DIFFUSION-ELECTRICAL PHENOMENA IN A PLASMA CONFINED IN A TOKAMAK MACHINE

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We calculate the particle and heat fluxes due to the presence of conduction current and of the current-inducing electric field in the Tokamak machine. The additional fluxes are directed to the interior of the plasma and therefore the process of diffusion recalls the well known phenomenon of pinching of a current-carrying plasma column. The plasma velocity in the interior differs from the velocity of classical diffusion by a factor β_I^{-1} ($\beta_I = 4\pi p/B_I^2$, p is the plasma pressure, and B_I is the field produced by the current). Simultaneously, an additional resistance to the current is produced. In a Coulomb plasma, the resistance differs little from that calculated by Spitzer. In a rarefied plasma, the longitudinal electric field causes drift of the "trapped" particles across the magnetic field and makes a comparable contribution to the self-compression of the column.

1. CHOICE OF MODEL AND BASIC PREMISES

IT was shown earlier that collisions of identical particles differ only in the character of their motion in a toroidal magnetic field[1)], in a plasma with non-uniform temperature and density, leads to the occurrence of additional flow of electrons relative to the ions; this current is proportional to the density and temperature gradients[1,2]. This means that, in accordance with the principle of symmetry of the kinetic coefficients[3], contributions to the particle and heat fluxes should exist in traps of the Tokamak type; these contributions are proportional to the current used in the Tokamak to maintain the plasma equilibrium in the toroidal magnetic field. In the present paper we calculate the values of these additional currents of particles and heat, and show that they are directed to the interior of the plasma column. We shall also show that at the same time the current experiences an additional resistance proportional to the frequency of the electron-electron collisions.

In this paper, just as in[1,4] we use the model of an axially symmetrical magnetic field with small toroidality and with a rotational conversion value (see the figure)

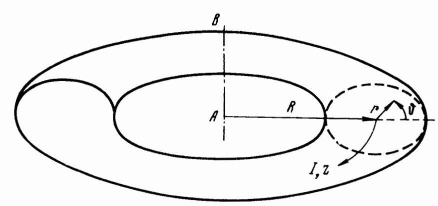
$$\mathbf{B} = B_0\{\mathbf{e}_z + \Theta(r)\mathbf{e}_\vartheta\} / (1 + \varepsilon\cos\vartheta), \qquad \varepsilon \equiv r/R \ll 1, \quad \Theta(r) \equiv B_I/B_z, \tag{1}$$

where B_I is the self-field of the current I, B_z is the toroidal magnetic field. The induction electric field causing the current to flow in the plasma is directed along the toroid:

$$\mathbf{E} = E_{0z}\mathbf{e}_z / (1 + \varepsilon\cos\vartheta). \tag{2}$$

In addition we confine ourselves to the case of a small Larmor radius of the particles and a low stream velocity:

$$\frac{r_{ci}}{\Theta n}\frac{dn}{dr} \ll 1, \qquad \frac{u_0}{v_{Te}} \ll 1, \tag{3}$$



Geometry of plasma pinch and coordinate system. AB–symmetry axis of the toroidal trap, z–coordinate along the minor axis of the toroid.

where n(r) is the particle-density profile, u_0 is the stream velocity of the electrons, $v_{Tj} = \sqrt{2T_j/m_j}$ is the thermal velocity, $r_{cj} = v_{Tj}/\omega_{cj}$ is the cyclotron radius, and $\omega_{cj} = e_jB_0/m_jc$ is the cyclotron frequency of the particles of type j. Assumption (3) makes it possible to neglect the inertia of the particles and the influence of the magnetic field on the motion of the particles, by virtue of the satisfaction in this case of the inequalities[5,6]

$$\frac{e\Phi_1(r,\vartheta)}{T_{0j}} \ll \varepsilon, \qquad \frac{v_E}{\Theta} \equiv \frac{c}{B_I^{(0)}}\frac{d\Phi_0}{dr} \ll v_{Ti}. \tag{4}$$

The potential of the self-consistent electric field is expanded here in a series in the small toroidal ratio

$$\Phi(r,\vartheta) = \Phi_0(r) + \Phi_1(r,\vartheta) + \dots.$$

In order not to complicate the exposition with calculations of the already known transport coefficients, we set the particle-density and particle-temperature gradients equal to zero and by the same token confine ourselves to only those parts of the particle and heat fluxes which result from the presence of a conduction current in the plasma. At the end of the paper we shall write out the total equations for the fluxes of particles and heat, and the values of the electric current, and verify the satisfaction of the principle of symmetry of the kinetic coefficients.

It is convenient to break up the solution of the problem into two parts in accordance with the methods used to describe the plasma. In the region of intermediate collision frequencies, when most particles can

1)We have in mind here "trapped" particles that oscillate in the bounded region where the magnetic field is weak, as well as "transiting" particles that circulate freely along the magnetic field.

already be regarded as collision-free, and the trapped particles do not have time to go from one "magnetic mirror" to the other during the time between collisions, we shall use the kinetic equation with the approximate collision term of Batnagar, Gross, and Krook (henceforth BGK)[7]. To describe a rarefied plasma it is necessary to use the more accurate form proposed by Landau for the collision integral[8]. The case of mean free paths shorter than the period of variation of the magnetic field will not be considered at all. Simple estimates show that in this collision-frequency region the influence of the current on the plasma diffusion decreases rapidly with decreasing free path.

2. CASE OF INTERMEDIATE COLLISION FREQUENCIES

To describe the plasma in the next interval of collision frequencies

$$v_{Tj}\Theta\varepsilon^{3/2}/r \ll \nu_j \ll v_{Tj}\Theta/r \tag{5}$$

we shall use the drift kinetic equation with the BGK collision term:

$$\begin{aligned}\frac{\partial f_j}{\partial t}+(\Theta v_\parallel+v_E)\frac{\partial f_j}{r\partial\vartheta}-\frac{\mu B_0/m_j+v_\parallel^2}{R}\sin\vartheta\left(\Theta\frac{\partial}{\partial v_\parallel}+\frac{1}{\omega_{cj}}\frac{\partial}{\partial r}\right)f_j\\+\frac{e_jE_{0z}}{m_j}\frac{\partial f_j}{\partial v_\parallel}=-\nu_j\left(\delta f_j-\frac{\delta n_j}{n}f_{0j}\right)+\sum_k\nu_{jk}\frac{m_j}{T_{0j}}\mathbf{v}\mathbf{u}_kf_{0j},\end{aligned}\tag{6}$$

where $\delta f_j = f_j - f_{0j}$ is the deviation of the particle distribution from a local Maxwellian distribution with density n and temperature T_{0j}, $\delta n_j = \int \delta f_j d^3v$ is the perturbation of the particle density, $u_j = \int \mathbf{v}\delta f_j d^3v$ is the mass velocity of the particles, and $\nu_j = \nu_{jj} + \nu_{jk}$.

In this article we impose the following limitation on the non-isothermal character of the plasma:

$$T_{0i}/T_{0e} < (m_i/m_e)^{1/3}$$

In this case the frequencies of the electron-ion and electron-electron collisions in (6) are connected by the following relation, which ensures a correct expression for the force of electron-ion friction[9]:

$$\nu_{ei}=\frac{m_i}{m_e}\nu_{ie}=0.51,\qquad \nu_{ee}=\frac{16\sqrt{\pi}\lambda e^4n}{3m_j^2v_{Tj}^3}.$$

The similarity of Eq. (6) to the equation for small oscillations of a plasma with azimuthal wave number m = 1 and frequency $\omega_0 = -v_E/r$ was noted many times[1,6]. We use this analogy to obtain the law of conservation of the particle momentum from the quasilinear equation that describes the reaction of a small oscillation on the plasma particles.

We first neglect this influence. Then the particle distribution in the electric field E_z can be expressed in the form of a series in Sonine polynomials $L_q^{(3/2)}$ [9]:

$$\begin{aligned}f_j^{(0)}=\frac{n}{\pi^{3/2}v_{Tj}^3}\exp\left[-\frac{m_j(v_\parallel-u_{0j})^2}{2T_{0j}}-\frac{\mu B_0}{T_{0j}}\right]\\\times\left\{1-\frac{m_j}{T_{0j}}v_\parallel u_{0j}\sum_{q=1}^{\infty}a_qL_q^{(3/2)}\left(\frac{m_j\mathbf{v}^2}{2T_{0j}}\right)\right\},\\u_{0i}\equiv 0,\qquad u_{0e}\equiv u_0.\end{aligned}$$

The coefficients a_q in this equation are calculated in accordance with[9] (see Sec. 3).

In the approximation linear in the toroidality $\epsilon \ll 1$, the deviation of the distribution of the particles from that described above was obtained in[1]:

$$\begin{aligned}f_j^{(1)}=-\frac{\mu B_0/m_j+v_\parallel^2}{R}\left[\Theta\frac{\partial}{\partial v_\parallel}+\frac{1}{\omega_{cj}}\frac{\partial}{\partial r}\right]\\\times f_j^{(0)}\left\{\mathrm{P}\frac{r\cos\vartheta}{\Theta v_\parallel+v_E}-\pi r\delta(\Theta v_\parallel+v_E)\sin\vartheta\right\}.\end{aligned}\tag{7}$$

The influence of the toroidality on the redistribution of the particles is an effect of second order of smallness and is taken into account by the following nonlinear equation:

$$\begin{aligned}\frac{\partial f_j^{(0)}}{\partial t}=\mathrm{St}_{QL}-\nu_j\left[\delta f_j^{(0)}-\frac{\delta n_j^{(0)}}{n}f_{0j}\right]+\sum_k\nu_{jk}\mathbf{v}\mathbf{u}_{0k}f_{0j},\\\mathrm{St}_{QL}=\left(\Theta\frac{\partial}{\partial v_\parallel}+\frac{1}{\omega_{cj}}\frac{\partial}{\partial r}\right)\frac{\pi\varepsilon^2}{2r}\left(\frac{\mu B_0}{m_j}\right)^2\\\times\delta(\Theta v_\parallel+v_E)\left(\Theta\frac{\partial}{\partial v_\parallel}+\frac{1}{\omega_{cj}}\frac{\partial}{\partial r}\right)f_j^{(0)}.\end{aligned}$$

The procedure of averaging over the "oscillation phases," which is needed in order to obtain this equation, is replaced here by averaging over the magnetic surface (i.e., over the variable ϑ). From this we can easily derive the law of variation of the particle momentum

$$\begin{aligned}m_jn\frac{\partial u_{0j}}{\partial t}=e_jnE_z-m_jn\frac{\pi^{1/2}|\Theta|v_{Tj}}{2r}\varepsilon^2\left(1.15u_{0j}+\frac{v_E}{\Theta}\right)\\+0.51m_en\nu_{ee}u_0\,\mathrm{sign}\,e_j.\end{aligned}\tag{8}$$

It is important here to note two circumstances. First, the plasma resistance differs somewhat from that calculated by Spitzer[10]. This difference can be readily noted in the Tokamak[11,12], since the presence of additional resistance due to toroidality ($\delta\sigma_\parallel \lesssim \epsilon^{1/2}\sigma_\parallel$) is completely masked by the "turbulent resistance." The difference may be appreciable however, in a system with strong inhomogeneity of the magnetic field or of the density[13].

Second, the quasistationary state is reached not because of a change in the directional velocities of the particles, but as the result of the appearance of a small radial electric field

$$\frac{c}{B_0}\frac{d\Phi_0}{dr}=1.15\sqrt{\frac{m_eT_{0e}}{m_iT_{0i}}}u_0\Theta.$$

It is precisely the presence of such a field which ensures also ambipolarity of the diffusion. Multiplying the correction to the unperturbed distribution function $f_j^{(1)}$ by the particle drift velocity, integrating the resulting expression over the entire velocity space, and averaging over ϑ, we obtain the particle flux

$$\langle nv_r\rangle^j=-\int_0^{2\pi}\frac{d\vartheta}{2\pi}\int\frac{\mu B_0/m_j+v_\parallel^2}{\omega_{cj}R}\sin\vartheta f_j^{(1)}d^3\mathbf{v}=-\frac{1.15D_e}{T_{0e}}\frac{IB_I^{(0)}}{c},\tag{9}$$

where

$$D_e=\frac{\sqrt{\pi}\varepsilon^2r_{ce}}{2|\Theta|r}\frac{cT_{0e}}{eB_0}$$

is the previously obtained particle diffusion coefficient[1]. We calculate the heat flux in perfect analogy:

$$q_j=\gamma_j^IT_{0j}\langle nv_r\rangle;\qquad \gamma_i^I=3,\qquad \gamma_e^I=3.77.$$

3. TRANSPORT COEFFICIENTS IN A RAREFIED PLASMA

We now consider a rarefied plasma, in which the "trapped" particles have time to cover their banana-like trajectory several times during the time between collisions. This takes place when

$$\nu_j \ll v_{Tj}\Theta\varepsilon^{3/2}/r. \tag{10}$$

If we neglect completely the influence of the electric field and of the rare collisions, then the distribution of the "trapped" particles depends only on three integrals of motion:

1) energy
2) the transverse adiabatic invariant
3) the generalized momentum

$$J = mv_{\|}(1+\varepsilon\cos\vartheta) - \frac{e}{c}\int_0^r B_I^{(0)}\,dr + \frac{e}{c}A_{0z}(t).$$

The transiting particles have one more integral of motion - the sign of the longitudinal velocity $\sigma = \operatorname{sign} v_{\|}$. Accordingly, the distribution function in the absence of an induction electric field and collisions is given by

$$f_{tj}^{(0)} = \frac{n_0}{\pi^{3/2}v_{Tj}^3}\exp\left[-x_j - 2\varepsilon\varkappa^2 x_j - \frac{u_j^2}{v_{Tj}^2}\right], \quad \varkappa^2 < 1, \tag{11}$$

$$f_{uj}^{(0)} = \frac{n_0}{\pi^{3/2}v_{Tj}^3}\exp\left[-x_j - 2\varepsilon\varkappa^2 x_j - \frac{u_j^2}{v_{Tj}^2} + 0.5\pi\sigma\sqrt{2x_j\varepsilon}\,\frac{u_j}{v_{Tj}}\int_1^{\varkappa^2}\frac{dt}{\sqrt{t}\,\mathrm{E}(t^{-1/2})}\right], \quad \varkappa^2 > 1,$$

where

$$x_j = \mu B_0/T_{0j}, \quad u_j = u_0\delta_{je} + v_E/\Theta,$$

$$v_{\|} = -\frac{v_E}{\Theta} + \sigma v_{Tj}\sqrt{2x_j\varepsilon\left(\varkappa^2 - \sin^2\frac{\vartheta}{2}\right)},$$

$\mathrm{E}(1/\kappa)$ is the complete elliptic integral of the second kind. The electron distribution function written out above is obtained directly from Eq. (30) of[1], if it is recognized that in the electron rest system the radial electric field differs from that in the laboratory system by an amount

$$E_{0r} - E_{0r}' = -\frac{1}{nec}[\mathbf{I}_z\times\mathbf{B}_I]. \tag{12}$$

Using (12), we can show that the particle distribution is invariant under Galilean transformations.

Allowance for the induction electric field and of the particle collisions leads to effects of two types. The first is that the "trapped" particles drift under the influence of this field inside the plasma pinch. The drift velocity can be obtained by averaging the drift equations of motion over the time of one period of revolution along its almost closed trajectory:

$$\langle dr/dt\rangle = -cE_{0z}/B_I. \tag{13}$$

On the other hand, there is collisional relaxation of the particle distribution. As we have noted in[2], in a toroidal magnetic field it has significant distinguishing features. First, the form of the distribution function of the "trapped" particles does not change under the influence of the collisions. Second, the collisions of the transiting particles with one another and with the "trapped" particles leads to a slow variation of the distribution parameters (in this case u_j) and have practically no effect on the particle diffusion.

We shall seek a solution of the kinetic equation everywhere outside the transition layer between the phase spaces of the "trapped" and transiting particles, using the method of successive approximations, and represent the particle distribution function in the form

$$f_j = f_j^{(0)}(\mu,\mathscr{E};J,\sigma)(1-\Phi_j^{(0)}u_0) + f_j^{(1)}(\mu,\mathscr{E};J,\sigma;\vartheta). \tag{14}$$

The factor Φ describes here the change of the particle distribution; this change exists even in a homogeneous collision plasma, if the plasma is acted upon by an external electric field. Allowance for this correction is necessary in order to calculate correctly the electron-ion friction force[9]. The correction $f_j^{(1)}$ describes the redistribution of the particles in an inhomogeneous magnetic field under the influence of rare collisions. In this paper, the explicit form of this part of the distribution function will not be needed, since the particle and heat fluxes can be expressed in terms of the discontinuities of the derivatives of the unperturbed function $f_j^{(0)}$ in the transition layer.

For greater clarity, we subdivide the collision term in the kinetic equation into two parts, one describing the particle collisions and neglecting the discontinuities of their distribution functions, and the second responsible for the collisions of the "trapped" and transiting particles

$$\mathrm{St}\{f_j\} = \mathrm{St}_{j\neq j'} + \mathrm{St}_{jj} + \sum_{j'}\{\mathrm{St}_{jt,j'u} + \mathrm{St}_{ju,j't}\}. \tag{15}$$

In the first approximation we neglect the inhomogeneity of the magnetic field and the discontinuities on the distribution function. Then, as is well known, we can expand the function Φ_j in a series in Sonine polynomials:

$$\Phi_e(v^2, v_{\|}) = \frac{m_e}{T_{0e}}\sum_{q=1}^{\infty}a_q L_q^{(3/2)}\left(\frac{m_e v^2}{2T_e}\right)v_{\|}; \quad \Phi_i \equiv 0 \tag{16}$$

and reduce the kinetic equation to the matrix equation for the coefficient a_q:

$$\sum_{q=1}^{\infty}(\alpha_{pq} + \alpha_{pq}')a_q = \alpha_{0p}', \tag{17}$$

where

$$\alpha_{pq} = -\frac{1}{3}\frac{m_e}{\nu_{ee}T_e}\int v_{\|}L_p^{(3/2)}\{\mathrm{St}[f_0 L_q^{(3/2)}v_{\|}, f_{0e}] + \mathrm{St}[f_{0e}, f_{0e}L_q^{(3/2)}v_{\|}]\}\,d^3\mathbf{v}$$

$$\alpha_{pq}' = -\frac{1}{3}\frac{m_e}{\nu_{ee}T_e}\int v_{\|}L_p^{(3/2)}\,\mathrm{St}[f_{0e}L_q^{(3/2)}v_{\|}, f_{0i}']\,d^3\mathbf{v},$$

f_{0e} and f_{0i}' are the locally Maxwellian electron and ion functions, shifted along the $v_{\|}$ axis by an amount equal to the current velocity of the electrons.

We shall need for the calculations only nine of the first matrix elements[9]:

$$\alpha_{pq} = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 3/4 \\ 0 & 3/4 & 45/16 \end{pmatrix}, \quad \alpha_{pq}' = \begin{pmatrix} 1 & 3/2 & 15/8 \\ 3/2 & 13/4 & 69/16 \\ 15/8 & 69/16 & 433/64 \end{pmatrix}. \tag{18}$$

In an inhomogeneous magnetic field, the condition for the solvability of the equation for $f_j^{(1)}$ requires that

$$\Phi_{ue}^{(0)} = \left(\frac{m_e}{2T_e}\right)^{1/2}\sum_{q=1}^{\infty}a_{qe}^{(0)}L_q^{(3/2)}(x_e)\,\sigma\sqrt{2x_e\varepsilon}\left(\frac{\pi}{2}\right)\int_1^{\varkappa^2}\frac{dt}{t^{1/2}\mathrm{E}(t^{-1/2})}; \tag{19}$$

$$\Phi_{te}^{(0)} = 0.$$

In addition, the friction of the transiting particles, which transport the current, against the particles from the transition layer causes an additional deceleration of the current. To write down the explicit expression for Ohm's law in the inhomogeneous magnetic field, it is necessary to take into account, in the matrix equation for the coefficients a_q, the additional term describing the action of the particles of the transition layer on the transiting particles. The parameter $\epsilon^{1/2}$ (the fraction of the "trapped" particles in the plasma) will be assumed small. Then the coefficients a_q can be sought in the form of expansions in this parameter. In the first approximation, the kinetic equation reduces to Eq. (17). In the next higher approximation, with allowance for the particles from the transition layer, we obtain

$$\nu_{ee} I \left\{ \alpha_{p0} - \sum_{q=1}^{\infty} (\alpha_{pq} + \alpha_{pq}')(a_{qe}^{(0)} + a_{qe}^{(1)}) - \sqrt{\varepsilon}\left[\alpha_{p0}^{ue,te} - \sum_{q=1}^{\infty} \alpha_{pq}^{ue,te} a_{qe}^{(0)}\right]\right\} = \frac{n_* e^2}{m_e} E_{0z}\delta_{p0}, \tag{20}$$

where $I = -n_* e u_0$ is the current carried by the transiting electrons, $n_* = n_0(1 - 2\sqrt{2\epsilon/\pi})$ is the density of the transiting particles averaged over the magnetic surface, and the matrix elements α_{pq} are calculated in the Appendix.

Obtaining from this the coefficients $a_{qe}^{(1)}$ and substituting them in the expression for the friction force, we write Ohm's law in the form

$$I = \sigma_{\parallel}^{(0)}\,[1 + 0.35\varepsilon^{1/2}]E_{0z}, \tag{21}$$

where $\sigma^{(0)} = n_* e^2/0.51\, m_e \nu_{ee}$ is the conductivity of the plasma of the transiting particles after Spitzer. The vanishing of the matrix elements α_{0p} makes it possible to obtain immediately the momentum-balance equation for the transiting ions. From this we determine the self-consistent electric field:

$$F_{ei} - m_e \nu_{ee} u_0 \left[\alpha_{00}^{iu,et} - \sum_{q=1}^{\infty} \alpha_{0q}^{iu,et} a_{qe}^{(0)}\right] n\varepsilon^{1/2} - n_* e E_{0z} = \nu_{ii} m_i n_* \alpha_{00}^{iu,it} \varepsilon^{1/2} \frac{v_E}{\Theta},$$

$$F_{ei} = -\nu_{ee} m_e n_* u_0 \left[\alpha_{00}' - \sum_{q=1}^{\infty} \alpha_{0q}' a_q\right], \tag{22}$$

where F_{ei} is the force of the electron-ion friction.

The particle flux averaged over the magnetic surface is given by

$$\langle nv_r\rangle^j = \frac{-(2\varepsilon)^{3/2}}{\omega_{cj}} \int_0^{2\pi} d\vartheta \sum_{\sigma=\pm 1} \int_0^{\infty} \left(\frac{\mu B_0}{m_j}\right)^{3/2} d\frac{\mu B_0}{m_j} \times \int_{\sin^2 \vartheta/2}^{\infty} \frac{[f_j^{(0)} + f_j^{(1)}]\, d\varkappa^2}{2\sqrt{2}[\varkappa^2 - \sin^2(\vartheta/2)]^{1/2}} \frac{\sin\vartheta}{r}. \tag{23}$$

We have used here relation (11), which connects the velocity with the new variables κ^2 and μ, in order to calculate the element of the phase volume in the new variables. Integration by parts with respect to the variable ϑ and subsequent utilization of the kinetic equation make it possible to express the current in terms of the already known moments of the collision term[1]

$$\langle nv_r\rangle = \frac{c}{e_j B_I} \sum_{j'} \int m_j v_{\parallel} \mathrm{St}_{tj,uj'}\{f_j^{(0)}(1 - \Phi_{uj} u_0), f_{0j'}\} d^3 v = -(D_e/T_{0e}) c^{-1} I B_I \sum \left[\alpha_{00}^{te,uj'} - \sum_{q=1}^{\infty} \alpha_{0q}^{te,uj'} a_{qe}^{(0)}\right], \tag{24}$$

where $D_e = \nu_{ee} r_{ce}^2 \epsilon^{1/2}/\Theta^2$.

The ambipolarity of the diffusion is ensured by the momentum balance equation of the transiting particles. The heat flux in turn is expressed in terms of the following moment of the collision term:

$$q_e = D_e c^{-1} I B_I \sum_{j'} \left[\alpha_{10}^{te,uj'} - \sum_{q=1}^{\infty} \alpha_{1q}^{te,uj'} a_{qe}^{(0)}\right] - \frac{5}{2} T_{0e}\langle nv_r\rangle, \tag{25}$$

$$q_i = \gamma T_{0i}\langle nv_r\rangle;\ \gamma^{-1} = \sqrt{2}[\sqrt{2} - \ln(1 + \sqrt{2})]. \tag{26}$$

4. SUMMARY OF RESULTS

For convenience in using the obtained formulas, let us write down the equations for the particle and heat fluxes with allowance for the previously obtained transport coefficients[1]. In addition, in order to verify the fulfillment of the principle of symmetry of the kinetic coefficients, we repeat here also the expression obtained in[2] for Ohm's law, corrected on the basis of the calculations of the present article. As a result we get

$$\langle nv_r\rangle = \langle nv_r\rangle^n + \langle nv_r\rangle^I - \alpha_{ei}\varepsilon^{1/2} n_0 c E_{0z}/B_I,$$

$$\langle nv_r\rangle^n = -\frac{\alpha_e D_e}{T_{0e}} \sum_j n T_{0j}\left[\frac{d\ln n}{dr} - \left(\frac{3}{2} - \gamma_j^n\right)\frac{d\ln T_{0j}}{dr}\right] \tag{27}$$

$$\langle nv_r\rangle^I = -\frac{\alpha_I D_e}{T_{0e}} \frac{I B_I}{e};$$

$$q_j = T_{0j}[\gamma_j^n \langle nv_r\rangle^n + \gamma_j^I \langle nv_r\rangle^I - \gamma_j^E \alpha_{ei}\varepsilon^{1/2} n_0 c E_{0z}/B_I] - \varkappa_j D_j n_0 \frac{dT_{0j}}{dr}. \tag{28}$$

$$I = \sigma_{\parallel}\left\{E_{0z} - \frac{\alpha_I D_e}{T_{0e}} \frac{B_I}{c} \sum_j n T_{0j}\left[\frac{d\ln n}{dr} - \left(\frac{3}{2} - \gamma_j^I\right)\frac{d\ln T_{0j}}{dr}\right]\right\}. \tag{29}$$

The coefficients α, γ, κ, D, and $\sigma_{\parallel}$ depend on the collision frequency:

a) for $\nu_j \ll v_{Tj}\Theta\epsilon^{3/2}/r$:

$$\alpha_{jj} = 0.37;\ \alpha_{ei} = 0{,}70;\ \alpha_I = 0.23;$$
$$\alpha_e = \alpha_{ee} + \alpha_{ei};\ \varkappa_e = 1{,}18;\ \varkappa_i = 0{,}57.$$
$$\gamma_e^n = 1{,}11;\ \gamma_i^n = 1{,}33 = \gamma_e^I;\ \gamma_i^I = 1{,}33 - 0.18\, T_e/T_i;\ \gamma_e^E = 2{,}04;\ \gamma_i^E = 1{,}44.$$
$$D_j = \nu_{jj} r_{cj}^2 \varepsilon^{1/2}/\Theta^2;\ \sigma_{\parallel} = \sigma_{\parallel}^{(0)}[1 + 0{,}35\varepsilon^{1/2}]^{-1}.$$

b) for $v_{Tj}\Theta\epsilon^{3/2}/r \ll \nu_j \ll v_{Tj}\Theta/r$:

$$\alpha_{jj} = 1;\ \alpha_{ei} = 0;\ \alpha_I = 1.15;\ \gamma_j^n = \varkappa_j = 3;\ \gamma_e^I = 3{,}77;\ \gamma_i^I = 3.$$
$$D_j = \frac{\sqrt{\pi}\varepsilon^2 r_{cj}}{2|\Omega| r} \frac{c T_{0j}}{e_j B_0};\quad \sigma_{\parallel} = \sigma_{\parallel}^{(0)}\left[1 + 1{,}5\frac{\pi^{1/2} v_{Te}|\Theta|}{\nu_{ee} r}\varepsilon^2\right]^{-1}.$$

We see that the relation for the symmetry of the kinetic coefficients is not satisfied in the case of a rarefied plasma. The reason is that a small group of "trapped" particles drift in the toroidal electric field and in the magnetic field of the current even in the absence of collisions. It is interesting to note also that in the hypothetical case $\nu_{ee} \gg \nu_{ei}$ (the frequency of the electron-electron collisions greatly exceeds the frequency of the electron-ion collisions) the aforementioned electric drift becomes small and the symmetry relation is satisfied also in a rare plasma. In accordance with the foregoing, we have separated in the expressions for the particle and heat fluxes the parts connected with the electric drift of the "trapped" particles.

We call attention also to the sign of the electric diffusion of the plasma. Namely, the plasma pinch is slowly compressed under the influence of the electric field and the current's own magnetic field. The relation between the rate of diffusion of the plasma to the outside and the self-contraction of the pinch is best expressed in terms of β_I—the current ratio. Assuming the current distribution over the cross section of the pinch to be homogeneous and confining ourselves to case (b), which is the most appropriate for the experimental conditions with TM-3 and T-3 apparatus[11,12], we obtain

$$\langle nv_r\rangle = -D_e\left\{\frac{d\ln n}{dr}+\frac{3}{2(1+\tau)}\left[\frac{d\ln T_{0e}}{dr}+\tau\frac{d\ln T_{0i}}{dr}\right]+\frac{2,3}{\beta_I r}\right\};\quad \tau=\frac{T_{0i}}{T_{0e}}. \tag{30}$$

In the region of small particle mean free paths, where the hydrodynamic description of the plasma is valid, the electrodiffusion current is proportional to the viscosity of the electrons and therefore decreases rapidly with decreasing mean free path.

In conclusion, the author thanks Academician R. Z. Sagdeev for valuable advice and a discussion of the work.

APPENDIX

CALCULATION OF MATRIX ELEMENTS

During the course of the calculations in Sec. 3 it is necessary to obtain the values of the integrals of the type

$$\alpha_{pq}^{jt,\,j'u} = \int \Psi_p \,\mathrm{St}_{jt,\,j'u}\{f_{0j}\Phi_q,\, f_{0j'}\}\, d^3\mathrm{v},$$
$$\alpha_{pq}^{ju,\,j't} = \int \Psi_p \,\mathrm{St}_{ju,\,j't}\{f_{0j},\, f_{0j'}\Phi_q\}\, d^3\mathrm{v}. \tag{A.1}$$

The roles of Ψ_p and Φ_q are played respectively by the functions

$$L_p^{(3/2)}\left(\frac{mv^2}{2T}\right)v_\parallel,\quad L_q^{(3/2)}\left(\frac{mv^2}{2T}\right)\sigma v\sqrt{2\varepsilon}\,\frac{\pi}{4}\int_1^{\varkappa^2}\frac{dt}{t^{1/2}E(t^{-1/2})},$$

where $L_p^{(3/2)}$ are Sonine polynomials (generalized Laguerre polynomials), which have the following generating function:

$$(1-\xi)^{-5/2}\exp\left[-\frac{t\xi}{1-\xi}\right]=\sum_{p=0}^{\infty}\xi^p L_p^{(3/2)}(t). \tag{A.2}$$

Instead of calculating the matrix elements (A.1) separately for each pair of values p and q, it is convenient to calculate the matrix element of the generated function (A.2). Expanding it then in powers of ξ and η we obtain the sought matrix elements in the form of the coefficients of ξ^p and η^q:

$$\alpha^{jt,\,j'u} \infty \sum_{p,q=0}^{\infty}\xi^p\eta^q\alpha_{pq}^{jt,\,j'u}. \tag{A.3}$$

In the first integral of (A.1) we can immediately integrate with respect to the variable v. In addition, it suffices to retain in it only the derivatives with respect to the longitudinal velocity and to change over to the variables x_j and κ^2[1]:

$$\alpha_{pq}^{jt,\,j'u} = \frac{\nu_{jj}\varepsilon^{-1/2}}{2\sqrt{2}}\int_0^{2\pi}\frac{d\vartheta}{2\pi}\sum_{\sigma=\pm1}\int_0^{\infty}A(x_{j'})x_j^{-1}\,dx_j\cdot$$
$$\times\int_{\sin^2(\vartheta/2)}^{\infty}d\varkappa^2\Psi_p\sigma\sqrt{\varkappa^2-\sin^2(\vartheta/2)}\frac{\partial}{\partial\varkappa^2}\left\{\sqrt{\varkappa^2-\sin^2(\vartheta/2)}\left(\frac{\partial}{\partial\varkappa^2}+2x_j\varepsilon\right)\right.$$
$$\left.-\sigma\frac{u_j}{v_{Tj}}\sqrt{2x_j\varepsilon}\right\}v_{Tj}^3 f_{0j}\Phi_q,$$

$$A(x_{j'})=\frac{3\sqrt{\pi}}{4}\left(\eta(x_{j'})+\eta'(x_{j'})-\frac{\eta(x_{j'})}{2x_{j'}}\right),$$

$$\eta(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t}\sqrt{t}\,dt,\qquad x_{j'}=\frac{2\mu B_0}{m_j v_{Tj'}^2}.$$

The integration of an expression of similar type with respect to the variable κ^2 was carried out in our paper[1]. Using the results of that paper, we obtain

$$\alpha^{jt,j'u}=\nu_{jj}\alpha_{ei}\varepsilon^{1/2}\left[\frac{\sqrt{B}}{\beta_{j'}^{1/2}\beta_j(1+x+y)}-\frac{1}{2\beta_{j'}}\ln\frac{\sqrt{B}+\beta_{j'}^{1/2}}{\sqrt{B}-\beta_{j'}^{1/2}}\right](1-\xi)^{-5/2}(1-\eta)^{-5/2} \tag{A.4}$$

where $\beta_j = m_j/2T_{0j}$; $x = \xi/(1-\xi)$; $y = \eta/(1-\eta)$;

$$B=\beta_{j'}+\beta_j(1+x+y);$$

$$\alpha_{ei}=\frac{3\pi}{8\sqrt{2}}\left[\frac{8}{\pi^2}-\int_1^{\infty}\frac{dt}{\sqrt{t}}\left(\frac{1}{\mathrm{E}(t^{-1/2})}-\frac{4}{\pi^2}K(t^{-1/2})\right)\right].$$

In calculating the second integral, it turns out to be more convenient to integrate first by parts, and then carry out the integration with respect to the variable v. After simple but cumbersome calculations we obtain

$$\alpha^{ju,\,j't}=-\nu_{jj}(\beta_{j'}/\beta_j)^{3/2}\frac{2T_{0j}}{m_{j'}}\left[\frac{\sqrt{A\beta_j(1+x)}}{\beta_{j'}(1+y)}-\frac{1}{2}\ln\frac{\sqrt{A}+\sqrt{\beta_j(1+x)}}{\sqrt{A}-\sqrt{\beta_j(1+x)}}\right.$$
$$\left.-\xi\left(\frac{\sqrt{A\beta_j(1+x)}}{\beta_{j'}(1+y)}+2\sqrt{\frac{\beta_j(1+x)}{A}}-\frac{3}{2}\ln\frac{\sqrt{A}+\sqrt{\beta_j(1+x)}}{\sqrt{A}-\sqrt{\beta_j(1+x)}}\right)\right](1-\eta)^{-5/2}, \tag{A.5}$$

where $A = \beta_j(1+x) + \beta_j'(1+y)$.

With the aid of (A.4) and (A.5) we can obtain expressions for all the necessary matrix elements. For electron-ion collisions, recognizing that $\beta_j \ll \beta_j'$, we have

$$\alpha^{et,\,iu}=\nu_{ee}\alpha_{ei}\varepsilon^{1/2}\beta_e^{-1}(1-\xi\eta)^{-1}(1-\xi)^{-3/2}(1-\eta)^{-3/2},$$
$$\alpha^{eu,\,it}=-\frac{2}{3}\nu_{ee}\alpha_{ei}\varepsilon^{1/2}\frac{2T_{0e}}{m_i}(1-\eta)^{-1}(1-\xi)^{-3/2}.$$

For ion-electron collisions, to the contrary, $\beta_j \gg \beta_j'$, and from (A.4) and (A.5) we obtain

$$\alpha^{it,\,eu}=\frac{2}{3}\nu_{ee}\alpha_{ei}\varepsilon^{1/2}\frac{m_e^2}{m_i^2}\beta_e^{-1}(1-\xi\eta)^{-3/2}(1-\eta)^{-1}(1-\xi)^{-1}$$
$$\alpha^{iu,\,et}=-\nu_{ee}\alpha_{ei}\varepsilon^{1/2}\frac{2T_{0e}}{m_i}(1-\eta)^{-3/2}.$$

For collisions of identical particles we have calculated only nine elements of the matrix α_{pq} used in the text:

$$\alpha_{pq}^{jt,ju}=\alpha_{jj}\begin{pmatrix}1 & 5/2-\gamma & 35/8-19/8\gamma\\ 5/2-\gamma & 25/4-11/4\gamma & 175/16-193/32\gamma\\ 35/8-19/8\gamma & 175/16-193/32\gamma & 1225/64-2893/256\gamma\end{pmatrix}, \tag{A.6}$$

$$\alpha_{pq}^{ju,jt}=-\alpha_{jj}\begin{pmatrix}1 & 5/2-\gamma & 35/8-19/8\gamma\\ 3(\gamma-1) & 27/4\gamma-15/2 & 357/32\gamma-105/8\\ 5/8\gamma & 35/32\gamma & 373/256\gamma\end{pmatrix}, \tag{A.7}$$

where

$$\alpha_{jj}=\alpha_{ei}(\sqrt{2}-\ln(1+\sqrt{2}));\quad \gamma^{-1}=\sqrt{2}(\sqrt{2}-\ln(1+\sqrt{2})).$$

The numerical values of the matrix elements for the collisions of unlike particles differ from those for a homogeneous plasma only by the factor α_{ei}:

$$\alpha_{pq}^{te,\,ui}=\alpha_{ei}\alpha_{pq}';\quad \alpha_{0q}^{ui,\,te}=-\alpha_{0q}^{te,ui}. \tag{A.8}$$

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