NONLINEAR EFFECTS CAUSED BY TRANSVERSE WAVES IN A PLASMA

B. CHAKRABORTI

Jadavpur University, India

Submitted July 15, 1969

Zh. Eksp. Teor. Fiz. 59, 457–463 (August, 1970)

The nonlinear Vlasov equations for a plasma are solved (neglecting the collision terms) to third order in a perturbation that represents a transverse wave in a neutral isotropic plasma. The second-order solution contains the longitudinal second harmonic and the stationary flow along the wave vector. The correction to the frequency in the initial-value problem and the correction to the wave vector in the boundary-value problem occur in the third-order solution; these two corrections are different in the two planes of elliptical polarization. Transverse waves corresponding to eight different frequencies are obtained. The increase in the number of waves leads to a nonlinear deflection of the radiation at the boundary of the plasma. For the zeroth correction, four of the waves correspond to the first harmonic and the other four to the third harmonic. In the third-order approximation this leads to the splitting of a single elliptically polarized wave into two plane-polarized waves with different phase and group velocities. The frequency shift is in the red direction. Numerical calculations for the nucleus Fe^{57} used in Mossbauer experiments yield a relative frequency shift of the order of only 10^{-23}.

1. INTRODUCTION

NONLINEAR effects up to third order that arise in wave propagation in a uniform isotropic plasma have been studied in a number of papers.[1-6]

We have solved the nonlinear Vlasov equations for a plasma[7] up to third order in a perturbation in the form of elliptically polarized electromagnetic waves; this problem has not been treated by other authors.

The initial nonlinear equations that describe the electron motion are

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)f+\frac{e}{m}\left\{\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right\}\nabla_{\mathbf{v}}f=0, \qquad (1.1)^*$$

$$\operatorname{rot}\mathbf{H}-\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}=\frac{4\pi e}{c}\int_{-\infty}^{\infty}\mathbf{v}(f-f_i)\,d\mathbf{v}, \qquad (1.2)$$

$$\operatorname{rot}\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}=0, \qquad (1.3)$$

$$\operatorname{div}\mathbf{E}=4\pi e\int_{-\infty}^{\infty}(f-f_i)\,d\mathbf{v}, \qquad (1.4)$$

where $f(\mathbf{r}, \mathbf{v}, t)$ and f_i are respectively the distribution functions for the electrons and ions, e is the negative charge of the electron, m is the electron mass, and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field vectors.

We assume the existence of a neutral isotropic equilibrium state for the plasma in the absence of a static magnetic field with the following distribution functions:

$$f_{0e}(v^2)=\left(\frac{m}{2\pi\varkappa T_e}\right)^{3/2}\exp\left(-\frac{mv^2}{2\varkappa T_e}\right),$$
$$f_{0i}=\left(\frac{M}{2\pi\varkappa T_i}\right)^{3/2}\exp\left(-\frac{Mv^2}{2\varkappa T_i}\right), \qquad (1.5)$$

where κ is the Boltzmann constant, M is the ion mass and T_e and T_i are the kinetic temperatures for the electrons and ions.

*$[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

This complete system of equations will be solved by a method of successive approximations given by Bogdanov[8] and developed further in the present paper.

In the first approximation the linearized equation (1.1) assumes the form

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)f_1+\frac{Ne}{m}\left\{\mathbf{E}_1+\frac{1}{c}[\mathbf{vH}_1]\right\}\nabla_{\mathbf{v}}f_0=0, \qquad (1.6)$$

where $f_0(v^2)$ is the Maxwellian electron distribution $f_{0e}(v^2)$ from (1.5). The solution $f_1(\mathbf{r}, \mathbf{v}, t)$, which corresponds to elliptically polarized electromagnetic waves, is of the form

$$f_1(\mathbf{r},\mathbf{v},t)=g_1(\mathbf{v})\cos\theta+g_2(\mathbf{v})\sin\theta, \qquad (1.7)$$

$$\theta=\mathbf{kr}-\omega t. \qquad (1.8)$$

Substituting this in (1.2) and (1.3) and simplifying we have

$$f_1(\mathbf{r},\mathbf{v},t)=-\frac{Ne}{m}\frac{f_0}{\dot\theta}\frac{m}{\varkappa T}(E_1v_1\cos\theta-E_2v_2\sin\theta), \qquad (1.9)$$

$$\dot\theta=\mathbf{kv}-\omega, \qquad (1.10)$$

where v_1 and v_2 are two orthogonal components of $\mathbf{v}$, which are perpendicular to $\mathbf{k}_1$, while E_1 and E_2 are the amplitudes of the two components of the perturbing electric field in these directions. The plane which contains $\mathbf{k}$ and v_i (i = 1, 2) will be called the $\perp i$ polarization plane (i = 1, 2) of the elliptically polarized wave. Actually, (1.9) is an integro-differential equation since

$$(E_1;E_2)=\frac{4\pi e\omega}{\Omega^2}\left\{\int v_1g_1\,d\mathbf{v};-\int v_2g_2\,d\mathbf{v}\right\}, \qquad (1.11)$$

where

$$\Omega^2=\omega^2-k^2c^2. \qquad (1.12)$$

The integrals in (1.11) are triple integrals in velocity space and the limits must encompass the entire space.

The equations given above yield the dispersion relation

$$1+\frac{\omega^2{}_0\omega}{\Omega^2}\int\frac{F(v_\parallel)\,dv_\parallel}{\dot\theta}=0, \tag{1.13}$$

where

$$F(v_\parallel)=\int f_0(v^2)\,dv_1\,dv_2, \tag{1.14}$$

and $v_\parallel$ is the component of $\mathbf{v}$ parallel to $\mathbf{k}$. The singularity that appears in (1.13) when $v_\parallel = \omega/k$ is not important for transverse waves.[9] Hence, in what follows we shall make use of the principal value of the integral in (1.13), expanding the reciprocal denominator in powers of $kv_\parallel/\omega$ and integrating Eq. (1.13) by parts. Other similar integrals which contain positive integral powers of $\dot\theta$ in the denominator are treated in the same way.

2. SECOND-ORDER APPROXIMATION

The second-order approximation to Eq. (1.1) can be written in the form

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)f_2+\frac{Ne}{m}\left(E_{2\parallel}\frac{\partial f_0}{\partial v_\parallel}+E_{2\perp_1}\frac{\partial f_0}{\partial v_1}+E_{2\perp_2}\frac{\partial f_0}{\partial v_2}\right)$$
$$+\,i(\delta\omega+\mathbf{v}\delta\mathbf{k})f_1=-\frac{e}{m}\left[\left(E_{1\perp_1}\frac{\partial f_1}{\partial v_1}+E_{1\perp_2}\frac{\partial f_1}{\partial v_2}\right)\right.$$
$$\left.+\frac{v_\parallel}{c}\left(H_{1\perp_1}\frac{\partial f_1}{\partial v_2}-H_{1\perp_2}\frac{\partial f_1}{\partial v_1}\right)f_1+\frac{1}{c}(v_1H_{1\perp_2}-v_2H_{1\perp_1})\frac{\partial f_1}{\partial v_1}\right],$$

where $E_{2\parallel}$ is the longitudinal field in second order while $E_{2\perp_1}$ and $E_{2\perp_2}$ are the two components of the elliptically polarized field in second order with polarizations $\perp_1$ and $\perp_2$ respectively. It is evident that terms that play the role of sources appear on the right side of this equation and consist of quadratic products of the first-order field quantities. Account is also taken of the possibility of nonlinear corrections to the frequency and wave vector, $\delta\omega$ and $\delta\mathbf{k}$, respectively. However, the sources on the right side of the equation cannot lead to the appearance of first harmonics, so that these cannot appear on the left side of the equation. It then follows that the last term on the left, composed of first harmonics of the field quantities, must vanish for all values of $\delta\omega$ and $\delta\mathbf{k}$. Thus, in the second approximation the quantities $\delta\omega$ and $\delta\mathbf{k}$ vanish, as has also been deduced in[1-6].

To solve the equations given above we write the first-order solution in the form

$$f_1(\mathbf{r},\mathbf{v},t)=\exp(-\gamma t)\,(g_1\cos\theta+g_2\sin\theta) \tag{2.1}$$

and take the limit $\gamma \to 0$. Using Eqs. (1.2) and (1.3) we find

$$f_2(\mathbf{r},\mathbf{v},t)=\Phi_1(\mathbf{v})\exp(-2\gamma t)+\Phi_2(\mathbf{v})\cos 2\theta+\Phi_3(\mathbf{v})\sin 2\theta, \tag{2.2}$$

where

$$\Phi_1=-\frac{Ne^2}{4m^2\omega}\frac{m}{\varkappa T}\left[\frac{\omega_0^4(E_1^2-E_2^2)kv_\parallel f_0}{4\dot\theta^2(1-{}^1/_4\,\omega_0^2\mathcal{F}_2)}\mathcal{F}_3\right.$$
$$\left.+\frac{(E_1^2v_1^2-E_2^2v_2^2)k}{\dot\theta^2}\frac{\partial}{\partial v_\parallel}\left(\frac{f_0}{\dot\theta}\right)-\frac{f_0}{\dot\theta}(\lambda_1E_1^2-\lambda_2E_2^2)\right], \tag{2.3}$$

$$\Phi_2=-\frac{Ne^2}{4m^2\omega}\frac{m}{\varkappa T}\frac{f_0}{\dot\theta}\left[\frac{k\omega_0^4(E_1^2-E_2^2)v_\parallel f_0}{4\dot\theta}\mathcal{F}_3\left(1-\frac{\omega_0^2}{4}\mathcal{F}_2\right)^{-2}\right.$$
$$\left.+\frac{k(E_1^2v_1^2-E_2^2v_2^2)}{\theta}\frac{\partial}{\partial v_\parallel}\left(\frac{f_0}{\dot\theta}\right)\right]+\frac{Ne^2}{4m^2\omega}\frac{m}{\varkappa T}\frac{f_0}{\dot\theta}(\lambda_1E_1^2-\lambda_2E_2^2), \tag{2.4}$$

$$\Phi_3=E_1E_2\frac{Ne^2}{2m^2\omega}\frac{m}{\varkappa T}\left[k\frac{\partial}{\partial v_\parallel}\left(\frac{f_0}{\dot\theta}\right)+\frac{m}{\varkappa T}f_0\right]\frac{v_1v_2}{\dot\theta};$$

$$\lambda_1=1-\frac{mv_1^2}{\varkappa T}\qquad\lambda_2=1-\frac{mv_2^2}{\varkappa T},\quad\mathcal{F}_m=\int\frac{F\,dv_\parallel}{\dot\theta^m}. \tag{2.5}$$

The separation of the charges and the induction of the currents are governed by the following relations:

$$\int(\mathbf{v}_\perp\Phi_1;\ \mathbf{v}_\perp\Phi_2;\ \Phi_1;\ \mathbf{v}\Phi_3;\ \Phi_3)\,d\mathbf{v}=0, \tag{2.6}$$

where $\mathbf{v}_\perp = \mathbf{v}_1 + \mathbf{v}_2$. These relations show that the transverse waves do not appear in the second approximation. The quantity $\Phi_1(\mathbf{v})$ is a correction which is independent of time and coordinates and hence changes the form of the background distribution $f_0(v^2)$. However, the third relation in (2.6) shows that this does not lead to a static separation of charges; the first relation shows that Φ_1 does not lead to the appearance of a stationary current transverse to $\mathbf{k}$. Only a stationary longitudinal current $\delta j_\parallel$ appears, where

$$\delta j_\parallel=\int ev_\parallel\Phi_1\,d\mathbf{v}=-\frac{(E_1^2+E_2^2)ke\omega_0^4}{\Omega^4}\mathcal{F}_1^2. \tag{2.7}$$

This quantity might be called the "wave wind"; it can be found by taking f_1 in the form (2.1) and then letting $\gamma \to 0$. A "wind" of this type has been predicted in a paper by Golovko[1] but only in the case of longitudinal waves and in first order. Other authors[2-6] did not note this effect. A review of nonlinear effects in the theory of orbits and particles moving under the effect of an electromagnetic wave has been given by Bloembergen.[10] The analysis of Eq. (2) of this work, after multiplication of the electromagnetic field by the factor $\exp(-\gamma t)$, leads to the appearance of additional terms which give in the second approximation a stationary flow that coincides with $\delta j_\parallel$ at zero temperature.

If we write the longitudinal electric field in second order in the form $E_l \sin 2\theta + E_{l_1} \cos 2\theta$ then

$$E_l=\frac{2\pi e}{k}\int\Phi_2\,d\mathbf{v}=-\frac{(E_1^2-E_2^2)ke\omega_0^4}{8m\omega(1-{}^1/_4\,\omega_0^2\mathcal{F}_2)}\mathcal{F}_3, \tag{2.8}$$

$$E_{l_1}=\frac{2\pi e}{k}\int\Phi_3\,d\mathbf{v}=0, \tag{2.9}$$

where E_l vanishes in the case of the circular polarization of the wave.

We note that the role of the coefficients for the quantities $\sin 2\theta$ and $\cos 2\theta$ are essentially different, but we can not present a physical explanation for this difference.

If S_l is the energy flux in the form of a longitudinal field, then[11], p. 284

$$|S_l|=\frac{E_l^2\omega(4\omega^2-\omega_0^2)\cos^2 2\theta}{4\pi k\omega_0^2}. \tag{2.10}$$

Comparing this result with the Poynting flux of the first-order electromagnetic wave we can determine the fraction of energy that is converted into longitudinal form. Furthermore, if S_0 is the energy density in the "wave wind," then

$$S_0\bigg/\left(\frac{(E_1^2+E_2^2)}{8\pi}\right)=\frac{\omega\omega_0^4\mathcal{F}_1(1+\omega\mathcal{F}_1)}{(\Omega^2-\gamma^2)^2+4\gamma^2\omega^2}, \tag{2.11}$$

which vanishes at zero temperature, showing that the transfer of energy to the "wave wind" is effected by microscopic thermal motion.

This motion is also responsible for the second-order pressure wave on the plane that contains the electric oscillations of the plane-polarized wave. To demonstrate this, it is necessary to compute all of the components of the pressure tensor p_{ij}, these being specified by the relation

$$p_{ij} = m \int v_i v_j f_2 \, d\mathbf{v}.$$

Putting $E_2 = 0$ we find

$$\frac{16\pi\omega}{E_l^2 k \omega_0^2} p_{22} = \left[\frac{\omega_0^2 \overline{\mathscr{F}_1} \mathscr{F}_3}{4(1 - {}^1/_4 \omega_0^2 \mathscr{F}_2)} - 3k \frac{\varkappa T}{m} \mathscr{F}_3 \right] \cos 20,$$

$$\mathscr{F}_1 = \int \frac{v_\| F \, dv_\|}{\dot{\theta}}; \tag{2.12}$$

the right side of equation (2.12) vanishes when $T = 0$.

3. THIRD-ORDER APPROXIMATION

The second-order solution does not contain increments to the frequency and wave number. However, they appear in the third-order solution, otherwise the results are meaningless. The increments are different for the $\perp_i$ polarizations ($i = 1, 2$) if the wave is not circularly polarized. If the wave vector (taking account of the increment) is $\mathbf{k}_i = \mathbf{k} + \delta\mathbf{k}_i$, the frequency is $\omega_i = \omega + \delta\omega_i$ and the phase is $\theta_i = (\mathbf{k}_i \cdot \mathbf{v}) - \omega_i$, then $\dot{\theta}_i = k_i v_\| - \omega_i$ and $\delta\dot{\theta}_i = \dot{\theta}_i - \dot{\theta}$. The four phases θ_1, $2\theta_2 - \theta_1$, $2\theta_1 - \theta_2$, and θ_2 give the phases of the first harmonic waves in first order when the increments vanish; the other four phases $3\theta_1$, $2\theta_1 + \theta_2$, $2\theta_2 + \theta_1$, and $3\theta_2$ are equal to 3θ in the case of zero increments. The mixed harmonics $2\theta_2 + \theta_1$ and $2\theta_1 \pm \theta_2$ appear as the consequence of the interaction between fields with two different polarizations and cause a rotational effect of the magnetic term of the Lorentz force $(\mathbf{v} \times \mathbf{H})/c$. The third-order approximation will represent a linear combination of sines and cosines of these arguments. The coefficients of these trigonometric relations are functions of $\mathbf{v}$. However, up to third order the difference between the cosines of θ_1, $2\theta_2 - \theta_1$, $2\theta_1 - \theta_2$, and θ_2 will be negligibly small and each function can be taken equal to $\cos\theta'$, where $\theta' = k' v_\| - \omega'$, $\mathbf{k}' = \mathbf{k} + \delta\mathbf{k}$, and $\omega' = \omega + \delta\omega$. Similarly, the sines of these arguments can be equated to $\sin\theta'$, while the arguments $3\theta_1$, $2\theta_1 + \theta_2$, $2\theta_2 + \theta_1$, $3\theta_2$ can be taken equal to $3\theta'$ for the purposes of the present discussion.

Thus, the third-order approximation to (1.1) can be written in the form

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\Delta\right) f_3 + \frac{Ne}{m}\left(E_{3\|}\frac{\partial f_0}{\partial v_\|} + E_{3\perp_1}\frac{\partial f_0}{\partial v_1} + E_{3\perp_2}\frac{\partial f_0}{\partial v_2}\right)$$
$$+ E_1 \frac{Ne}{m}\frac{m}{\varkappa T}\sin\theta_1 \left\{\frac{\delta\omega_1}{\omega}\left(\frac{\omega^2 + k^2c^2}{\Omega^2} - \frac{\omega}{\dot{\theta}}\right) - \delta k_1\left(\frac{2kc^2}{\Omega^2} - \frac{v_\|}{\dot{\theta}}\right)\right\} v_1 f_0$$
$$+ E_2 \frac{Ne}{m}\frac{m}{\varkappa T}\cos\theta_2 \left\{\frac{\delta\omega_2}{\omega}\left(\frac{\omega^2 + k^2c^2}{\Omega^2} - \frac{\omega}{\dot{\theta}}\right)\right.$$
$$\left. - \delta k_2\left(\frac{2kc^2}{\Omega^2} - \frac{v_\|}{\dot{\theta}}\right)\right\} v_2 f_0 = -\frac{e}{m} E_{2\|}\frac{\partial f_1}{\partial v_\|} - \frac{e}{m}\left\{\mathbf{E}_1 + \frac{1}{c}[\mathbf{v}\mathbf{H}_1]\right\} \nabla_{\mathbf{v}} f_2.$$

For a field with polarization $\perp_1$ in the case of the initial-value problem we have $\delta k_1 = 0$ and the frequency increment is

$$\delta\omega_1\left[\omega^2\mathscr{F}_2 + \frac{\omega^2 + k^2c^2}{\omega_0^2}\right] = -E_1^2 \frac{k^2c^2}{4m^2\omega}\mathscr{F}_3$$
$$+ \frac{(E_1^2 - E_2^2) k^2 e^4 \mathscr{F}_3}{32 m^2 \omega (1 - {}^1/_4 \omega_0^2 \mathscr{F}_2)}\{4 + \omega_0^2\omega\mathscr{F}_3 - \omega_0^2\mathscr{F}_2\}$$
$$- \frac{E_1^2 \Omega^2 k^2 e^2 \mathscr{F}_2}{m^2\{(\Omega^2 - \gamma^2)^2 + 4\gamma^2\omega^2\}} - \frac{(3E_1^2 + E_2^2)\Omega^2 k^2 e^2}{m^2\{(\Omega^2 - \gamma^2)^2 + 4\gamma^2\omega^2\}}\frac{\varkappa T}{m}\mathscr{F}_4$$
$$+ (3E_1^2 - E_2^2)\frac{3k^4c^2}{8m^2\omega}\frac{\varkappa T}{m}\mathscr{F}_5. \tag{3.1}$$

In the case of boundary-value problem, $\delta\omega_1 = 0$ and δk_1 can be determined from the equation

$$\frac{\delta k_1}{k}\left\{\omega^2\mathscr{F}_2 + \frac{2\omega^2 - 3\Omega^2}{\omega_0^2}\right\} = -\frac{\delta\omega_1}{\omega}\left\{\omega^2\mathscr{F}_2 + \frac{\omega^2 + k^2c^2}{\omega_0^2}\right\} \tag{3.2}$$

making use of (3.1). Similarly, the corresponding quantity for the field polarization $\perp_2$ is obtained by interchanging E_1 and E_2 in (3.1) and (3.2).

The amplitude E_{31} of the third harmonic of the electric field with polarization $\perp_1$ is

$$E_{31} = E_1 \frac{k^2\pi^2 N e^4}{288\Omega^2 m^3 \omega}\left[-8E_1^2\mathscr{F}_3 - (3E_1^2 - E_2^2)\frac{12k^2\varkappa T}{m}\mathscr{F}_5\right.$$
$$\left. - \frac{(E_1^2 - E_2^2)\mathscr{F}_3\{4 - \omega_0^2(3\omega\mathscr{F}_3 + \mathscr{F}_2)\}}{1 - {}^1/_4\omega_0^2\mathscr{F}_2}\right]. \tag{3.3}$$

The amplitude E_{32} of the third harmonic with polarization $\perp_2$ is obtained by reversing the sign of each term and interchanging E_1 and E_2. The quantity $1 - (1/4)\omega_0^2\mathscr{F}_2$ in the denominator in the third term on the right side of (3.3) cannot vanish because of the dispersion relation (1.13). The increment δE_1 to the amplitude E_1 for $T = 0$ is given by

$$\delta E_1 = E_1^3 \frac{e^2(16\omega^4 - 3\omega^2\omega_0^2 + 2\omega_0^4)}{2m^2c^2\omega^2\omega_0^2(4\omega^2 - \omega_0^2)}. \tag{3.4}$$

4. DISCUSSION

Expanding the obtained expressions in powers of ω_0^2/ω^2 and $\kappa T/mc^2$ and keeping only the first nonvanishing powers of these quantities, we have

$$\frac{\delta\omega_1}{\omega} = -\frac{e^2}{16m^2c^2\omega^4}\left[\left\{8E_1^2\left(1 + \frac{7\varkappa T}{mc^2}\right) + 8E_2^2\frac{\varkappa T}{mc^2}\right\}\omega^2\right.$$
$$\left. - E_1^2\omega_0^2\left(1 - \frac{3\varkappa T}{mc^2}\right) - E_2^2\omega_0^2\left(1 + \frac{9\varkappa T}{mc^2}\right)\right].$$

The quantity $\delta\omega_2$ can be found in similar fashion. For a plane-polarized wave in the polarization plane $\perp_1$ we have $E_2 = 0$. Consequently $\delta\omega_1/\omega < 0$, so that the frequency shift is in the red direction. The magnitude of the change in the red shift with temperature $(\omega^{-1}\partial(\delta\omega_1)/\partial T)$ is also negative, indicating that the frequency is shifted in large degree in the red direction with increasing temperature. Furthermore, the shift is different for the two planes of elliptical polarization. The increments $\partial(\delta\omega_1)/\partial k$ and $\partial(\delta\omega_2)/\partial k$ of the group velocity are also different in the two planes, so that the wave is split into two waves that move with different group and phase velocities in the two polarization planes.

Taking $m = 9 \times 10^{-28}$ g and $e = -4.8 \times 10^{-10}$ esu for the electron, we find at $E_2 = 0$, $E_1 \equiv E$, and $T = 0$

$$\frac{\delta\omega}{\omega} = -E^2\frac{15.5 \cdot 10^{13}}{\omega^2} \quad \frac{E_l}{E} = -E\frac{2.2 \cdot 10^{[illegible]}\omega_0^2}{\omega^3}, \qquad \delta j = E \cdot 7.4 \cdot 10^5.$$

The nucleus Fe^{57} used in Mossbauer experiments ($h\nu = 14.4$ keV) can serve as a source of γ rays. At such high frequencies the coupling of the electrons with their own nuclei is very weak so that the electrons can be regarded as comprising a plasma. Taking $\omega = 2.2 \times 10^{19}$ and $N = 4 \times 10^{23}$, we get $\omega_0 = 4.58 \times 10^{16}$. Assuming that the source delivers a flux of 10^8 photons/sec we have $\delta\omega/\omega = -2.8 \times 10^{-23}$ which is much smaller than the red shift, of the order of 10^{-14}, observed in the Mossbauer effect.

As a second example we consider the propagation of ultraviolet rays in alkali metals. Taking E = 10 esu and $\omega = 10^{15}$ Hz, in the case of sodium $N = 26.3 \times 10^{19}$ cm^{-3} and an electrical conductivity $\sigma = 9.3 \times 10^{16}$ cgs esu, we find $\delta j/\sigma E = 0.75$ and $\delta j^{(2)}/\sigma E = 0.094$, where $\delta j^{(2)}$ is the amplitude of the second harmonic $(= e \int v_{\|} \Phi_2 dv)$. In this case $\delta\omega/\omega = -15.5 \times 10^{-14}$.

A third example is provided by the ionosphere, where typical values are $E = (1/10)$ V/m, $\omega = 2\pi \times 5 \times 10^7$ Hz, and $N = 10^6$ cm^{-3}. In this case $\delta\omega/\omega = -1.7 \times 10^{-14}$ and $E_l/E = 2 \times 10^{-7}$.

The calculation for the nucleus used in the Mossbauer experiment yields a negligibly small shift. The relativistic corrections can lead to an additional red shift. This is expected since the relativistic correction[4] to the frequency shift[2] leads to a change in the sign of the shift from the blue to the red.

The shift in the wave vector δk in the boundary-value problem leads to an interesting nonlinear deflection of the radiation at the plane of separation between the plasma and another medium.[12]

In conclusion the author would like to express his thanks for the help given by Professor A. A. Vlasov, who made it possible for the author to work on this problem during his residence in the Soviet Union.

[1] V. A. Golovko, Zh. Eksp. Teor. Fiz. 47, 1765 (1964) [Sov. Phys.-JETP 20, 1189 (1965)].

[2] D. Montgomery and D. A. Tidman, Phys. Fluids 7, 242 (1964).

[3] D. A. Tidman and H. M. Stainer, Phys. Fluids 8, 345 (1965).

[4] F. W. Slütter and D. Montgomery, Phys. Fluids 8, 551 (1965).

[5] A. H. Nayfeh, Phys. Fluids 8, 1896 (1965).

[6] P. A. Sturrock, Proc. Roy Soc. 242A, 277 (1957); ibid 242A, 1230 (1957).

[7] A. A. Vlasov, Zh. Eksp. Teor. Fiz. 8, 291 (1938).

[8] A. V. Bogdanov, Uchenye zapiski MOPI 33, 63 (1955); ibid 33, 47 (1955).

[9] V. N. Gershman, Zh. Eksp. Teor. Fiz. 24, 453 (1953).

[10] N. Bloembergen, Proc. I.E.E.E. 51, 121 (1963).

[11] A. A. Vlasov, Teoriya mnogikh chastits (Theory of Many Particles) Gostekhizdat, 1950.

[12] B. Chakraborti, ZhETF Pis. Red. 8, 636 (1968). [JETP Lett. 8, 383 (1968)].

Translated by H. Lashinsky
53