ABSORPTION OF SOUND IN METALS IN AN INCLINED MAGNETIC FIELD

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Sound absorption in metals with a closed Fermi surface is studied both theoretically and experimentally in the case of strong magnetic fields as a function of the change in the angle of inclination φ of the vector H relative to a direction perpendicular to the sound propagation. The absorption coefficient $\Gamma(\varphi)$ undergoes a sharp increase under the conditions $\varphi = \varphi_{cr}$, which is determined by the condition $k\bar{v}_{H\,max}\sin\varphi_{rs} = s$. It is shown that measurement of $\Gamma(\varphi)$ and $\partial\Gamma(\varphi)/\partial\varphi$ at the point $\varphi = \varphi_{cr}$ allows us to determine the electron velocities in the vicinity of the reference point and also the relaxation time and the magnitude of the deformation potential. The results of the theoretical calculations are compared with experimental data obtained for Bi.

In metals placed in a strong magnetic field H, a sharp increase is observed in the sound absorption as a function of the angle of inclination of the vector H relative to the direction of the sound propagation. This effect was first discovered by Reneker in Bi.[1] The qualitative explanation of the phenomenon reduces to the following.

In the region of strong magnetic fields, when the spatial inhomogeneity of the sound wave is unimportant,

$$kR \ll 1, \quad l \gg R, \tag{1}$$

the resonant sound absorption is brought about by electrons for which the phase relation

$$\overline{\mathbf{kv}} = \omega \tag{2}$$

is satisfied (Landau damping). Here k and ω are the wave vector and the sound frequency; v, l and R are the velocity, free path length and the cyclotron radius of the orbit of the conduction electron; the bar indicates averaging over the period of rotation of the electrons in the field H. In metals with a closed Fermi surface, in a magnetic field perpendicular to the vector k, we have $\mathbf{k}\cdot\bar{\mathbf{v}} = 0$ ($\bar{v}$ is equal in magnitude to the mean velocity $\bar{v}_H$ of the electron along the magnetic field). The phase relation (2) is not satisfied and the absorption is small.

For inclination of the vector H by an angle φ from the perpendicular $\mathbf{k}\cdot\mathbf{H} = 0$, the current carriers acquire a drift component along k, equal to $\bar{v}_H \sin\varphi$. For small φ, the value of $\bar{v}_H \sin\varphi$ is small in comparison with the phase velocity of sound $s = \omega/k$ for electrons with maximum velocity $\bar{v}_{H\,max}$. For some critical angle of inclination φ_{cr},

$$\sin\varphi_{cr} = s / \bar{v}_{H\,max} \tag{3}$$

electrons with the maximum velocity $\bar{v}_{H\,max}$ from the vicinity of the elliptical limiting point on the Fermi surface satisfy the resonance condition (2). Consequently, for $\varphi \gtrsim \varphi_{cr}$ there is a group of electrons moving in phase with the wave and effectively absorbing its energy. Near the angle φ_{cr}, the absorption coefficient $\Gamma(\varphi)$ increases sharply—the inclination effect. Since collisions disrupt the phase relation (2) between the current carriers and the wave, the necessary condition for a sharp increase in $\Gamma(\varphi)$ close to $\varphi = \varphi_{cr}$ is the high frequency condition

$$\omega \gg \nu, \quad \nu = \tau^{-1} \tag{4}$$

(τ is the relaxation time).

The relation (3) permits us to determine the electron velocities $\bar{v}_{H\,max}$ and to establish the Fermi surface of the metal. Unfortunately, it is possible to measure the angles of inclination experimentally only for metals with a sufficiently small Fermi energy ϵ_f. The semimetals belong to this group, along with metals possessing portions on the Fermi surface with anomalously small velocities. At the present time, the inclination effect has been observed in Bi,[1,2] Sb[3] and, more recently, Ga.[4]

The present work is devoted to the theoretical and experimental investigation of the sound absorption in an inclined magnetic field. A theory of the inclination effect is constructed and it is shown that the measurements of the absorption coefficient $\Gamma(\varphi)$ and its derivative $\partial\Gamma(\varphi)/\partial\varphi$ for angles $\varphi \approx \varphi_{cr}$ allow us to determine the relaxation time and the value of the deformation potential for electrons from the vicinity of the limiting points. The results of the theoretical calculations are compared with the experimental data obtained for Bi.

1. THEORY

The sound absorption in metals in the presence of a constant magnetic field H is connected with two mechanisms of interaction of the conduction electrons with acoustic oscillations—deformation and induction.[5,6] The acoustic oscillations produce a deformation of the lattice. This deformation leads to modulation of the energy of the electrons and the appearance of a variable electric field E, which also produces absorption of the sound energy. The induction mechanism is associated with the induced electric field $\mathbf{G} = [\mathbf{u} \times \mathbf{H}]/c$, which appears in the intersection of the lines of force of the magnetic field of the deformed conductor ($\mathbf{u} = \mathbf{u}_0 \exp\{i\mathbf{k}\cdot\mathbf{r} - i\omega t\}$ is the displacement vector).

We shall investigate the sound absorption Γ for closed Fermi surfaces in the region of strong magnetic fields (1). In the region of the fields (1), one must generally take into account both the mechanisms of electron interaction with sound. The sound absorption coefficient Γ is described by the following expression:[6]

$$\Gamma(H, \omega, \nu, \varphi) = \frac{1}{(2\pi\hbar)^3} \frac{1}{W} \operatorname{Re} \int_{\varepsilon=\varepsilon_F} \frac{m\, dp_H}{\Omega} \int_0^{2\pi} dt\, g^*(p_H, t) \int_{-\infty}^{t} dt_1\, g(p_H, t_1) \times \exp\left[\int_t^{t_1} \frac{\nu - i\omega + i\mathbf{k}\mathbf{v}}{\Omega} dt_2\right]. \tag{5}$$

Here $W = \frac{1}{2}\rho\omega^2 |u_0|^2 s$ is the energy density of the sound wave; m and Ω are the cyclotron mass and the frequency of the electron; **p** is its momentum, $p_H = \mathbf{p}\cdot\mathbf{H}/H$; t is the dimensionless time (phase) of the motion of the electron in orbit; $\varphi = \frac{1}{2}\pi - \vartheta$; ϑ is the angle between the vectors **k** and **H**; g is the rate of change of the energy of the electron in the sound field:

$$g(p_H, t) = e(\mathbf{E} + \mathbf{G})\mathbf{v} + \Lambda_{ik}\dot{u}_{ik}; \tag{6}$$

$\Lambda_{ik}(p)$ is the symmetric deformation potential tensor, which vanishes in averaging over the Fermi surface; u_{ik} is the deformation tensor. The variable electric field **E** is found from the Maxwell equations

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \quad \operatorname{div}\mathbf{j} = 0, \tag{7}$$

in which the current is represented by the sum

$$\mathbf{j} = \mathbf{j}_\Lambda + \hat{\sigma}(\mathbf{E} + \mathbf{G}). \tag{8}$$

The deformation current j_Λ is determined in the following way:

$$j_{\Lambda i} = \frac{2e}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} \frac{m\, dp_H}{\Omega} \int_0^{2\pi} dt\, v_i(p_H, t) \int_{-\infty}^{t} dt_1\, g_\Lambda(p_H, t_1) \times \exp\left[\int_t^{t_1} \frac{\nu - i\omega + i\mathbf{k}\mathbf{v}}{\Omega} dt_2\right]. \tag{9}$$

The conductivity tensor $\hat{\sigma}$ has the form

$$\sigma_{ik} = \frac{2e^2}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} \frac{m\, dp_H}{\Omega} \int_0^{2\pi} dt\, v_i(p_H, t) \int_{-\infty}^{t} dt_1 v_k(p_H, t_1) \times \exp\left[\int_t^{t_1} \frac{\nu - i\omega + i\mathbf{k}\mathbf{v}}{\Omega} dt_2\right]. \tag{10}$$

We choose the set of coordinates xyz so that the z axis is directed along the vector **H**, and the y axis is perpendicular to the plane in which the vectors **k** and **H** are located. Further, we introduce the coordinates ξyζ with the ζ axis parallel to the vector **k**. It is convenient to solve the Maxwell equation (7) in this set of coordinates, eliminating the longitudinal field and introducing the renormalized quantities

$$\tilde{\sigma}_{ab} = \sigma_{ab} - \frac{\sigma_{a\zeta}\sigma_{\zeta b}}{\sigma_{\zeta\zeta}}, \quad \tilde{j}_{\Lambda a} = j_{\Lambda a} - \frac{\sigma_{a\zeta}}{\sigma_{\zeta\zeta}} j_{\Lambda\zeta}, \quad a, b = \xi, y. \tag{11}$$

Solution of Eqs. (7) is elementary and we shall not write out the rather cumbersome formulas.

We represent the absorption coefficient Γ in the form of a sum:

$$\Gamma = \Gamma_\Lambda + \Gamma_1 = \Gamma_\Lambda + \frac{1}{2W}\operatorname{Re}\sum_{a,b=\xi,y} \tilde{\sigma}_{ab} E_a'^* E_b' + \frac{1}{W}\operatorname{Re}\sum_{a,b=\xi,y} \tilde{j}_{\Lambda a} E_a'^*. \tag{12}$$

The first component Γ_Λ, due to the "deformation," is determined by Eq. (5), in which we need only write the component $g_\Lambda = \Lambda_{ik}\dot{u}_{ik}$ in place of the function $g(p_H, t)$. The second component is the Joule heat loss in the variable electric fields

$$\mathbf{E}' = \mathbf{E} + \mathbf{G}. \tag{13}$$

For the study of the absorption coefficient $\Gamma(\varphi)$, it is necessary to compare the quantities Γ_Λ and Γ_1 and estimate the role of the variable electric fields **E** and **G**. The solutions of Maxwell's equations (7) contain the parameter

$$\delta \sim c(4\pi\omega\, |\, \tilde{\sigma}_{ab}\, |)^{-1/2}, \tag{14}$$

which has the meaning of the "penetration depth" of the electromagnetic field at the sound frequency ω. It is not difficult to show that if the penetration depth δ is much greater than the sound wavelength λ,

$$\delta \gg \lambda, \tag{15}$$

then the electric fields **E** are small in comparison with the induced fields **G**. Consequently,

$$\mathbf{E}' \approx \mathbf{G}. \tag{15a}$$

In the opposite limiting case,

$$\delta \ll \lambda, \tag{16}$$

the induced fields are small and

$$\mathbf{E}' \approx \mathbf{E}. \tag{16a}$$

We compute the coefficient of deformation absorption $\Gamma_\Lambda(\varphi)$. It is convenient to do this in the set of coordinates xyz connected with the magnetic field. In the integration over t and t_1, we use the condition (1). We have

$$\Gamma_\Lambda = \sum_\alpha \frac{2\pi}{(2\pi\hbar)^3 W} \operatorname{Re} \int_{\varepsilon=\varepsilon_F} m\, dp_z |\bar{g}_\Lambda(p_z)|^2\, \tilde{\nu}^{-1}. \tag{17}$$

Here $\tilde{\nu} = \bar{\nu} - i\omega + ik\bar{v}_z \sin\varphi$, α is the number of group of carriers of the multiply connected Fermi surface.[7] For the estimates (15) and (16), we must know the values of the components of the conductivity tensor σ_{ik}. Integration over t and t_1 reveals the order of smallness of the quantities σ_{ik} in the sense of the criteria (1) and (4):

$$\sigma_{zz} = \sum_\alpha \frac{4\pi e^2}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} m\, dp_z \bar{v}_z^2 \tilde{\nu}^{-1},$$

$$\sigma_{yy} = \sum_\alpha \frac{4\pi e^2}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} \frac{m\, dp_z}{\Omega}\left\{\frac{\tilde{\nu} - i\omega}{\Omega}\overline{p_x^2} + 2k^2\cos^2\varphi \frac{\overline{p_y v_y^2}}{\tilde{\nu}\Omega}\right\},$$

$$\sigma_{xx} = \sum_\alpha \frac{4\pi e^2}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} \frac{m\, dp_z}{\Omega} \frac{\tilde{\nu} - i\omega}{\Omega} \overline{p_y^2},$$

$$\sigma_{xy} = \frac{2ec(N_1 - N_2)}{H} + O[(kR)^2]. \tag{18}$$

Here $N_1(N_2)$ is the number of electrons (holes) and is determined by the volume bounded by the surface $\epsilon(p) = \epsilon_F$, inside which the energy is smaller (larger) than ϵ_F. The remaining components of σ_{ik} are small and do not have to be considered. The component of the current $j_{\Lambda x}$ turns out to be small in comparison with the components $j_{\Lambda z}$ and $j_{\Lambda y}$. The expressions for the latter have the form

$$j_{\Lambda z} = \sum_\alpha \frac{4\pi e}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} m\, dp_z \bar{g}_\Lambda \bar{v}_z \tilde{\nu}^{-1}, \quad j_{\Lambda y} = \sum_\alpha i \frac{4\pi e k_x}{(2\pi\hbar)^3} \int_{\varepsilon=\varepsilon_F} \frac{dp_z}{\Omega} \overline{g_\Lambda p_y v_y}\, \tilde{\nu}^{-1}. \tag{19}$$

In the calculation of the component $j_{\Lambda z}$ it is necessary to take it into account that $\Lambda_{ik}(p)$ vanishes in averaging over the Fermi surface. The renormalized components of the tensor $\tilde{\sigma}_{ab}$ are connected with Eqs. (18) by the following relations:

$$\tilde{\sigma}_{\xi\xi}=\frac{\sigma_{zz}\sigma_{xx}}{\sigma_{\zeta\zeta}},\quad \tilde{\sigma}_{yy}=\sigma_{yy}-\cos^2\varphi\frac{\sigma_{yz}\sigma_{zy}}{\sigma_{\zeta\zeta}},\qquad (20)$$

$$\sigma_{\zeta\zeta}=\sigma_{xx}\cos^2\varphi+\sigma_{zz}\sin^2\varphi$$

(we note that the effect of inclination takes place in the region of small angles φ).

Renormalization of the conductivity (20) leads to different results for metals for which $N_1 \neq N_3$ and for compensated metals with $N_1 = N_2$. This is caused by the different values of the Hall conductivity σ_{xy} for these two cases.

1. In the case $N_1 \neq N_2$, as follows from (18) and (20), the renormalized components of the conductivity $\hat{\sigma}_{yy}$ and $\tilde{\sigma}_{\xi\xi}$ have the same order of magnitude. Estimates show that the parameters of penetration depth

$$\delta_1 \sim c(4\pi\omega|\tilde{\sigma}_{yy}|)^{-1/2},$$
$$\delta_2 \sim c(4\pi\omega|\tilde{\sigma}_{\xi\xi}|)^{-1/2}$$

are practically always small in comparison with the sound wavelength λ for metals (in magnetic fields H up to several tens of kilo-oersteds). Here $E'_\alpha \approx E_\alpha$ are determined from the equations

$$\left(k^2-\frac{4\pi i\omega}{c^2}\tilde{\sigma}_{aa}\right)E_a=\frac{4\pi i\omega}{c^2}j_{\Lambda a}.$$

The component Γ_1 is small in comparison with $\Gamma_\Lambda(\Gamma_1/\Gamma_\Lambda \sim (kR)^2 \ll 1)$. The absorption is determined by the deformation term and is equal to

$$\Gamma(\varphi)=\sum_\alpha\frac{2\pi}{(2\pi\hbar)^3W}\cdot\mathrm{Re}\int\frac{m\,dp_z|g_\Lambda(p_z)|^2}{\bar{\nu}-i(\omega-k\bar{v}_z\sin\varphi)}.\qquad (21)$$

We consider the limiting case of very pure metals, when $\bar{\nu}\to 0$. We replace the resonance denominator by the δ function:

$$\mathrm{Re}\frac{1}{\bar{\nu}-i(\omega-k\bar{v}_z\sin\varphi)}\to\pi\delta(\omega-k\bar{v}_z\sin\varphi)\qquad (21a)$$

and obtain

$$\Gamma_\alpha(\varphi)=\begin{cases}\Gamma_\Lambda^{(0)}\pi\dfrac{s}{|\bar{v}_{z\,\max}\sin\varphi|}, & |\sin\varphi|\geqslant\dfrac{s}{\bar{v}_{z\,\max}},\\ 0, & |\sin\varphi|<\dfrac{s}{\bar{v}_{z\,\max}},\end{cases}$$

$$\Gamma_\Lambda^{(0)}=\frac{2\pi}{(2\pi\hbar)^3W}\frac{mp_0|\bar{g}_\Lambda(p_z)|^2}{\omega}\bigg|_{p_z=\tilde{p}_z},\qquad (22)$$

$$\tilde{p}_z=p_0\frac{s}{\bar{v}_{z\,\max}\sin\varphi}.$$

(We have assumed that the function $g_\Lambda(p_z)$ does not have delta-like singularities over the characteristic width of the δ function; $p_{max} = -p_{min} = p_0$ is the momentum of the electron at the reference points of the Fermi surface; $\bar{v}_{z\,max} = \bar{v}_z(p_0)$ is its mean velocity along the magnetic field at the limiting points). For angles $\varphi \geq \varphi_{cr}$ a sharp increase takes place in the sound absorption, and the maximum value of $\Gamma(\varphi)$ for the limiting case $\bar{\nu} \to 0$ is achieved for $\varphi = \varphi_{cr} = \arcsin(s/\bar{v}_{z\,max})$. Figure 1 shows the graph of the function $\Gamma(\varphi)$ as $\bar{\nu} \to 0$.

For exact calculation of the integral (21) over p_z for collision frequency $\bar{\nu}$ different from zero, it is necessary to know the dependence of the deformation potential on the momentum. We cannot write down the detailed expression for an arbitrary dispersion law of electrons $\epsilon(p)$, but it is easy to obtain it for the

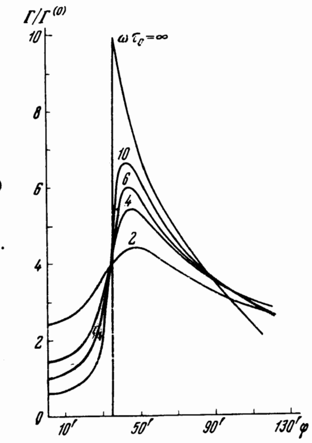

FIG. 1. Theoretical curves of $\Gamma/\Gamma^{(0)}$ for different values of $\omega\tau_0$ ($\bar{v}_{z\,max}$ = 2 × 10^7 cm/sec, s = 2.02 × 10^5 sm/sec).

quadratic isotropic case $\epsilon = p^2/2m$. The deformation potential tensor $\Lambda_{ik}(p)$ has the form

$$\Lambda_{ik}(p)=\Lambda(n_i n_k-{}^1\!/_3\delta_{ik}),\quad n=p/p_0.$$

The absorption coefficient Γ, with accuracy up to small terms of the order ν^2/ω^2, is described by the formula

$$\Gamma=\Gamma_u^{(0)}\frac{s}{v_0\sin\varphi}\left\{\mathrm{arctg}\frac{kv_0\sin\varphi+\omega}{\nu}+\mathrm{arctg}\frac{kv_0\sin\varphi-\omega}{\nu}\right\}.\qquad (23)$$

The amplitude of $\Gamma_u^{(0)}$ for longitudinal sound is equal to

$$\Gamma_u^{(0)}=\frac{2\pi k^2|u|^2mp_0}{(2\pi\hbar)^3W\omega}\left\{\Lambda^2\left[\frac{1}{6}-\frac{\omega^2}{2(kv_0\sin\varphi)^2}\right]^2\right\}.\qquad (24)$$

The expression in the curly brackets is identical with the value of the longitudinal component of the deformation potential tensor $\overline{\Lambda}^2_{\zeta\zeta} \approx \overline{\Lambda}^2_{xx}$ for $p_z = p_0 s/v_0\sin\varphi$.

For an arbitrary convex Fermi surface, in analogy with the limiting case $\bar{\nu} \to 0$ (22) and the case $\epsilon = p^2/2m$ (23), we take out the function $|\bar{g}_\Lambda(p_z)|^2$ from under the integral at the value of the argument $p_z = \tilde{p}_z \approx p_0$. Then the absorption coefficient has the form

$$\Gamma=\sum_\alpha\Gamma_{\Lambda\alpha}^{(0)}\frac{s}{\bar{v}_{z\,max}\sin\varphi}Y,$$

$$Y=\mathrm{arctg}\left[\omega\tau_0\left(\frac{\bar{v}_{z\,max}\sin\varphi}{s}-1\right)\right]+\mathrm{arctg}\left[\omega\tau_0\left(\frac{\bar{v}_{z\,max}\sin\varphi}{s}+1\right)\right],\qquad (25)$$

where τ_0 is the relaxation time of the electrons from the neighborhood of the limiting points. Equation (25) describes the change in the sound absorption coefficient for inclination of the vector H from the perpendicular direction $k \cdot H = 0$. The formula is symmetric relative to change of sign in φ.

2. In the case of compensated metals with $N_1 = N_2$, the Hall conductivity σ_{xy} is small in comparison with the diagonal elements of the tensor σ_{ik}. Therefore the renormalized components $\tilde{\sigma}_{yy}$ and $\tilde{j}_{\Lambda y}$, with accuracy to within small components (at least, of the order of $(kR)^2 \ll 1$) are equal to σ_{yy} and $j_{\Lambda y}$, respectively [(18), (19)]. It is seen from formulas (18) and (20) that the component $\tilde{\sigma}_{yy}$ is small in comparison with the component $\tilde{\sigma}_{\xi\xi}(\tilde{\sigma}_{yy}/\tilde{\sigma}_{\xi\xi} \sim \nu^2/\Omega^2)$. The penetration depth parameter $\delta_1 \sim c(4\pi\omega|\tilde{\sigma}_{yy}|)^{-1/2}$ significantly exceeds $\delta_2 \sim c(4\pi\omega|\tilde{\sigma}_{\xi\xi}|)^{-1/2}$ and realization of the limiting case $\delta_1 \gg \lambda$ is possible. This limiting case is satisfied for metals with small concentrations of

carriers, for example, semimetals of the type Bi, Sb, and so on, in the magnetic fields (1). The absorption coefficient $\Gamma(\varphi)$ has the form

$$\Gamma = \Gamma_\Lambda + W^{-1}\,\mathrm{Re}\,\{{}^1/{}_2\sigma_{yy}|G_y|^2 + j_{\Lambda y}G_y^*\}. \qquad (26)$$

Since the inclination effect is due to electrons with the maximum velocity $\bar{v}_{z\,max}$ from the neighborhood of the limiting points, then the transverse components of the conductivity σ_{yy} and the current $j_{\Lambda y}$ are not large. Therefore, one must expect that the amplitude of the induction absorption Γ_1 will be small.

In the limiting case of pure metals, $\bar{\nu} \to 0$, and upon integration over p_z in the expressions for the conductivity σ_{yy} (18) and the current $j_{\Lambda y}$ (19), we get the formula (21a). The induction absorption is equal to

$$\Gamma_{1\alpha} = \begin{cases} (\Gamma_G^{(0)} + \Gamma_{\mathrm{int}}^{(0)})\dfrac{\pi s}{|\bar{v}_{z\,max}\sin\varphi|}, & |\sin\varphi| \geqslant \dfrac{s}{\bar{v}_{z\,max}}, \\ 0, & |\sin\varphi| < \dfrac{s}{\bar{v}_{z\,max}}. \end{cases}$$

$$\Gamma_G^{(0)} = \frac{4\pi}{(2\pi\hbar)^3 W}\frac{mp_0}{\omega}\left[k\cos\varphi\cdot\overline{p_y v_y}\Big|_{z=\tilde{p}_z}\frac{|[\dot{u}H]_y|}{H}\right]^2,$$

$$\Gamma_{\mathrm{int}}^{(0)} = \frac{4\pi}{(2\pi\hbar)^3 W}\frac{mp_0}{\omega}k\cos\varphi\cdot\overline{p_y v_y}\Big|_{p_z=\tilde{p}_z}\frac{|\bar{g}_\Lambda[\dot{u}H]_y^*|}{H}. \qquad (27)^*$$

In what follows, we shall be interested in the application to semimetals, for which the dispersion law for electrons can, in first approximation, be regarded as quadratic. For a quadratic dispersion law,

$$\frac{\overline{p_y v_y}}{p_0 v_0}\Big|_{p_z=\tilde{p}_z} = \left(1 - \frac{\sin^2\varphi_{cr}}{\sin^2\varphi}\right) \quad (\varphi \neq 0),$$

and is small in the region of angles near φ_{cr}. Consequently, the amplitude of the induction absorption in the region of angles of the inclination effect is close to zero. The absorption is due to the deformation component Γ_Λ (22). For an arbitrary dispersion law, the sound absorption coefficient can be written in the form

$$\Gamma_\alpha = \begin{cases} \Gamma_\alpha^{(0)}\dfrac{\pi s}{|\bar{v}_{z\,max}\sin\varphi|}, & |\sin\varphi| \geqslant \dfrac{s}{\bar{v}_{z\,max}} \\ 0, & |\sin\varphi| < \dfrac{s}{\bar{v}_{z\,max}} \end{cases} \qquad (28)$$

where the amplitude value of $\Gamma_\alpha^{(0)}$ is equal to

$$\Gamma^{(0)} = \Gamma_\Lambda^{(0)} + \Gamma_G^{(0)} + \Gamma_{\mathrm{int}}^{(0)}.$$

In the case of arbitrary collision frequency $\bar{\nu}$, the calculation of the integrals over p_z for the components σ_{yy} and $j_{\Lambda y}$ can be carried out for specific models of the Fermi surface. For an ellipsoidal surface, the calculations are elementary. Upon satisfaction of the criterion (4), the expressions for σ_{yy} and $j_{\Lambda y}$ reduce to the following:

$$\mathrm{Re}\,\sigma_{yy} = \sigma\frac{s}{\bar{v}_{z\,max}\sin\varphi}\left(1 - \frac{\sin^2\varphi_{cr}}{\sin^2\varphi}\right)^2 Y,$$

$$\mathrm{Re}\,j_{\Lambda y}G_y^* = j|G_y|\frac{s}{\bar{v}_{z\,max}\sin\varphi}\left(1 - \frac{\sin^2\varphi_{cr}}{\sin^2\varphi}\right)Y, \qquad (29)$$

$$\sigma = \frac{4\pi}{(2\pi\hbar)^3}\left[\frac{\varepsilon_F ck\cos\varphi}{H}\right]^2\frac{mp_0}{\omega}, \quad j = \frac{4\pi}{(2\pi\hbar)^3}\frac{\varepsilon_F ck\cos\varphi}{H}\frac{mp_0}{\omega}.$$

Thanks to the factor $1 - \sin^2\varphi_{cr}/\sin^2\varphi$ in the range of angles of sharp increase in $\Gamma(\varphi)$, these expressions are small. Thus, with accuracy up to terms of higher order of smallness, the sound absorption coefficient $\Gamma(\varphi)$ for semimetals is described by (25).

*$[\dot{u}H] \equiv \dot{u} \times H$.

2. EXPERIMENT

The study of sound absorption in an inclined field was performed with an ultrasonic spectrometer operating in a continuous regime. To obtain smooth variation of inclination of the magnetic field, a solenoid was placed between the poles of an electromagnet, the magnetic field of the solenoid being normal to the field of the electromagnet H = const ($H \perp k$). As a consequence of the application of two mutually perpendicular fields, one of which, H', is changed linearly in time, the resultant vector H'' is inclined at a determined angle φ. Regulation of the current in the solenoid was carried out so that the magnetic field of the solenoid changed from some value $-H_0'$ to $+H_0'$. The resulting vector H'' varies in inclination from $-\varphi$ $+\varphi$. Such a sweep of the field does not require extraordinarily accurate establishment of the electromagnetic field H perpendicular to the vector k. Special attention was paid to the accuracy of determination of the orientation of the crystallographic axes of the Bi specimen.

After use of an electric spark cutter and subsequent lapping of the surface of the samples to parallelism to within 0.5 μ, the samples had the shape of discs of diameter 8–9 mm and thickness 2 mm. Bismuth of the type Bi-000 was used in the experiments, subject to twenty zone recrystallizations.

We used a $LiNbO_3$ plate as an ultrasonic source, with a fundamental frequency of 165 MHz. The plate could be excited in the odd harmonics, so that frequencies of 495 and 840 MHz were obtained. A detailed description of the experimental method will be given in a subsequent paper.

3. DISCUSSION OF RESULTS

Figure 1 shows the dependence of $\Gamma_\alpha(\varphi)/\Gamma_\alpha^{(0)}$ for various values of $\omega\tau_0$ and $\bar{v}_{z\,max} = 2 \times 10^7$ cm/sec. This velocity corresponds to the velocity of electrons of group α at the limiting point for the case $k \parallel C_3$ and $\measuredangle(H, C_1) \approx 15°$. The quantity $\bar{v}_{z\,max}$ is computed in the approximation of an isotropic quadratic relation $\epsilon(p) = {}^1/{}_2\alpha_{ik}p_i p_k$; α_{ik} is the tensor of inverse effective mass. The value of the Fermi energy and the component of the effective mass tensor are taken from[8]. At $\varphi = 0$, the absorption changes according to the law $2\omega\tau/1 + (\omega\tau)^2$ (the expression is obtained from Eq. (25) in the limiting case $\varphi \to 0$) and tends to zero as $\omega\tau \to \infty$. With increase in $\omega\tau_0$, the curves are characterized by a more rapid growth of $\Gamma(\varphi)$ near φ_{cr}. This agrees with experiment (Fig. 2, continuous curves). The amplitude decreases upon decrease in $\omega\tau_0$ and the position of the maxima is shifted in the direction of larger φ. It is of interest to note that all the curves intersect near $\varphi = \varphi_{cr}$. The point $\varphi = \varphi_{cr}$ is not a special point (point of maximum, inflection, and so on) of the curve $\Gamma(\varphi)$. However, for large value of $\omega\tau_0 > 1$, the position $\varphi = \varphi_{cr}$ is very close to the location of the inflection point of the curve $\Gamma(\varphi)$. Therefore, the value of $\bar{v}_{z\,max}$ at large $\omega\tau_0$ can be determined from the position of the turning point.

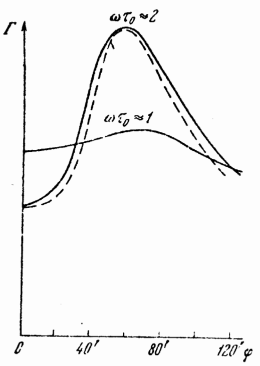

FIG. 2. Comparison of the theoretical and experimental curves for the parameters $\omega\tau_0 \approx 2$, $\bar{v}_{z\,max} = 1.3 \times 10^7$ cm/sec, s = 2.02×10^7 cm/sec; $\mathbf{k} \parallel C_3$. Continuous curves—experiment, dashed curve, theoretical. For comparison of the character of growth of $\Gamma(\varphi)$ as a function of $\omega\tau_0$, the experimental curves are given for $\omega\tau_0 \approx 1$.

Figure 2 shows the result of comparison of the theoretical curve $\Gamma(\varphi)/\Gamma^{(0)}$ with the experimental one. The experimental curve was obtained for $\mathbf{k} \parallel C_3$, $\bar{v}_{z\,max} = 1.3 \times 10^7$ cm/sec, $\measuredangle$ (H, C_1) $\approx 30°$. The value of $\omega\tau_0$ was estimated from geometric resonance and is close to two; the sound frequency is ω = 495 MHz, and T = 1.4°K. It is interesting that the absorption is the total effect of all groups of carriers. The direction of the field H was so chosen that the velocities of the electron at the reference points of the various electron groups, $v_{H\,max}$ were close to one another. The theoretical curve was computed for the parameters $\omega\tau_1$ = 2, $v_{z\,max} = 1.3 \times 10^7$cm/sec (which corresponds to the direction $\measuredangle$ (H, C_1) = 30°), and s_{C_3} = 2.0 $\times 10^5$cm/sec.

As is seen from the drawing, there is excellent agreement between theory and experiment. The location of the maxima is identical within several minutes of arc. The line shape of the experimental curve repeats the theoretical one, although it is somewhat broader than the latter. The half-widths of the curves differ by 6′—7′, which is within the limits of experimental error.

The study of the "inclination effect" allows us to measure not only the velocities of the electron from the neighborhood of the limiting point, but also to determine its relaxation time τ_0 and to estimate the value of the deformation potential. As was pointed out above, upon increase in $\omega\tau_0$ the curves become characterized by a sharp increase in $\Gamma(\varphi)$. The inclination angle β of the tangent to the curve $\Gamma(\varphi)/\Gamma^{(0)}$ at the point $\varphi = \varphi_{cr}$ is described by the expression

$$\operatorname{tg}\beta = \frac{d}{d\varphi}\left(\frac{\Gamma(\varphi)}{\Gamma^{(0)}}\right) = \operatorname{ctg}\varphi_{cr}\left[\omega\tau_0\frac{2+4(\omega\tau_0)^2}{1+4(\omega\tau_0)^2} - \operatorname{arctg}2\omega\tau_0\right],$$
$$\Gamma(\varphi_{cr}) = \Gamma_0 \operatorname{arctg} 2\omega\tau_0.$$

By plotting the dependence of tan β on $\omega\tau_0$ for a given value of $\bar{v}_{z\,max}$, we can determine $\omega\tau_0$ for the experimentally measured inclination angle β. Thus the inclination effect is a suitable method for the measurement of the relaxation times of electrons of the vicinity of the reference points for all Fermi surfaces.

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