω which are equal to

$$\omega \lessapprox \omega_{max} = \pi j^2\eta_2. \tag{9}$$

For $\omega < \omega_{max}$, the expansion in this parameter gives the following values of the roots k_1 and k_2 of Eq. (8):

$$\begin{aligned} k_1 &= \frac{2\pi j}{c}\sqrt{\frac{\eta_2}{\eta}} + i\left[\frac{\eta_1(\mathbf{kj})}{4c\eta k} + \frac{\omega}{2\pi cj\sqrt{\eta\eta_2}}\right], \\ k_2 &= \frac{2\pi j}{c}\sqrt{\frac{\eta_2}{\eta}} + i\left[\frac{\eta_1(\mathbf{kj})}{4c\eta k} - \frac{\omega}{2\pi cj\sqrt{\eta\eta_2}}\right]. \end{aligned} \tag{10}$$

It is seen from (10) that for frequencies less than

$$\frac{j^2}{2}\frac{\sqrt{\eta_2\eta_1^2}}{\eta} = \frac{1}{2\pi}\sqrt{\frac{\eta_1^2}{\eta\eta_2}}\,\omega_{max} < \omega_{max}, \tag{11}$$

the signs of Im k_1 and Im k_2 are alike and coincide with the sign of $\eta_1(\mathbf{k}\cdot\mathbf{j})$, while for $\omega > \omega_{min}$, the signs of Im k_1 and Im k_2 are different.

For $\omega < \omega_{min}$ and $\eta_1(\mathbf{k}\cdot\mathbf{j}) > 0$, two damped waves are propagated in the cylinder; if $\omega < \omega_{min}$ and $\eta_1(\mathbf{k}\cdot\mathbf{j}) < 0$, both waves grow. Since Im $k_{1,2}$ < Re $k_{1,2}$, then the condition for the validity of (8), $kR \ll 1$, takes the form $2\pi c^{-1} jR\sqrt{\eta_2/\eta} \ll 1$ or, upon substitution for η and η_2, $2\pi jR\mu_-/c^2 \ll 1$.

If an electromagnetic wave of frequency ω is incident at one end of the cylinder, of frequency ω such that $\omega_{max} > \omega > \omega_{min}$, then, as can be shown analytically,[4] the amplitude of the wave that comes out from the other end is smaller than the amplitude of the incident wave in the ratio

$$\frac{\omega}{j}\sqrt{\frac{\eta}{\eta_2}}\exp\frac{\omega L}{2\pi cj\sqrt{\eta\eta_2}},$$

if the length L of the cylinder is such that the exponent here is larger than unity. The amplitude decrease is essentially different than in the absence of a current, when it is determined by the skin effect and has the form $\exp(\omega L/2\pi\eta c^2)^{1/2}$.

If $\omega < \omega_{min}$, then amplification of the incident wave is possible. The ratio of the field of the emerging wave to the field of the incident wave (with accuracy to $ck\eta < 1$) is

$$2\pi j\sqrt{\eta\eta_2}\exp\left[\frac{\eta_1 jL}{c\eta}\right]\left\{\exp\left[i\frac{2\pi jL}{c}\sqrt{\frac{\eta_2}{\eta}}+\frac{\omega L}{jc\sqrt{\eta\eta_2}}\right]\right.$$
$$\left.-\exp\left[-i\frac{2\pi jL}{c}\sqrt{\frac{\eta_2}{\eta}}-\frac{\omega L}{jc\sqrt{\eta\eta_2}}\right]\right\}^{-1}. \quad (12)$$

For $L > (c\eta/\eta_1 j)\ln|2\pi j\sqrt{\eta\eta_2}|$, the modulus of (12) is greater than unity. For frequencies much smaller than $jc\sqrt{\eta\eta_2}/L$, the exponent in the denominator of (12) can be neglected and for

$$\frac{jL}{c}\sqrt{\frac{\eta_2}{\eta}}=p, \quad p=1,2,\ldots \quad (13)$$

a sharp maximum occurs in the amplification, for which the increase in the amplitude is equal to $\exp(p\sqrt{\eta_1^2/\eta\eta_2})$. From the condition $kR < 1$, we get the result that $p_{max} < L/R$, i.e., the greatest amplification is less than $\exp(LR^{-1}\sqrt{\eta_1^2/\eta\eta_2})$; for $L/R \approx 10$, an amplification by a factor of ten or greater is possible.

We estimate the characteristic values ω_{max}, ω_{min}. For tin antimonide at liquid nitrogen temperatures, current density 3×10^3 amp/cm^2, and concentration 10^{15} cm^{-3}, we have $\omega_{max} \approx 3\times 10^8$ sec^{-1} and $\omega_{min} \approx 10^8$ sec^{-1}; in bismuth at liquid helium temperatures and the same current densities, $\omega_{max} \approx 10^7$ sec^{-1}, and in typical metals with both types of carriers, $\omega_{max} \approx 10^2$ sec^{-1}.

In the presence of a strong external magnetic field $c/\mu_+ > H_0 > c/\sqrt{\mu_-\mu_+}$ parallel to the current, the dispersion relation has the form

$$k^2+\frac{4\pi(\mathbf{kj})}{cH_0}-\frac{4\pi\omega}{\eta_1 H_0 c^2}\left[1-\frac{i\eta}{|\eta_1 H_0|}\right]+\frac{4\pi i\eta_2}{\eta_1 H_0}\left(\frac{2\pi j_0}{c}\right)^2=0. \quad (14)$$

In the absence of dissipation, the frequency is

$$\omega=\mathbf{kv}+\frac{ck(\mathbf{kH}_0)}{4\pi ne},$$

i.e., it corresponds to waves that are more complicated than drift (for H = 0) or helicoidal (for j = 0) waves. We call these waves galvanomagnetic. As also for H = 0, the presence of a constant current does not show itself in the values of the wave vector at frequencies greater than $\omega^*_{max} = |\eta_1| j^2/H_0$. If the same frequency is less than this value, then

$$k_1=-\frac{4\pi j}{cH_0}\left[1-\frac{ic}{4\mu_- H_0}\right], \quad k_2=\frac{2\pi j}{cH_0}\left[\frac{\omega}{\omega^*_{max}}-\frac{ic}{2\mu_- H_0}\right].$$

Im k_1 and Im k_2 have opposite signs, and the incident wave is damped. A singular case is that of frequencies close to

$$\omega^*_{min}=\frac{\eta_2}{\eta}|\eta_1 H_0|\left(\frac{2\pi j}{c}\right)^2;$$

for $\omega = \omega^*_{min}$ the imaginary terms in (14) cancel, $k_{1,2}$ are purely real, and the damping of the wave is absent; for $|(\omega - \omega^*_{min})/\omega^*_{min}| \ll 1$, the damping of the waves is small.

We now consider the case of an external magnetic field parallel to the current. If this field is perpendicular to the current, then the phenomena which takes place at H = 0 are preserved. However, in order to avoid the mathematical complications, we consider a plate (of thickness much less than its other dimensions), with current flowing on the surface, and with an impressed magnetic field $c/\mu_+ > H_0 > c/\sqrt{\mu_-\mu_+}$ in this same plane perpendicular to the surface. It is seen that for $2\pi j > ckH_0$, the results described by Eqs. (9)—(13) are generally unchanged. For the opposite inequality,

$$\omega^*_{max}=ckH_0 j_0\eta_2, \quad \omega^*_{min}=\left[ckH_0 j^3\frac{\eta_1^2\eta_2}{\eta}\right]^{1/2}.$$

Amplification of the incident wave is possible for $\omega < \omega^*_{min}$, if

$$L>\frac{c\eta}{|\eta_1 j|}\ln|2ck\eta_1 H_0|;$$

rapid increase in the wave is possible for these same frequencies as before if

$$L\left[\frac{\eta_2}{\eta}\frac{kH_0 j_0}{c}\right]^{1/2}=p, \quad p=1,2,\ldots$$

We now consider the amplification of coupled galvanomagnetic-acoustic waves. In this case, in addition to the condition $\omega < \omega_{min}$, the condition of quasi-resonance (6) is also necessary. These conditions are compatible if the current density is

$$j>\left[\frac{2\pi(p+{}^3/_4)s}{R}\right]^{1/2}\left[\frac{\eta}{\eta_2\eta_1^2}\right]^{1/4}.$$

Taking into account the condition of validity of our considerations found previously, we obtain the result that the amplification of the coupled waves is possible if

$$\sigma<\frac{c^2}{Rs}\frac{1}{(p+{}^3/_4)(2\pi)^3}.$$

Here two cases are possible. If

$$j^2<j_1^2=2\left(p+\frac{3}{4}\right)\frac{c^2}{Rl_{ph}}\sqrt{\frac{\eta^3}{\eta_2\eta_1^2}}$$

then the ratio of the density of acoustic energy to the magnetic is determined by Eq. (7). For the opposite case, it decreases by the factor j^2/j_1^2.

4. EFFECT OF THE TRANSVERSE MAGNETIC FIELD ON THE CRITICAL ELECTRIC FIELD

In Sec. 2, we considered the case of a plate put in a magnetic field that was located in its plane and perpendicular to the current flowing in it. In this case, it was assumed that the concentration of current carriers was distributed uniformly in the Hall direction. This is true only in the case in which *the thickness of* the plate is so large that recombination in the volume establishes the equilibrium concentration at every point. Here we consider the opposite limiting case of the plate, when one can neglect recombination in the volume, i.e., the thickness $d \ll L_D$ (L_D is the diffusion length). We simultaneously assume that the recombination rate is small. Then the concentration of current carriers is

$$n(x)=\bar{n}\frac{d}{L_{EH}}\exp\frac{x-d/2}{L_{EH}}. \qquad (15)$$

Here $x = \pm d/2$ corresponds to the edges of the plate; $L_{EH} = [D_-(H) + D_+(H)]E^{-1}[\mu_-(H) + \mu_+(H)]^{-1}$; in the limit of strong and weak fields, $L_{EH} \sim H^{-1}$. For sufficiently strong fields $L_{EH} < d$, and the sample is divided into a region of high concentration (of thickness L_{EH}) and a region of low concentration. In the first region, the current is increased by the factor d/L_{EH} for the same electric field, i.e., j_{cr} can be achieved for the least electric field. If now $L_D \lesssim d$, then the dependence of the concentration on the coordinate is more complicated than in (15); but even in this case the current density near the boundary is higher than for $H = 0$. Therefore, in a transverse magnetic field, the instability sets in for a smaller electric field E_{cr} than for $H_0 \parallel j$ or $H_0 = 0$. This result agrees with the experimental data.[3] However, the dependence of E_{cr} on $H_\perp$ is nonmonotomic. For a magnetic field larger or smaller than the ones that were considered in Secs. 2 and 3 and in[1], and, to be specific, for

$$c/\mu_- < H_\perp < c/\sqrt{\mu_-\mu_+},$$

the coefficient $\eta_2 \sim H^{-2}$, if we neglect quantities of the order of $\sqrt{\mu_+/\mu_-}$, in its calculations. In this case the calculation is similar to that given in[1], but with account of the indicated dependence, leads (in our approximation) to the vanishing in the dispersion relation of the term that produces wave excitation. Therefore, in the interval

$$c/\sqrt{\mu_-\mu_+} > H_0 > c/\mu_-$$

the critical electric field should increase sharply. Consequently, it should have a minimum as a function of the magnetic field near $H_0 \approx c/\mu_-$, which was observed in a number of experiments.

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