

MUON SPIN PRECESSION IN MESIC MOLECULES

V. M. SAMOĬLOV

Institute of Theoretical and Experimental Physics

Submitted January 16, 1970

Zh. Eksp. Teor. Fiz. 58, 2202-2209 (June 1970)

Nuclear spin interaction in a diatomic molecule consisting of muonium and an atom with a nuclear spin of $1/2$ is considered in a magnetic field H directed perpendicular to the quantization axis. Two cases are considered, that of a "fastened" and that of a "rotating" molecule. The exact values of the terms and the time dependence of the muon polarization vector are obtained for the "rotating" molecule. Approximate solutions for an arbitrary orientation (which go over to the exact ones in absence of a field) and exact solutions for some selected orientations of the molecule are obtained for a "fastened" molecule. Numerical estimates of possible precession frequencies for the MuH molecules are presented.

THE behavior of muonium in condensed media was considered in detail in [1, 2]. One of the possible mechanisms of depolarization is the one in which the μ^+ meson is in a chemically bound state in a diatomic molecule ("mesic molecule") as one of the atoms. Let us assume that at a certain instant of time $t = 0$ there was produced in the medium such a molecule, consisting of muonium and some other atom (we shall henceforth confine ourselves to molecules in which the spin of the nucleus of the second atom is equal to $1/2$). If such a molecule is diamagnetic (the total magnetic moment of all the electrons is equal to zero), then in the absence of external factors the magnetic moment of the muon interacts only with the magnetic moment of the second nucleus. The energy of such an interaction is of the order of $m_1 m_2 r^{-3}$, where m_1 and m_2 are the magnetic moments of the muon and the nucleus and r is the distance between the nuclei. Neglecting the vibrations of the nuclei, this quantity can be regarded as a constant and we can put $m_1 m_2 r^{-3} = g$. Such an interaction leads to spin precession with frequency $\omega_{pr} \approx g$ (we are using throughout a system of units with $\hbar = 1$). The exact value of ω_{pr} can be determined by writing down the interaction Hamiltonian and solving the corresponding Schrödinger equation.

1. Let a polarized μ^+ meson with a polarization vector \mathbf{P} directed along the positive direction of a certain axis Z (the quantization axis), after falling in the medium, produce via the muonium stage a mesic molecule with the properties listed above. We assume that the entire system is placed in an external constant and homogeneous magnetic field \mathbf{H} directed perpendicular to the Z axis (we choose this direction to be the X axis, i.e., $\mathbf{H} = (H, 0, 0)$). Choosing the third axis Y orthogonal to the first two, we obtain a natural system of coordinates, in which we can write out an expression for the Hamiltonian of the system.

The Hamiltonian of the system of spins of the nuclei of such a molecule is

$$\mathcal{H} = g[(\sigma_1 \sigma_2) - 3(\sigma_{1n})(\sigma_{2n})] + H(m_1 \sigma_{1x} + m_2 \sigma_{2x}); \quad (1)$$

Here $g = m_1 m_2 r^{-3}$ is the spin-interaction constant, σ_1 and σ_2 are the spin operators with components that are Pauli matrices in the usual representation, $\mathbf{n} = (\alpha,$

$\beta, \gamma)$ is a unit vector along the molecule axis, where $\alpha, \beta,$ and γ are the corresponding direction cosines relative to the coordinate axis, and the indices 1 and 2 pertain respectively to the muon and to the second nucleus.

The behavior of such a spin system at any instant of time is determined completely by the spin wave function $S(t)$, expressed, as usual (see [3]) in terms of the basis vectors:

$$S(t) = A(t)|1/2, 1/2\rangle + B(t)|1/2, -1/2\rangle + C(t)|-1/2, 1/2\rangle + D(t)|-1/2, -1/2\rangle, \quad (2)$$

and the normalization condition is satisfied for each t , i.e.,

$$|A(t)|^2 + |B(t)|^2 + |C(t)|^2 + |D(t)|^2 = 1. \quad (3)$$

The function $S(t)$ satisfies the Schrödinger equation

$$i\partial S / \partial t = \mathcal{H}S \quad (4)$$

with a Hamiltonian in the form (1).

If the polarization vector at any instant t is measured along the z axis, then

$$P_z(t) = \langle \sigma_{1z} \rangle = \text{Sp}\{S^\dagger(t)\sigma_{1z}S(t)\} = |A|^2 + |B|^2 - |C|^2 + |D|^2. \quad (5)$$

We determine the functions $A, B, C,$ and D by solving Eq. (4) with specified initial conditions, and we obtain the final solution of the problem from (5). The solution of Eq. (4) turns out to be essentially different for "fastened" and "rotating" molecules. Let us consider each of these cases separately.

2. "Fastened" molecule. The vector \mathbf{n} does not vary in time, and Eq. (4) yields for $A, B, C,$ and D a system of linear differential equations of first order with constant coefficients, obtained by equating the coefficients of identical basis vectors. We write out this system in a representation in which the unknowns are the linear combinations $A + D, A - D, B + C,$ and $B - C$. We have

$$\begin{aligned} i(A + D) &= 2g(3\beta^2 - 1)(A + D) - 6ig\alpha\beta(A - D) \\ &\quad + 6g(i\beta\gamma + HM/6g)(B + C), \\ i(A - D) &= 6ig\alpha\beta(A + D) + 2g(3\alpha^2 - 1)(A - D) \\ &\quad - 6g\alpha\gamma(B + C) + Hm(B - C), \end{aligned} \quad (6)$$

$$\begin{aligned} i(B + C) &= 6g(-i\beta\gamma + HM/6g)(A + D) - 6g\alpha\gamma(A - D) \\ &\quad + 2g(3\gamma^2 - 1)(B + C), \\ i(B - C) &= Hm(A - D); \end{aligned}$$

We put here in addition $M = m_1 + m_2$ and $m = m_1 - m_2$.

We seek the solution of such a system, as usual, in the form

$$A = \sum C_i^A \exp \lambda_i t, \quad B = \sum C_i^B \exp \lambda_i t \text{ etc.}$$

where the eigenvalues λ_i are the roots of the corresponding characteristic equation. In this problem this equation is

$$(\lambda)^4 - 2[6g^2 + (m_1^2 + m_2^2)H^2](\lambda)^2 + 4g[2m_1 m_2 H^2(3\alpha^2 - 1) - 4g^2]i\lambda + (m_1 - m_2)^2 H^2[(m_1 + m_2)^2 H^2 + 4g^2(3\alpha^2 - 1)] = 0. \quad (7)$$

The solution of such an algebraic equation of fourth order is quite cumbersome, and this makes the solution of the entire problem extremely laborious. It becomes much simpler, however, for some values of α and H .

a) For certain orientations of the molecule, Eq. (7) has a sufficiently compact solution. In particular, this takes place when the molecule axis is oriented along the coordinate axis X, Y, or Z.

For the cases of Z orientation ($\gamma = 1, \alpha = \beta = 0$), Eq. (7) has the following roots:

$$i\lambda_{1,2} = -g \pm \sqrt{g^2 + H^2 m^2}, \quad i\lambda_{3,4} = g \pm \sqrt{(3g)^2 + H^2 m^2}. \quad (8)$$

For the X-orientation case ($\alpha = 1, \beta = \gamma = 0$), the roots of (7) are

$$i\lambda_{1,2} = -2g \pm \sqrt{(2g)^2 + H^2 m^2}, \quad i\lambda_{3,4} = 2g \pm HM. \quad (9)$$

For the Y-orientation case ($\beta = 1, \alpha = \gamma = 0$) the roots coincide with (8).

We present by way of an illustrative example, without writing out the general solution of this system (6), an expression for the polarization vector in the case of Z orientation. If at the initial instant $t = 0$ the spins were parallel (ortho state), i.e., $A = 1, B = C = D = 0$, then

$$\langle \sigma_{1z} \rangle_{\text{ortho}} = \frac{1}{4I_1 I_2} \{ [H^2 M m + (3g + R_2)(-g + R_1)] \cos(2g + R_2 - R_1)t - [H^2 M m + (3g + R_2)(g + R_1)] \cos(2g + R_2 + R_1)t - [H^2 M m + (3g - R_2)(-g + R_1)] \cos(2g - R_2 - R_1)t + [H^2 M m + (3g - R_2)(g + R_1)] \cos(2g - R_2 + R_1)t \}; \quad (10)$$

$$R_1 = \sqrt{g^2 + H^2 m^2}, \quad R_2 = \sqrt{(3g)^2 + H^2 m^2}.$$

If at the initial instant $t = 0$ the spins were antiparallel (para state), i.e., $B = 1, A = C = D = 0$, then

$$\langle \sigma_{1z} \rangle_{\text{para}} = \frac{1}{4R_1 R_2} \{ [H^2 M m + (-3g + R_2)(g + R_1)] \cos(2g + R_2 - R_1)t - [H^2 M m + (-3g + R_2)(g - R_1)] \cos(2g + R_2 + R_1)t + [-H^2 M m + (3g + R_2)(g + R_1)] \cos(2g - R_2 - R_1)t + [H^2 M m - (3g + R_2)(g - R_1)] \cos(2g - R_2 + R_1)t \}. \quad (11)$$

The cases of X and Y orientations give for $\langle \sigma_{1z} \rangle$ expressions having the same structure as (10) and (11).

b) For the case of arbitrary orientation of the molecule, a compact result can be obtained for $\langle \sigma_{1z} \rangle$ only for weak fields, when $H(m_1 + m_2) \ll g$. Solving Eq. (7) in the zeroth approximation in H (putting $H = 0$), we find the zeroth approximation of the solution of Eq. (7):

$$i\lambda_1^0 = 4g, \quad i\lambda_2^0 = i\lambda_3^0 = -2g, \quad i\lambda_4^0 = 0. \quad (12)$$

Assuming further that $\lambda_i = \lambda_i^0 + \Delta\lambda_i$, we solve the equation with the small parameter HM/g relative to $\Delta\lambda_i$. The first nonvanishing corrections in powers of H turn out to be of second order for $\Delta\lambda_1$ and $\Delta\lambda_4$ and of the

first order for $\Delta\lambda_2$ and $\Delta\lambda_3$. If we confine ourselves to first-power corrections in H , the eigenvalues turn out to be:

$$i\lambda_1 = 4g, \quad i\lambda_4 = 0, \\ i\lambda_2 = -2g + H^{2/3}(m_1^2 + m_2^2) - \alpha^2(m_1 - m_2)^2]^{1/2} = -2g + H\delta, \\ i\lambda_3 = -2g - H^{2/3}(m_1^2 + m_2^2) - \alpha^2(m_1 - m_2)^2]^{1/2} = -2g - H\delta. \quad (13)$$

The solution of the system (6) in the same approximation, with allowance for the relations between the coefficients C_i , is

$$A + D = C_1 e^{i(-4g)t} + C_2 e^{i(2g-H\delta)t} + C_3 e^{i(2g+H\delta)t}, \\ A - D = -\frac{\alpha i}{\beta} C_1 e^{i(-4g)t} + \frac{\beta^2 + \gamma^2}{M\gamma + i\beta\delta} \frac{\delta}{\alpha} C_2 e^{i(2g-H\delta)t} - \frac{\beta^2 + \gamma^2}{M\gamma - i\beta\delta} \frac{\delta}{\alpha} C_3 e^{i(2g+H\delta)t}, \\ B + C = -\frac{\gamma i}{\beta} C_1 e^{i(-4g)t} + \frac{\gamma\delta + i\beta M}{M\gamma + i\beta\delta} C_2 e^{i(2g-H\delta)t} + \frac{-\gamma\delta + i\beta M}{M\gamma - i\beta\delta} C_3 e^{i(2g+H\delta)t}, \\ B - C = C_4. \quad (14)$$

The final choice of C_i and of the polarization vector $\langle \sigma_{1z} \rangle$ is determined by the initial conditions. When the spins are parallel at the instant $t = 0$, i.e., $A = 1, B = C = D = 0$ (ortho state), we have

$$\langle \sigma_{1z} \rangle_{\text{ortho}} = \frac{2\gamma^2 - \alpha^2\beta^2}{2} + \frac{(\alpha^2\beta^2 + \gamma^2)(\beta^2 + \gamma^2 - 1)}{2(\beta^2 + \gamma^2)} \cos 2H\delta t \\ + \frac{1}{M\alpha\delta(\beta^2 + \gamma^2)} \{ [(\alpha\beta^2(\beta^2 + \gamma^2) + \alpha^2\beta^2(\alpha + \gamma))(\beta^2 + \gamma^2) + \alpha^2\gamma(\alpha^2\beta^2 + \gamma^2)] M\delta + \alpha^2\beta\gamma(\delta^2 - \alpha^2 M^2) + \gamma(\beta^2 + \gamma^2)(\beta^2\delta^2 + \alpha^2\gamma M^2) \} \cos(6g - H\delta)t \\ + \{ [\alpha\beta^2(\beta^2 + \gamma^2) + \alpha^2\beta^2(\alpha + \gamma)](\beta^2 + \gamma^2) + \alpha^2\gamma(\alpha^2\beta^2 + \gamma^2) \} M\delta - \alpha^2\beta^2\gamma(\delta^2 + \alpha^2 M^2) - \gamma(\beta^2 + \gamma^2)(\beta^2\delta^2 + \alpha^2\gamma M^2) \} \cos(6g + H\delta)t \\ - \{ [-\alpha^2\beta(2\beta^2 + \gamma^2 - \alpha\gamma)(\beta^2 + \gamma^2) + \alpha\beta\gamma(\alpha^2\beta^2 + \gamma^2)] M\delta + \alpha^2\beta^2\gamma^2(\delta^2 - \alpha^2 M^2) - \beta\gamma(\beta^2 + \gamma^2)(\alpha\delta^2 + \gamma\alpha^2 M^2) \} \sin(6g - H\delta)t \\ - \{ [-\alpha^2\beta(2\beta^2 + \gamma^2 - \alpha\gamma)(\beta^2 + \gamma^2) + \alpha\beta\gamma(\alpha^2\beta^2 + \gamma^2)] M\delta - \alpha^2\beta^2\gamma^2(\delta^2 - \alpha^2 M^2) + \beta\gamma(\beta^2 + \gamma^2)(\alpha\delta^2 + \gamma\alpha^2 M^2) \} \sin(6g + H\delta)t \}. \quad (15)$$

For spins antiparallel at the instant $t = 0$, i.e., $B = 1, A = C = D = 0$ (para state), we obtain

$$\langle \sigma_{1z} \rangle_{\text{para}} = -\frac{\alpha^2\gamma(\alpha^2\gamma^2 M^2 + \beta^2\delta^2) + \beta^3\gamma(\beta^2 + \gamma^2)\delta^2}{2M\alpha\delta(\beta^2 + \gamma^2)} \cos(6g - H\delta)t \\ + \frac{\alpha^2\gamma(\alpha^2\gamma^2 M^2 + \beta^2\delta^2) + \beta^3\gamma(\beta^2 + \gamma^2)\delta^2}{2M\alpha\delta(\beta^2 + \gamma^2)} \cos(6g + H\delta)t + \gamma^2 \cos 4gt \\ - \frac{\beta\gamma(\alpha^2\gamma^2 M^2 + \beta^2\delta^2) - \alpha\beta\gamma^2(\beta^2 + \gamma^2)M\delta}{2M\delta} \sin 2H\delta t \\ + \frac{\alpha^2\gamma^2 + \beta^2}{2(\beta^2 + \gamma^2)} [\cos(2g - H\delta)t + \cos(2g + H\delta)t] \\ + \frac{\beta\gamma^2(\alpha^2 M^2 - \delta^2)}{2M\delta(\beta^2 + \gamma^2)} [\sin(2g + H\delta)t - \sin(2g - H\delta)t] \\ + \frac{\beta\gamma^2(\alpha^2 M^2 - \delta^2)}{2M\delta(\beta^2 + \gamma^2)} [\sin(6g + H\delta)t - \sin(6g - H\delta)t]. \quad (16)$$

If there is no external field ($H = 0$), expressions (15) and (16) simplify greatly. In spherical coordinates with polar axes along X and with the azimuth reckoned from the Z axis, the corresponding expressions are

$$\langle \sigma_{1z} \rangle_{\text{ortho}} = \frac{tg^2 \vartheta \cos^2 \varphi + (tg^2 \vartheta \sin^2 \varphi + 1) \cos 6gt}{1 + tg^2 \vartheta}, \quad (17)$$

$$\langle \sigma_{1z} \rangle_{\text{para}} = \frac{(tg^2 \vartheta \sin^2 \varphi + 1) \cos 2gt + tg^2 \vartheta \cos^2 \varphi \cos 4gt}{1 + tg^2 \vartheta}. \quad (18)$$

It is seen from (17) that when the molecule is oriented along the Z axis, i.e., at $\varphi = 0$ and $\vartheta = \pi/2$, the ex-

pression for $\langle \sigma_{1z} \rangle_{\text{ortho}}$ is identically equal to unity and consequently there is no precession.

Inasmuch as the molecules can be formed with different orientations, the observable quantity will be the polarization vector averaged over ϑ and φ with appropriate statistical weight. If the different orientations are equally probable, such an averaging of expressions (17) and (18) can be carried out elementary in yields

$$\langle \overline{\sigma_{1z}} \rangle_{\text{ortho}} = \frac{1 + 2 \cos 6gt}{3}, \quad (19)$$

$$\langle \overline{\sigma_{1z}} \rangle_{\text{para}} = \frac{2 \cos 2gt + \cos 4gt}{3}. \quad (20)$$

3. "Rotating" molecule. The vector \mathbf{n} depends on the time and is a periodic function with repetition frequency ω_{rot} . In the ground state $\omega_{\text{rot}} = I^{-1}$, where the moment of inertia is $I = M_1 M_2 r^2 / (M_1 + M_2) \approx M_1 r^2$ (M_1 and M_2 are respectively the masses of the muon and of the proton, with $M_1 \ll M_2$).

It is easy to verify that for any rotational state of such a molecule we always have $\omega_{\text{rot}} \gg \omega_{\text{pr}}$, where, as before, ω_{pr} is the precession frequency of the interacting spins in the absence of an external field. For example, for the molecule MuH (the analog of H_2) we have $I \approx 10^{-41}$, $\omega_{\text{rot}} \approx 10^{14}$, and $\omega_{\text{pr}} \approx g/\hbar = 0.3 \times 10^6$ (in cgs units esu). Therefore it is perfectly permissible to first average expression (1) over the "rotation" of the vector \mathbf{n} , and then solve the Schrödinger equation with such an averaged time-independent Hamiltonian.

We note that for a system of spins each of which is equal to $1/2$, expression (1) can be replaced by another equivalent expression

$$\overline{\mathcal{H}} = g(\sigma_{1z}) (1 - 3 \cos^2 \vartheta) + \text{terms with } cH; \quad (21)$$

here ϑ is the angle between the vector \mathbf{n} and the quantization axis Z . Averaging of this expression over the rotation means the determination of the average of the operator $\overline{\mathcal{H}}$ over the wave functions of the rotator in the state $|J, M\rangle$, where J is the eigenvalue of the angular momentum of the rotator and M is its projection on the quantization axis. Denoting the wave function of the rotator by $\Theta_{JM}(\cos \vartheta)$, we write the expression for the Hamiltonian averaged over the rotator:

$$\overline{\mathcal{H}} \approx g(\sigma_{1z}) \int_{-1}^{+1} (1 - 3 \cos^2 \vartheta) \Theta_{JM}^2(\cos \vartheta) d \cos \vartheta = gC(J, M) (\sigma_{1z}); \quad (22)$$

we have omitted here terms with H , and the integration limits correspond to the interval over which the eigenfunctions of the rotator are orthogonal.

The rotator functions $\Theta_{JM}(\cos \vartheta)$ coincide with the orthonormalized associated Legendre polynomials $P_J^M(\cos \vartheta)$, and therefore the numerical value of the integral $C(J, M)$ is proportional to the Clebsch-Gordan coefficient expressed in terms of the Wigner $3j$ -symbol:

$$C(J, M) \approx \begin{pmatrix} J & J & 2 \\ M & -M & 0 \end{pmatrix} = \frac{2 [3M^2 - J(J+1)]}{[(2J-1)2J(2J+1)(2J+2)(2J+3)]^{1/2}} \quad (23)$$

(since $1 - 3 \cos^2 \vartheta$ coincides, apart from a numerical factor, with $P_2^0(\cos \vartheta)$, see [41]).

Putting $g^* = gC(J, M)$, we write the total expression for the Hamiltonian averaged over the rotator:

$$\overline{\mathcal{H}} = g^*(\sigma_{1z}) + H(m_1 \sigma_{1x} + m_2 \sigma_{2x}). \quad (24)$$

The terms with H do not change form, since they do not depend on the angle ϑ .

The obtained Hamiltonian and the entire subsequent treatment of the problem are very similar to those in the problem with hyperfine splitting in hydrogen or muonium in the 1s state.

We seek again the wave function $S(t)$ from the Schrödinger equation with Hamiltonian in the form (24):

$$i\partial S / \partial t = \overline{\mathcal{H}} S. \quad (25)$$

For the function $S(t)$, relations (2)–(5) are valid as before. The corresponding system of equations is

$$\begin{aligned} i(A + D) &= g^*(A + D) + H(m_1 + m_2)(B + C), \\ i(B + C) &= g^*(B + C) + H(m_1 + m_2)(A + D), \end{aligned} \quad (26)$$

$$\begin{aligned} i(A - D) &= g^*(A - D) + H(m_2 - m_1)(B - C), \\ i(B - C) &= -3g^*(B - C) + H(m_2 - m_1)(A - D). \end{aligned}$$

The general solution of the system (26) is sought in the form

$$A(t) = \sum C_i^A \exp \lambda_i t, \quad B(t) = \sum C_i^B \exp \lambda_i t \text{ etc.}$$

The roots of the corresponding secular equation are

$$i\lambda_{1,2} = g^* \pm H(m_1 + m_2), \quad i\lambda_{3,4} = -g^* \pm \sqrt{(2g^*)^2 + H^2(m_1 - m_2)^2} \quad (27)$$

The solution of the system (26) for a spin orientation parallel at $t = 0$ (ortho state, $A = 1$, $B = C = D = 0$) is

$$A + D = \cos HMt, \quad A - D = \cos Rt - \frac{2ig^*}{R} \sin Rt, \quad (28)$$

$$B + C = -i \sin HMt, \quad B - C = \frac{iHm}{R} \sin Rt.$$

Here, as above, we put $M = m_1 + m_2$, $m = m_1 - m_2$, and $R = \sqrt{(2g^*)^2 + H^2 m^2}$. The corresponding expression for the polarization vector is

$$\begin{aligned} \langle \sigma_{1z} \rangle_{\text{ortho}} &= \left(\cos 2g^* t \cos Rt + \frac{2g^*}{R} \sin 2g^* t \sin Rt \right) \cos HMt \\ &\quad + \frac{Hm}{R} \cos 2g^* t \sin HMt \sin Rt. \end{aligned} \quad (29)$$

The solution of the system (26) for a spin orientation antiparallel at $t = 0$ (para state, $B = 1$, $A = C = D = 0$) is

$$\begin{aligned} A &= -\frac{i}{2} \left[e^{ig^* t} \sin HMt + \frac{Hm}{R} e^{-ig^* t} \sin Rt \right], \\ D &= -\frac{i}{2} \left[e^{ig^* t} \sin HMt - \frac{Hm}{R} e^{-ig^* t} \sin Rt \right], \\ B &= \frac{1}{2} \left[e^{ig^* t} \cos HMt + \frac{e^{-ig^* t}}{R} (R \cos Rt - 2ig^* \sin Rt) \right], \\ C &= \frac{1}{2} \left[e^{ig^* t} \cos HMt - \frac{e^{-ig^* t}}{R} (R \cos Rt - 2ig^* \sin Rt) \right], \end{aligned} \quad (30)$$

and the corresponding expression for the polarization vector is

$$\begin{aligned} \langle \sigma_{1z} \rangle_{\text{para}} &= \cos 2g^* t \left[\cos Rt \cos HMt + \frac{Hm}{R} \sin Rt \sin HMt \right] \\ &\quad - \frac{2g^*}{R} \sin 2g^* t \sin Rt. \end{aligned} \quad (31)$$

In the absence of an external field, $H = 0$, expressions (29) and (31) go over into

$$\langle \sigma_{1z} \rangle_{\text{ortho}} = 1, \quad \langle \sigma_{1z} \rangle_{\text{para}} = \cos 4g^* t. \quad (32)$$

Let us refine the dependence of g^* on J and M .

In the ground state of the rotator $|0, 0\rangle$ expression

(23) vanishes, for in this case the degree of the numerator in (23) is higher than the degree of the denominator, and therefore $g^* = 0$, i.e., there is no spin interaction.

In the first excited state of the rotator $|1, 0\rangle$ and $|1, \pm 1\rangle$, the corresponding numbers can be readily obtained by direct integration in (22), and turned out to be

$$C(1, 0) = -4/5, \quad C(1, \pm 1) = 2/5.$$

Further increase of J and M leads to a monotonic decrease of the absence value of g^* to zero. Consequently, in the absence of an external field, the maximum value of the precession frequency corresponds to the rotator state $|1, 0\rangle$. With increasing J and M , the interaction of the spins and the precession frequency decrease to zero. This corresponds to the quasiclassical case and coincides with the conclusion drawn in [1].

If the molecule was produced in a state with a definite J , then inasmuch as states with different M at given J are equally probable, the experimentally observed quantity will be not $\langle\sigma_{1z}\rangle$, but the sum of different $\langle\sigma_{1z}\rangle$ corresponding to all possible M at a given J . In particular, at $J = 1$ it is possible to have $M = -1, 0$, and $+1$, and we obtain

$$\langle\sigma_{1z}\rangle = 1/3(\cos^{10}/5gt + 2\cos^8/5gt).$$

This formula and similar expressions for other J can be compared with data obtained from an experiment performed in accordance with the procedure of [5].

4. Numerical estimates of the precession frequency. Since muonium differs from the hydrogen atom only in the mass of the nucleus, the energy levels and the radii of the Bohr orbits of either of them differ only by fractions of one per cent. Therefore it is perfectly natural to assume that the dimension r of the MuH molecule is equal to the dimension $r = 0.74 \times 10^{-8}$ cm of the hydrogen molecule. From this we get

$$g = 2.79e^2\hbar^2 / 4M_1M_2c^2r^3 = 1.56 \cdot 10^{-24} \text{ erg}, \quad g/\hbar = 0.234 \text{ MHz},$$

The frequencies of the fastened molecules are

$$2g/\hbar = 0.468 \text{ MHz}, \quad 4g/\hbar = 0.936 \text{ MHz}, \quad 6g/\hbar = 1.404 \text{ MHz}$$

and the frequencies for the rotating molecule (for $J = 1$) are

$$^{10}/5g/\hbar = 0.745 \text{ MHz}, \quad ^8/5g/\hbar = 0.373 \text{ MHz}.$$

It is seen from the foregoing estimates that in each of the foregoing cases the effect is observable experimentally, since the precession period does not exceed the muon lifetime $\tau = 2.2 \mu\text{sec}$.

5. The formation of a fastened mesic molecule of the type considered here is apparently possible in a solid constituting a crystal lattice. Both nuclei of the molecule are retained in a fixed position by the Coulomb field of the crystal lattice. In this case formation of a mesic molecule from muonium and some impurity atom (for example, hydrogen) is possible. The main obstacle to experiment will in this case be apparently the formation of molecules with different distances r between atoms, which should lead to a superposition of spectra with slightly differing precession frequencies and their mutual "extinction."

The formation of a rotating molecule is apparently possible in a gas, liquid, or a molecular crystal. The distance r for all the molecules is in this case the same. However, if different molecules are produced with strongly different J , this also leads to a smearing of the picture, owing to the superposition of a large number of harmonics with different frequencies. In this case it is apparently advantageous to perform the experiment under conditions such that the integral of the possible values of ΔJ be as narrow as possible. It is desirable to obtain all molecules with the same value of J , and the best variance is the one in which $J = 1$. An exception is only the case when each molecule is in the state $J = 0$, in which case there is no spin precession at all caused by their interaction.

In conclusion the author is indebted to G. A. Lobov, Yu. V. Obukhov, and V. G. Firsov for initiating the work and for constant advice during the course of his performance, and also to V. G. Nosov for pointing out the existence of an error in the first draft of this paper.

¹V. G. Nosov and I. V. Yakovleva, Zh. Eksp. Teor. Fiz. **43**, 1750 (1962) [Sov. Phys.-JETP **16**, 1236 (1963)].

²I. G. Ivanter and V. P. Smilga, ibid. **54**, 559 (1968) and **55**, 1521 (1968) [27, 301 (1968) and 28, 796 (1969)].

³R. P. Feynman, R. Leighton, and M. Sands, Feynman Lectures in Physics,

⁴L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963 [Addison-Wesley, 1965].

⁵G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov and V. G. Firsov, Zh. Eksp. Teor. Fiz. **53**, 451 (1967) [Sov. Phys.-JETP **26**, 298 (1968)].