THEORY OF RADIATION EMISSION FROM CRYSTALS TRAVERSED BY A CURRENT

L. É. GUREVICH and I. V. IOFFE

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

Submitted December 11, 1969

Zh. Eksp. Teor. Fiz. 58, 2047—2053 (June, 1970)

Linear and nonlinear theories are developed for microwave emission from crystals traversed by a current in the absence or presence of an external magnetic field parallel or perpendicular to the current. The situation encountered in experiments on microwave emission from indium antimonide is considered. In a current-carrying conducting medium there exists a special branch of electromagnetic transverse oscillations whose excitation leads to emission. In the absence of an external field the frequency is linear relative to the wave vector; in a strong magnetic field an additional term appears which is quadratic with respect to the wave vector. The frequency and critical value of the current density are determined and found to agree with the experimental values. The dependence of the oscillation amplitude on the current density is deduced. The radiation intensity is calculated and it is shown that in a strong external magnetic field it increases in agreement with the experiments.

1. The emission of electromagnetic waves from indium antimonide traversed by a strong current has been observed in a number of experiments.[1-3] The emission occurred in the absence and in the presence of a magnetic field H_0, parallel or perpendicular to the current, the magnetic field lowering inappreciably the current density required for emission and having a weak effect on the frequency. However, in sufficiently strong magnetic fields ($H_0 > c/\mu$—where c is the speed of light and $\mu_\pm$ the electron and hole mobility) the intensity of the emission increased considerably. We shall present below a linear and nonlinear theory of this phenomenon for samples of cylindrical shape with a radius R much smaller than the length L in the cases $H_0 = 0$ and $\mathbf{H}_0 \parallel \mathbf{j}_0$ (j_0 is the density of the stationary current), or for a platelet one of whose dimensions d is much smaller than the other two dimensions L in the case $\mathbf{H}_0 \perp \mathbf{j}_0$ (the magnetic field and current are directed along the large sides of the platelet).

We shall show that in a medium traversed by a current there exists a branch of transverse electromagnetic oscillations and the emission is connected with the excitation of this branch. Qualitatively this can be understood as follows. Let an alternating magnetic azimuthal field appear in the crystal in a plane transverse to the current (the current is along the z axis). The presence of the current will lead for finite transverse dimensions of the sample to a Hall oscillatory electric field in the radial direction. This electric field together with the intrinsic magnetic field H_c of the stationary current will produce an alternating Hall current along the z axis which will give rise to new alternating magnetic azimuthal fields, but in other locations, etc. Only the nondissipative Hall conductivity participates in this process, i.e. we have not taken into account the attenuation of the wave. The wave appearing in this way can be called galvanomagnetic in analogy with the thermomagnetic wave[4] existing in the presence of a temperature gradient.

A galvanomagnetic wave is only possible in a conducting medium in the presence of a direct current. The velocity of such a wave is much smaller than the speed of light and the oscillatory magnetic field in the wave is therefore much larger than the electric oscillatory field, just as in thermomagnetic and helicoidal waves.[5] In a medium in which a strong magnetic field is also present there is in addition to a galvanomagnetic wave also a helicoidal wave. We shall see below that the frequencies of these two waves in experimental situations are close and the resulting oscillation constitutes a coupled wave.

Let us now consider dissipative processes. A finite conductivity σ leads to the usual attenuation $\approx c^2K^2/4\pi\sigma$ (K is the wave vector). However, the presence of an intrinsic magnetic field (or of an external magnetic field, if there is such) leads to the appearance of a focusing current. The sign of the focusing resistance can be negative and therefore for a sufficiently strong direct current the "antidissipation" connected with the focusing current may exceed the attenuation. We note that in an external strong constant magnetic field it is possible to explain qualitatively the excitation of the wave as Cerenkov radiation: the instability condition can be represented in the form of the equality of the carrier drift velocity and wave velocity. Such a graphic explanation is unsuitable in the general case.

Before we proceed to a calculation, we shall indicate the main results of the theory. The instability of galvanomagnetic waves appears at a current density (accurate to within a factor of order unity) $j_0 > j_{cr} \approx c^2/\mu_- L$.

The frequency of the resulting oscillations is then of the order of $\omega \approx 2\pi c^2/\sigma_0 L^2$ (σ_0 is the conductivity in the absence of a magnetic field). These values, as well as the increase of the critical current with decreasing length and the frequency increase agree with the experimental values of the order of $j_{cr} \approx 3 \times 10^3 - 10^4$ A/cm^2; $\omega \approx 2\pi(3 \times 10^9 - 10^{10})$ sec^{-1}.

We note that for such a current the intrinsic magnetic field is weak: $H_c < c/\mu_-$. For a sample located in an ideal waveguide with inner radius R_W the expressions for the critical current and frequency must be multiplied by $\max[1, L/R_W]$. For $R_W \approx R$ the obtained results are correct in order of magnitude.

The component of the oscillatory field in the wave directed along the direct current is smaller by a factor of r/L (r is the instantaneous radius) than the components perpendicular to the current. For a small excess of current over the critical current the amplitude of the oscillatory magnetic field transverse to the current is H_1 in a weak magnetic field and H_2 in a strong magnetic field

$$H_1 = \frac{c}{\mu_-}\sqrt{\frac{j_0 - j_{cr}}{j_{cr}}}, \quad H_2 = H_0\sqrt{\frac{j_0 - j_{cr}}{j_{cr}}}.$$

Thus the presence of a strong magnetic field increases the wave amplitude by a factor of $\mu_- H_0/c$. The wave is emitted in the direction of the current, i.e., from a surface perpendicular to the current; with this, for the same amplitude of the wave inside the crystal in an external magnetic field the portion of the energy emitted outside is larger by a factor of $(\mu_- H_0/c)^2$ than for $H_0 < c/\mu_-$.

For $H_0 < c/\mu_-$ the efficiency is

$$\frac{c}{8\pi}\left(\frac{c}{\sigma_0 L}\right)^2\left(\frac{c}{\mu_-}\right)^2\frac{\sigma_0}{Lj_{cr}^2}\frac{j_0 - j_{cr}}{j_{cr}}$$

and for $H_0 > c/\mu_-$ it is by a factor of $(\mu_- H_0/c)^4$ larger and can under experimental conditions be $\sim 10^{-2}$.

2. For $H_0 = 0$ there is no helicoidal wave and the galvanomagnetic wave appears most clearly. The equation for the oscillatory field H' is of the form ($H = H_c + H'$)

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{c^2}{4\pi}\operatorname{rot}\{\eta \operatorname{rot}\mathbf{H} + \eta_1[\operatorname{rot}\mathbf{H}\cdot\mathbf{H}] + \eta_2\mathbf{H}(\mathbf{H}\operatorname{rot}\mathbf{H})\}. \quad (1)^*$$

Here H_c is the intrinsic magnetic field of the direct current and the coefficients η, η_1 and η_2 are given for the general case in[4]. We shall see below that the intrinsic magnetic field of the current is such that $H_c < c/\mu_-$. In this case (σ_0 is the conductivity for H = 0, σ_1 and σ_2 are the Hall and focusing conductivities)

$$\eta = \sigma_0^{-1}, \quad \eta_1 = -\sigma_1\sigma_0^{-2}, \quad \eta_2 = (\sigma_1^2 - \sigma_0\sigma_2)\sigma_0^{-3}.$$

As has been shown in[6], for the currents required by us the carrier concentration and the mobilities depend only weakly on the current (a fact explained by the energy scattering of the electrons by optical phonons). We shall, therefore, not take into account the dependence of the kinetic coefficients on the direct current. We have also neglected the displacement current, since the frequency ω of the resulting oscillations is much less than that of the conductivity.

We linearize (1) over the oscillatory quantities $H' = H - H_c$, $j' = j - j_0$. In the case of a sample of cylindrical shape (the length L is much larger than the radius R) it is convenient to introduce a cylindrical system of coordinates r, φ, and z, and to set all oscillatory quantities $\sim \exp[ikz + im\varphi - i\omega t]f(r)$.

It can be shown that for the azimuthal number m = 0 and $|m| \gg 1$ there is no excitation. We shall restrict ourselves to the case $|m| = 1$ when the excitation conditions are least rigorous. Neglecting quantities of the order of $R\mu_- H_c/L_1 c$ and $\sigma_1^2/4\sigma\sigma_2 < 1$ (in indium antimonide the latter quantity does not exceed 0.2 and for $\sigma_{0+} \approx \sigma_{0-}$ it does not exceed 0.1; the subscripts $\mp$ denote quantities referring to electrons and holes), we find

$$\left[i(\omega + c\eta_1(kj_0)) + \frac{c^2\eta}{4\pi}\Delta\right]\mathbf{H}' - 2\pi\eta_2 j_0(j_0\mathbf{H}') - \frac{ic\eta_1 j_0}{2}\operatorname{rot}\mathbf{H}' = 0. \quad (2)$$

*[rot H × H] ≡ curl H × H, (H rot H) ≡ H·curl H.

[Here $H' = H'(r, \varphi, z)$ and the operators Δ and curl have the usual meaning.]

Applying to (2) the operation curl_z and substituting in the obtained expression the value of $\operatorname{curl}_z H'$ found from (2), we obtain

$$(\Delta + \varkappa_1^2)(\Delta + \varkappa_2^2)H_z' = 0, \quad (3)$$

$$\varkappa_1^2 = \frac{4\pi i}{c^2\eta}(\omega + c\eta_1(kj_0)) - \frac{4\eta_2}{\eta}\left(\frac{2\pi j_0}{c}\right)^2 \quad (4)$$

$$\varkappa_2^2 = \frac{4\pi i}{c^2\eta}(\omega + c\eta_1(kj_0)).$$

Taking into account the finiteness of H' for r = 0, we find from (3)

$$H_z' = C_1J_1(\sqrt{\varkappa_1^2 - k^2}\, r) + C_2J_2(\sqrt{\varkappa_2^2 - k^2}\, r) \quad (5)$$

(J and $\mathscr{H}$ are Bessel and Hankel functions). Employing (2), (5), and div H' = 0, one can find H_r' and H_φ'.

Outside the sample $\partial^2 H'/\partial t^2 = c^2\Delta H'$. We shall see below that $\omega \ll ck$. Therefore, taking into account that for $r \to \infty$, $H' \to 0$, we find for $r \geq R$

$$H_z'(r) = -iC_3\left(k - \frac{\omega^2}{c^2 k}\right)\mathscr{H}_1\left(i\sqrt{k^2 - \frac{\omega^2}{c^2}}\,r\right),$$

$$H_r' = C_3\frac{\partial}{\partial r}\mathscr{H}_1\left(i\sqrt{k^2 - \frac{\omega^2}{c^2}}\,r\right)$$

$$H_\varphi' = \frac{iC_3}{r}\mathscr{H}_1\left(i\sqrt{k^2 - \frac{\omega^2}{c^2}}\,r\right). \quad (6)$$

We note that the field outside the cylinder decreases like e^{-kr}. The condition of continuity of H' for r = R leads to a system of equations for $C_{1,2,3}$ whose determinant is zero. This is in fact the dispersion equation. In general form its solution is only possible numerically. We take into account the fact that at the boundaries of the crystal z = 0, L one can assume that $H' \to 0$, since in the conductor closing the circuit the field decreases exponentially at the depth of the skin layer $c/\sqrt{\omega\sigma} \ll L$. One can therefore set $k \approx \pi p/L$, p = 1, 2, ... This equation is only approximate since even for finite L the quantities describing the stationary state depend weakly on z (for example, because of injection). For $kR \ll 1$ there is then the solution $\kappa R \ll 1$. In fact, making use of the values of the Bessel and Hankel functions for small values of the arguments and $|m| = 1$, we find

$$\varkappa_1^2 = 2k^2, \quad (7)$$

so that indeed $\kappa R \ll 1$.

Substituting (4) in (7), we obtain the condition that the frequency is real

$$j_0 = j_{cr}(H_0 = 0) = \frac{ck}{2\pi\sqrt{2}}\sqrt{\left|\frac{\eta}{\eta_2}\right|} \approx \frac{c^2 p}{2\sqrt{2}\,\mu_- L} \quad (8)$$

(here use has been made of the fact that $|\eta/\eta_2|$ has a minimum for $\sigma_{0-} \approx \sigma_{0+}$). At the same time

$$\operatorname{Re}\omega = 2\pi\frac{p^2}{2\sqrt{2}}\frac{c^2}{\sigma_0 L^2} \quad (9)$$

For $j_c > j_{cr}$ the magnitude of $\operatorname{Im}\omega > 0$ and the oscillations grow. We note that with decreasing L the values of j_{cr} and ω grow, a fact which coincides with the experimental result. For $L \approx 1$ cm and $\sigma_0 = 10^{11}$ sec^{-1}, $\mu_- \approx 10^8$ absolute units, and $j_{cr} \approx 3 \times 10^3$ A/cm^2, $\omega \approx 2\pi \times 3 \times 10^9$ cps which is also close to the experimental results.

Using $\kappa r \ll 1$ we find that $|H_z'/H_{r,\varphi}'| \approx kr$ and that with an accuracy to $(kr)^2$ the values of $H_{r,\varphi}'$ do not depend on r. We note that for $j_0 = j_{cr}$ we have $\mu_- H_c/c \approx R/L \ll 1$ and the previous simplifications of the coefficients η are legitimate. The neglect of the displacement current is also legitimate, since $\omega/\sigma \approx (c/\sigma_0 L^2) \ll 1$. Finally, $\omega/ck = c/\sigma_0 L \ll 1$ and it is therefore seen from $\partial H'/\partial t = -c \operatorname{curl} E'$ that $|E'| \ll |H'|$.

If the sample is placed in a cylindrically shaped waveguide of radius R_w (the axis of the waveguide is parallel to the current), then one must use the usual conditions on the surface of the waveguide.[7] This leads to excitation conditions on the surface of the waveguide.[7] This leads to excitation conditions $j_{crw} = j_{cr} \max[1, L/R_w]$ and a frequency $\omega_w = \max[1, L/R_w]$. Therefore the results do not change for $L \gg R_w$ and for $R = R_w$, ω and j_{cr} increase. For $R \approx R_w$ the value of $\mu_- H_c/c$ reaches unity, i.e. the intrinsic magnetic field of the current ceases to be weak. In this case our results are correct in order of magnitude.

3. In the presence of an external magnetic field parallel to the current the coefficients η, η_1, and η_2 in our calculation change. However, under the condition $\mu_- H_c/c < 1$ we again obtain an equation of the form (3) but with other values of κ:

$$\varkappa_1^2 = \frac{4\pi i}{c^2\eta}(\omega + c\eta_1(\mathbf{k}\mathbf{j}_0)) - \frac{ik^2 H_0\eta_1}{\eta} - \frac{4\eta_2}{\eta}\left(\frac{2\pi j_0}{c}\right)^2 + 2\left(\frac{2\pi \mathbf{j}_0\mathbf{k}}{cH_0}\right). \quad (10)$$

We have no need for the explicit form of κ_2 ($|\kappa_2| \gg |\kappa_1|$). The dispersion equation has the same form (7), whence it follows that

$$\operatorname{Re}\omega = -c(\mathbf{k}\mathbf{j}_0)\eta_1 + \frac{c^2 k\eta_1(\mathbf{k}\mathbf{H}_0)}{4\pi}. \quad (11)$$

This is the dispersion equation of a helicon when the second term in (11) is larger than the first, and the equation of a galvanomagnetic wave when the first term dominates. It is readily seen that the condition that $\operatorname{Im}\omega$ is positive, i.e. the instability condition, coincides in this case with the condition for the Cerenkov emission of a wave with the frequency of (11). Here $j_c \approx ckH_0/4\pi$ and $\omega \approx 2\pi c H_0/nEL^2$ (e is the charge of the electron). In a weak field such a coincidence of the instability condition and the condition for Cerenkov emission does not take place. The former turns out to be more rigorous than the latter.

The excitation of waves with dispersion equation (11) was considered in[8-10]. First, the one-dimensional problem was considered in these papers when all the quantities only depended on the coordinate in the direction of the current. No boundary conditions were set in these papers in directions transverse to the current and therefore values of the wave vector were not obtained in these directions, i.e. the excitation conditions of the oscillations were in fact not determined. Secondly, no allowance was made in[8-10] for the intrinsic magnetic field of the current whereas for the transverse dimensions of the sample satisfying the requirements of these papers the intrinsic magnetic field becomes strong. Finally, the problem of the amplitude of the oscillations and their emission from the sample was not investigated in[8-10].

In the case of a magnetic field perpendicular to the current we shall consider a slab in which the length L of two sides (the Y and Z axes) is much larger than the length d of the third side (the X axis). Let the current and the magnetic field be directed along the Z and Y axes. In this case it is convenient to seek a solution of the form $\exp[ik_y y + ik_z z]\tilde{f}(x)$. For the same conditions as before we obtain for f(x) a fourth-order equation with constant coefficients. Its solution is of the form

$$f(x) = \sum_{i=1}^{4} C_i \exp ik_i x, \qquad k_i = \pm\left\{-k_y^2 - k_z^2 \pm\left[4\pi\omega\left(c^2(\mathbf{k}\mathbf{H}_0)\eta_1 - c\left(\frac{\mathbf{k}\mathbf{j}_0}{k}\right)\eta_1\right)^{-1}\right]^2\right\}^{1/2}.$$

The boundary conditions are satisfied with an accuracy within $(d/L)^2$ if

$$\omega \lesssim \left\{(k_y^2 + k_z^2)\left(\frac{c\eta_1}{4\pi}\right)^2 \left[c(\mathbf{k}\mathbf{H}_0) - j_0\right]^2\right\}^{1/2},$$

i.e., if $|k_x| \approx |k_{y,z}|$. Making use of this condition, we find that the expressions for the critical current and the frequency are the same as in the case of a magnetic field parallel to the current.

4. Let us now proceed to consider the nonlinear theory. The limitation on the growth of the amplitude is connected with the fact that the coefficient η which determines the attenuation of the wave increases with increasing field. Let us first consider a sample, infinite in the direction of the current, in which a traveling wave is propagated. Taking into account that $H_z' \ll H_{r,\varphi}'$ and that accurate to within $(kr)^2$ the magnitudes of $H_{r,\varphi}'$ do not depend on the radius, we find that $(\mu_- H'/c)^2 = \mu_-^2 H_\perp'^2/c^2$; $H_\perp'^2 = H_r'^2 + H_\varphi'^2$ depends neither on the coordinates nor on the time. Let us now set $H = H_c + H' + H'' + \ldots$, where H' is the solution of the linearized equation (3) with $\kappa_{1,2}$ determined by (4); H'' and H''' are the solutions of the equations, the left-hand part of which coincides with (3) when H' is replaced by H'' or H''' and whose right-hand side is obtained from (3) and (4) by expansion in H' up to terms H'^2 and H'^3. Solution shows that

$$\frac{\mu_-}{c} H''\left(\frac{c}{\mu_- H'}\right)^2 \ll 1, \quad (12)$$

and analogously for H'''. We shall therefore neglect H'' and H''' compared with H'^2 and H'^3. We obtain then Eq. (3) with η replaced by $\eta[1 + (\mu_- H_\perp'/c)^2]$. In analogy with Subsection 2 we find that $\operatorname{Im}\omega = 0$ for $j_0 = j_{cr}[1 + (\mu_- H_\perp'/c^2)^2]$, or

$$H_\perp'^2 = \frac{c^2}{\mu_-^2}\frac{j_0 - j_{cr}}{j_{cr}}.$$

The amplitude of the wave increases as the square root of the excess over the critical current. The calculation is valid so long as $(j_0 - j_{cr})/j_{cr} < 1$. The linear theory is not valid for large current densities.

In the case of a sample of finite length it must be taken into account that a new wave appears when the wave is reflected from the boundary. When a wave with a field $\sim \cos(kz + \varphi - \omega t)$ is reflected there appears a wave with a field $\sim \cos(-kz + \varphi - \omega t)$. In this case too, as can be shown, (12) is fulfilled and the equation of the type of Eq. (3) will include terms proportional to $\cos(\pm kz + \varphi - \omega t)$, $\cos(\pm kz + \varphi - \omega t)\cos 2(\varphi - \omega t)\ldots$

Since the equation must be satisfied for all z and φ, the equation splits into several independent equations. The terms proportional to $\cos(\pm kz + \varphi - \omega t)$ satisfy the same equations as those in the case of a traveling wave. Waves with fields $\sim \cos 2(\varphi - \omega t)\cos(\pm kz + \varphi - \omega t)$, and $\cos 2kz \cos(\pm kz + \varphi - \omega t)$ ... have amplitudes smaller by a factor of $[(j_0 - j_{cr})/j_{cr}]^{1/2}$ than waves with fields $\sim \cos(\pm kz + \varphi - \omega t)$.

If, on the other hand, the external magnetic field is strong ($H_0 > c/\mu_-$), then the value of $(\mu_- H'_\perp/c)^2$ in the coefficients η must be compared with $(\mu_- H_0/c)^2$ and not with unity and the expansion is in H'/H_0. A calculation shows that in this case

$$H_\perp'^2 = H_0^2 (j_0 - j_{cr})/j_{cr},$$

i.e., for the same supercriticality the amplitude of the wave is larger by a factor of $\mu_- H_0/c$.

5. Let us consider the emission of oscillations from the crystal. Two emission mechanisms are possible: a magnetic-dipole mechanism connected with oscillations of the resultant magnetic moment, and the emission of waves from the crystal in the direction of their propagation, i.e. in the direction of the current. It can be shown that in our case the first mechanism leads to a radiation intensity smaller by a factor of $(\omega R/c)^4$ than the first. Making use of the boundary conditions for z = 0, L, we find that the wave vector outside the crystal along the z axis is much smaller than k. A larger portion of the wave energy will be reflected by the boundary of the crystal and only a fraction $(ck/\sigma_0)^2$ for $H_0 < c/\mu_-$ or a fraction $(\eta_1 j_0)^2$ for $H_0 > c/\mu_-$ of the Poynting vector of the wave is emitted outside, i.e., in the case of a strong magnetic field the fraction of the emitted energy increases by a factor of $(\mu_- H_0/c)^2$ for the same amplitude inside the crystal. (The reflected portion of the wave propagates in the crystal in the opposite direction, becoming reinforced, since the expression for Im ω is even in k. Reinforcement of the wave by multiple reflection from the boundaries for z = 0, L is therefore possible. We shall not investigate this problem.)

In the absence of an external magnetic field the value of the Poynting vector outside is

$$S = \frac{c}{8\pi}\left(\frac{ck}{2\pi\sigma_0}\right)^2 H_\perp'^2,$$

and in strong magnetic field

$$S = \frac{c}{8\pi}(\eta_1 j_0)^2 H_\perp'^2.$$

The ratio of the emitted energy to

$$\int \frac{j_0^2}{\sigma} r\,dr\,dz\,d\varphi,$$

i.e., the efficiency, is in the absence of a magnetic field

$$\frac{c}{8\pi}\left(\frac{ck}{2\pi\sigma_0}\right)^2\left(\frac{c}{\mu_-}\right)^2\frac{\sigma_0}{Lj_0^2}\frac{j_0 - j_{cr}}{j_{cr}},$$

and in a strong external magnetic field

$$\frac{c}{8\pi}\frac{\eta_1^2\sigma_0}{L}H_0^2\frac{j_0 - j_{cr}}{j_{cr}}.$$

As is seen, the latter expression is larger by a factor of $(\mu_- H_0/c)^4$ than the preceding expression. In a strong magnetic field the efficiency may reach a magnitude $\sim 10^{-2}$.

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Translated by Z. Barnea
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