SOUND ABSORPTION IN A SUPERFLUID LIQUID UNDER CONDITIONS OF DAMPING OF THE NORMAL COMPONENT

T. A. KARCHEVA and D. G. SANIKIDZE

Institute of Cybernetics, Georgian Academy of Sciences

Submitted December 16, 1969

Zh. Eksp. Teor. Fiz. 58, 1843–1847 (May, 1970)

The absorption coefficients of first and second sound in superfluid liquid helium are calculated. These are connected with the surface and volume dissipative processes under the specific conditions of sound propagation in narrow channels, involving the damping of the normal component of the liquid. It is shown that the contribution of various dissipative processes in the sound absorption depends on the degree of damping of the normal component.

In a number of theoretical and experimental researches[1-4], the propagation of waves have been studied in superfluid helium under the conditions of partial or complete damping of the normal component. Such a situation arises in the propagation of sound in superfluid helium filling narrow channels. The character of the wave propagation is determined by the ratio of the transverse dimensions of the channel d to the penetration depth of the viscous wave $\lambda_v = (2\eta/\omega\rho_n)^{1/2}$, $\delta = d/\lambda_v$. If $\delta \gg 1$, then the normal component oscillates freely, and first and second sound propagate in the superfluid. For $\delta \ll 1$, the normal component is entirely damped and the first sound is modified into fourth sound while the second sound becomes a rapidly damped heat wave.[1,2] For the case $\delta \sim 1$, a strong dispersion of the sound velocities takes place, depending on the parameter δ.

The absorption of sound in superfluid helium is brought about by various dissipative processes. They can be tentatively divided into two types: volume—associated with the viscous coefficients (η, ζ_1, ζ_2, ζ_3) and the thermal conductivity, and surface—associated with the friction of the normal component of the liquid with the walls of the channels and with the heat flow through the wall.

From the fact that η and ζ_2 make a contribution to the absorption of first sound ($\delta \gg 1$) (generally, this statement is valid when small effects proportional to the coefficient of thermal expansion of helium, are not taken into account), and ζ_3 and κ appear in place of them in the absorption of fourth sound ($\delta \ll 1$),[1] it follows that the contribution of the various mechanisms to the absorption of the sound should change, depending on the degree of damping of the normal component. The present paper is devoted to the clarification of this problem.

We shall solve the complete set of equations of hydrodynamics (retaining the dissipative terms) together with the equation of thermal conduction of the wall with account of the following boundary conditions at the wall: vanishing of the normal component of the total current and the tangential component of the velocity of the normal part of the liquid, continuity of the heat flux:

$$(\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n, \mathbf{n}) = 0, \quad [\mathbf{v}_n, \mathbf{n}] = 0, \qquad (1)^*$$

$$(T\sigma\rho\mathbf{v}_n + \varkappa\nabla T, \mathbf{n}) = -\varkappa_w(\nabla T_w, \mathbf{n}), \quad -\varkappa_w(\nabla T_w, \mathbf{n}) = \alpha(T - T_w),$$

where ρ_s, ρ_n, v_s, v_n the density and velocity of the superfluid and normal components of He^4; T and κ are the temperature and coefficient of thermal conductivity of the liquid; T_w, κ_w, C_w are the temperature, thermal conductivity coefficient and heat capacity, respectively, of the wall; α is the heat resistances of the boundary between the wall and the liquid helium. By solving the set of hydrodynamic equations in the usual way,[1,2] we find the dispersion equation from the boundary conditions. This equation determines the waves traveling along the channels.

For clarification of the contribution of viscous processes to the sound absorption, we neglect the thermal conductivity of the walls κ_w and the heat resistance of the boundary; then the dispersion equation takes the form

$$k_\|^4 - k_\|^2\left\{k_1^2 + k_2^2 + ir\frac{\rho}{\rho_n}\frac{P_1k_2^2 - P_2k_1^2}{P_1 - P_2}\right\} + \left(1 + ir\frac{\rho}{\rho_n}\right)k_1^2k_2^2 -$$

$$- i\frac{\varkappa}{C_{He}}\frac{\omega}{u_2^2}(k_2^2 - k_\|^2)\left[k_1^2\left(1 + ir\frac{\rho}{\rho_n}\right) - k_\|^2\left(1 + ir\frac{\rho_s}{\rho_n}\right)\right] = 0.$$

where

$$\times k_1^2 = \frac{\omega^2}{u_1^2}\left\{1 + i\frac{\omega}{u_1^2\rho}\left(\frac{4}{3}\eta + \zeta_2\right)\right\},$$

$$k_2^2 = \frac{\omega^2}{u_2^2}\left\{1 + i\frac{\omega}{u_2^2}\left[\frac{\rho_s}{\rho_n\rho}\left(\frac{4}{3}\eta + \zeta_2 - 2\zeta_1\rho + \zeta_3\rho^2\right) + \frac{\varkappa}{C_{He}}\right]\right\}.$$

$$P_1 = 1 + \frac{i\omega}{\rho_n(u_1^2 - u_2^2)}\left(\frac{4}{3}\eta + \zeta_2 - \zeta_1\rho\right)$$

$$\times P_2 = -\frac{\rho_n}{\rho_s} + \frac{i\omega}{\rho_s(u_1^2 - u_2^2)}\left(\frac{4}{3}\eta + \zeta_2 - \zeta_1\rho\right).$$

Equation (2) is similar to the equation obtained in[2], only the dissipative terms are taken into account here. It should be noted that the equation remains valid for any shape of the channels along which the sound is propagated. The form of the channel determines the dependence of the function r on δ, which determines the degree of damping of the normal component of the liquid. A similar dispersion equation can be obtained

*$[\mathbf{v}_n, \mathbf{n}] \equiv \mathbf{v}_n \times \mathbf{n}$.

if we solve the set of hydrodynamic equations without boundary conditions, but by introducing the additional damping term $-\rho\omega r v_n$ in the equation of motion.[1,4]

For a cylindrical channel,

$$r=\frac{\rho_n}{\rho}(m_1+im_2)=-i\frac{\rho_n}{\rho}\frac{2I_1(k_3d)}{k_3dI_0(k_3d)-2I_1(k_3d)},$$

where $k_3^2 = i\omega\rho_n/\eta$. The absorption of first sound, due to friction of the liquid against the wall, is equal to[3]

$$\gamma_\delta^{(1)}=\frac{1}{2}\frac{\omega}{u_{1\delta}}\frac{a\rho_n/\rho}{1-b\rho_n/\rho};\tag{3}$$

here $u_{1\delta}^2 = u_1^2(1 - b\rho_n/\rho)$, a and b are functions of δ and as $\delta \to \infty$, both approach zero, and as $\delta \to 0$, $a \to 0$, $b \to 1$. Since the character of the functional dependence of a and b on δ depends weakly on the shape of the channel, we write down the results for the cylindrical capillary, where

$$a=\mathrm{Im}\frac{2I_1(k_3d)}{k_3dI_0(k_3d)},\qquad b=\mathrm{Re}\frac{2I_1(k_3d)}{k_3dI_0(k_3d)}.$$

The volume absorption of first sound, associated with η and ζ_2, changes identically for change in δ and has the form

$$\gamma_{\eta,\zeta_2}^{(1)}=\frac{1}{2}\left(\frac{4}{3}\eta+\zeta_2\right)\frac{\omega^2}{\rho u_{1\delta}^3}\frac{(1-b)^2-a^2}{1-b\rho_n/\rho}.\tag{4}$$

For $\delta \gg 1$, we get the well-known result from (4) for first sound.[5] Upon decrease in δ the contribution to the absorption of first sound of η and ζ_2 decreases and even becomes negative, remaining here less than the absorption associated with the slipping past of the normal component, since the frequencies $\omega < 10^6$ sec^{-1} can be propagated at small δ. But at large δ ($\delta > 3$), propagation of higher frequencies is possible and, inasmuch as the volume absorption is proportional to ω^2, while the absorption associated with the slipping is proportional to ω, it becomes of the same order as $\gamma_\delta^{(1)}$ and in the limit of first sound ($\delta \gg 1$) becomes the fundamental mechanism determining the absorption of first sound. A graph is shown in the drawing of the dependence of $\gamma_1 = \gamma_{\eta,\zeta_2}^{(1)}/\gamma^{(1)}$ on δ, where $\gamma^{(1)}$ is the volume absorption of first sound as $\delta \to \infty$ (curve 1).

As follows from the dissipative function,[5] the sound absorption associated with η and ζ_2 is proportional to $[\mathrm{div}\, \mathbf{v}_n]^2$; therefore in the increase in the degree of damping of the normal component of the liquid, the contribution of these terms to the sound absorption should decrease.

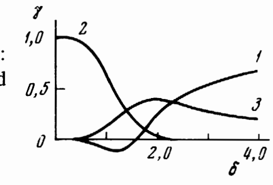
The absorption of sound associated with ζ_3 has the form

$$\gamma_{\zeta_3}^{(1)}=\frac{1}{2}\zeta_3\frac{\rho_s^2}{\rho}\frac{\omega^2}{u_{1\delta}^3}\frac{b^2-a^2}{1-b\rho_n/\rho}.\tag{5}$$

In the limit of fourth sound ($\delta \ll 1$) this absorption reaches a maximum value[1] and should decrease rapidly upon increase in δ, vanishing in the limit of first sound.

The figure shows a plot of the relation $\gamma = \gamma_{\zeta_3}^{(1)}/\gamma_{\zeta_3}^{(4)}$, where $\gamma_{\zeta_3}^{(4)} = \rho_s\omega^2\zeta_3/2u_4^3$ is that part of the absorption coefficient of fourth sound which is connected with ζ_3 (curve 2). A similar dependence on δ is obtained also for absorption due to the thermal conductivity of the liquid:

$$\gamma_\varkappa^{(1)}=\frac{\varkappa}{2C_{\mathrm{He}}}\frac{\rho_s\rho_n}{\rho^2}\frac{\omega^2}{u_{1\delta}^3}\frac{u_2^2}{u_{1\delta}^2}\frac{b^2-a^2}{1-b\rho_n/\rho}.\tag{6}$$



Dependence of the relative of sound absorption coefficient on δ for T = 1.8°K: 1—sound absorption associated with η and ζ_2; 2—sound absorption associated with ζ_3; 3—sound absorption associated with ζ_1.

The quantity ζ_1 makes a contribution to the absorption of first sound in the presence of damping of the normal component:

$$\gamma_{\zeta_1}^{(1)}=-\zeta_1\frac{\rho_s}{\rho}\frac{\omega^2}{u_{1\delta}^3}\frac{b(1-b)+a^2}{1-b\rho_n/\rho}.\tag{7}$$

As is seen from the drawing (curve 3, $\gamma_3^{(1)} = -\gamma_{\zeta_1}^{(1)}(\zeta_1\rho_s\omega^2/\rho u_1^3)^{-1}$), this part of the absorption is important in the region of strong dispersion and in the limiting cases ($\delta \to \infty$ and $\delta \to 0$) it generally disappears. This is connected with the fact that the sound absorption is proportional to $\mathrm{div}\, \mathbf{v}_n\, \mathrm{div}\, \rho_s(\mathbf{v}_n - \mathbf{v}_s)$ and in the limit of fourth sound ($\delta = 0$) the velocity of the normal component of the liquid $\mathbf{v}_n = 0$, and in the limit of first sound ($\delta \to \infty$) the normal and superfluid parts oscillate as a single unit, and the velocity of the relative motion $\mathbf{v}_n - \mathbf{v}_s$ is equal to zero.

As we see from the considerations given above, in the presence of damping of the normal component, all dissipative processes begin to play a role in the absorption of first sound. Here the situation is similar to that existing in solutions of He^3—He^4, where, from the presence of He^3 atoms, the relative velocity $\mathbf{v}_n - \mathbf{v}_s$ in the propagation of ordinary first sound differs from zero and all the coefficients of second viscosity make a contribution to the sound absorption.[6] We note that the contribution of volume mechanisms to the sound absorption increases with decrease in temperature, w whereas the absorption connected with the friction of the normal component with the walls of the channel decreases.

For the coefficient of second sound, we get the following result:

$$\gamma^{(2)}=\gamma_\delta^{(2)}+\frac{1}{2}\frac{\omega^2}{u_{2\delta}^3}\left[\frac{\rho_s}{\rho_n\rho}\left(\frac{4}{3}\eta+\zeta_2-2\zeta_1\rho+\zeta_3\rho^2\right)+\frac{\varkappa}{C_{\mathrm{He}}}\right],\tag{8}$$

where $\gamma_\delta^{(2)}$ is the absorption due to friction of the normal component of the liquid with the wall[2] and heat transfer through the wall.

As we shall see, all the coefficients of second viscosity and the coefficient of thermal conductivity enter into the absorption coefficient of second sound, independent of the degree of damping of the normal component. As $\delta \to 0$, the coefficient $\gamma^{(2)}$ goes to infinity ($u_{2\delta} \to 0$) and as $\delta \to \infty$ ($u_{2\delta} \to u_2$, $\gamma_\delta^{(2)} \to 0$) we get the well-known result for the absorption coefficient of second sound.

For account of the heat conduction of the walls and the heat resistance of the boundary, we can neglect the volume mechanisms of dissipation. Then the dispersion equation takes the form

$$k_\parallel^4-k_\parallel^2[k_1^2(1+ir)+k_2^2(1+M)(1+ir\rho_s/\rho_n)] + (1+M)k_1^2k_2^2(1+ir\rho/\rho_n)=0,\tag{9}$$

where

$$M=R_1+iR_2=\frac{i}{d\omega C_{\mathrm{He}}}\left(\frac{1}{\alpha}+\frac{1+i}{\gamma\varkappa_{\mathrm{w}}}\right)^{-1},\quad \gamma=\sqrt{\frac{2\omega C_{\mathrm{w}}}{\varkappa_{\mathrm{w}}}},$$

and for the squares of the wave vectors of first and second sound, we get the following results:

$$k_{1\delta}^2 = k_1^2 \frac{1 + ir\rho/\rho_n}{1 + ir\rho_s/\rho_n} \left[1 + \frac{k_1^2}{k_2^2} \frac{\rho_s r^2}{\rho_n (1+M)(1 + ir\rho_s/\rho_n)^2} \right], \quad (10)$$

$$k_{2\delta}^2 = k_2^2 (1 + ir\rho_s/\rho_n)(1+M). \quad (11)$$

As is seen from these formulas, in the presence of large heat conduction of the walls, there is a strong dispersion only for the velocity of second sound. So far as first sound is concerned, only the terms proportional to u_2^2/u_1^2 are decreased here, and in the limit when $\delta = 0$ (the limit of fourth sound), we get the result of[1].

The absorption of first sound, which is connected with the heat flow through the wall, is equal to

$$\gamma_{\varkappa\, w}^{(1)} = \frac{1}{2} \frac{\omega}{u_{1\delta}} \frac{u_2^2}{u_{1\delta}^2} \frac{\rho_n \rho_s}{\rho^2} \frac{R_2}{(1+R_1)^2 + R_2^2} \frac{b^2 - a^2}{1 - b\rho_n/\rho}. \quad (12)$$

It depends on δ and, similar to the absorption due to ζ_3 (curve 2), it is maximal in the limit of fourth sound and vanishes in the limit of first sound. The factor $R_2/[(1 + R_1)^2 + R_2^2]$ is maximal when $R_1 \ll R_2 \sim 1$, as is easy to see. Such a case can arise for sufficiently narrow channels filled with superfluid helium in the case of strongly heat-conducting walls. For copper at $T = 1.3^\circ$K, the heat resistance of the boundary, associated with the Kapitza discontinuity, $\alpha = 0.12$ W/cm^2-deg.[7] For capillaries with a diameter $d \approx 3 \times 10^{-4}$ cm at the sound frequency $\omega \approx 4 \times 10^4$ sec^{-1}

$$\gamma_{\varkappa\, w}^{(1)} \approx 2 \cdot 10^{-4}\ \text{cm}^{-1}, \quad \gamma_\delta^{(1)} \approx 2 \cdot 10^{-3}\ \text{cm}^{-1}.$$

In the limit of fourth sound, when $\delta \ll 1$, we get from (12)

$$\gamma_{\varkappa\, w}^{(4)} = \frac{1}{2} \frac{\omega}{u_4} \frac{u_2^2}{u_4^2} \frac{\rho_n}{\rho} \frac{R_2}{(1+R_1)^2 + R_2^2}. \quad (13)$$

Then, for R_1 and $R_2 \ll 1$ and R_1 and $R_2 \gg 1$, we get, respectively,[1]

$$\gamma_{\varkappa\, w}^{(4)} = \frac{1}{2} \frac{\omega u_2^2}{u_4^3} \frac{\rho_n}{\rho} R_2, \quad (14)$$

$$\gamma_{\varkappa\, w}^{(4)} = \frac{1}{2} \frac{\omega^2 u_2^2}{u_4^3} \frac{\rho_n}{\rho} \left(\frac{1}{\alpha} + \frac{1}{\gamma\varkappa_w} \right) C_{He} \rho d. \quad (15)$$

For the velocity and absorption of second sound, we get from (11)

$$u_{2\delta} = \sqrt{2} u_2 \left\{ \left[\left(1 - \frac{\rho_s}{\rho} m_2\right)^2 + m_1^2 \frac{\rho_s^2}{\rho^2} \right]^{1/2} [(1+R_1)^2 + R_2^2]^{1/2} + \left(1 - \frac{\rho_s}{\rho} m_2\right)(1 + R_1) - \frac{\rho_s}{\rho} m_1 R_2 \right\}^{-1/2}, \quad (16)$$

$$\gamma_\zeta^{(2)} = \frac{\omega}{\sqrt{2} u_2} \left\{ \left[\left(1 - m_2 \frac{\rho_s}{\rho}\right)^2 + m_1^2 \frac{\rho_s^2}{\rho^2} \right]^{1/2} [(1+R_1)^2 + R_2^2]^{1/2} - \left(1 - \frac{m_2 \rho_s}{\rho}\right)(1 + R_1) + R_2 m_1 \frac{\rho_s}{\rho} \right\}^{1/2}. \quad (17)$$

As follows from these considerations, the presence of a large heat conduction of the walls increases the velocity dispersion and the absorption of second sound, produced by the damping of the normal component.

The authors are grateful to Prof. M. I. Kaganov for discussion of the results of the research.

1. D. G. Sanikidze, I. N. Adamenko and M. I. Kaganov, Zh. Eksp. Teor. Fiz. 52, 584 (1967) [Soviet Phys.-JETP 25, 383 (1967)].
2. I. N. Adamenko and M. I. Kaganov, Zh. Eksp. Teor. Fiz. 53, 615 (1967) [Soviet Phys.-JETP 26, 394 (1968)].
3. B. N. Esel'son, N. E. Dyumin, E. Ya. Rudavskii and I. A. Serbin, ZhETF Pis. Red. 6, 723 (1967) [JETP 6, 203 (1967)].
4. G. L. Pollack and J. R. Pellam, Phys. Rev. 137, A1676 (1965).
5. I. M. Khalatnikov, Vvedenie v teoriyu sverkhtekuchesti (Introduction to the Theory of Superfluidity) (Nauka Press, 1965).
6. T. A. Karchava and D. G. Sanikidze, Zh. Eksp. Teor. Fiz. 57, 1349 (1969) [Soviet Phys.-JETP 30, 731 (1970)].
7. Kwan Wei-Yang, Zh. Eksp. Teor. Fiz. 42, 921 (1962) [Soviet Phys.-JETP 15, 635 (1962)].

Translated by R. T. Beyer
223