EFFECT OF DISLOCATIONS ON THE DENSITY OF ELECTRON STATES IN A MAGNETIC FIELD

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The influence of parallel linear dislocations on the spectrum and density of electron states in a crystal located in an external stationary uniform magnetic field is investigated. It is shown that the appearance of quasistationary states in the electron spectrum is induced by the dislocations. The respective singularities in the electron state density are studied, and in particular the dependence of the position of the points on the angle between the magnetic field and dislocations is investigated.

1. BESIDES point defects of the impurity type, real crystals always contain extended defects—dislocations. These defects may exert an appreciable influence on the electronic properties of solids. The influence of point defects on these properties has been the subject of an appreciable number of experimental and theoretical investigations. At the same time, the influence of the dislocations on the electron spectrum and on the density of the electron state is hardly considered in the literature. We know of only one paper by Kosevich and Tanatorov[1] devoted to a study of the electronic spectrum of a crystal with dislocations in a magnetic field.

Owing to the interaction of the electrons with the dislocations, additional bound states are produced, which should become manifest both in the thermodynamic and in the kinetic characteristics. The role of the dislocations becomes particularly clearly manifest in the case when the crystal is placed in an external quantizing magnetic field, and the temperature is low enough. In this case, a system of quasistationary states is produced, with energies close to the corresponding Landau levels. We investigate in this paper the electron spectrum and the density of states of the electrons in a crystal containing parallel dislocations and plates in an external magnetic field.

Let an electron with a quadratic isotropic dispersion law $E = p^2/2m$ move in a crystal situated in a constant homogeneous magnetic field **H** and containing linear crystal-lattice defects (dislocations). If there are no dislocations, then the density of the electronic states in a magnetic field is well known. We are interested, on the other hand, by the changes of the state density, which which are brought about by the presence of dislocations in the crystal.

Generally speaking, dislocations are curves of broken lines. However, if the radius of curvature of the dislocation line is much larger than the magnetic length $r_H = (c\hbar/eH)^{1/2}$, then the dislocation can be regarded as linear. Further, if the dislocation concentration ρ_D is small, then we can first consider the influence of one dislocation on the density of states, and then multiply the result by the concentration. We shall thus consider a "single-particle" problem.

The geometry of the problem is shown in Fig. 1. The dislocation is directed along the z axis. The magnetic

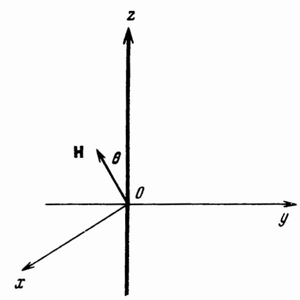

FIG. 1

field lies in the xz plane. We assume that the dislocation is associated with a perturbation of the crystal-lattice potential; this perturbation is independent of the coordinate z. In the xy plane, this perturbation represents a "smeared" δ-function, and the radius of the smearing is of the order of the interatomic distance a.

A similar model was proposed earlier by Kosevich and Tanatorov,[1] who obtained exhaustive results in the case of $\theta = 0$. We shall solve the problem by the method of degenerate regular perturbations, developed by Lifshitz.[2]

Since the perturbation is independent of the coordinate z, the unperturbed Hamiltonian $\mathscr{H}_0 = (\hat{\mathbf{p}} - e\mathbf{A}/c)^2/2m$ is conveniently chosen in a form such that it is independent of the coordinate z (and also of the coordinate x). This can be done by using the gauge invariance of the vector potential **A**. We choose **A** in the form

$$A_x = -yH_z, \quad A_y = 0, \quad A_z = yH_x,$$

i.e.,

$$A_x = -yH\cos\theta, \quad A_y = 0, \quad A_z = yH\sin\theta.$$

With such a gauge of the vector potential, the conserved quantities are p_z and p_x; the eigenfunctions and the eigenvalues of the unperturbed Schrödinger equation are

$$\psi_\alpha(x, y, z) = \frac{1}{2\pi\hbar\sqrt{2^n n!}}\left(\frac{eH}{\pi c\hbar}\right)^{1/4}\exp\left[\frac{i}{\hbar}(xp_x + zp_z) - \frac{(y-y_0)^2}{2r_H^2}\right]H_n\left(\frac{y-y_0}{r_H}\right),$$

$$E_\alpha = \hbar\Omega\left(n+\frac{1}{2}\right)+\frac{1}{2m}(p_x\sin\theta+p_z\cos\theta)^2,$$

where $y_0 = c(p_z \sin\theta - p_x \cos\theta)/eH$, $\Omega = eH/mc$ is the cyclotron frequency, $H_n(\xi)$ is a Hermite polynomial, α denotes the complete set of quantum numbers n, p_x and p_z. The set of functions ψ_α is orthonormal.

We note, first, that the dependence of the perturbed wave function on the coordinate z is determined by the factor $\exp(izp_z/\hbar)$, since the perturbation potential does not depend on z. We shall therefore denote henceforth by ψ the two-dimensional wave function $\psi(x, y)$. The action of the perturbing potential U on the ψ-function is determined by the relation

$$U\psi = -\beta u(x, y)(\psi, u). \tag{1}$$

In (1), the function u(x, y) denotes a "smeared" δ-function, (ψ, u) is the scalar product, the constant β represents the "intensity" of the perturbation and is equal in order of magnitude to U_0a^2, where U_0 is the characteristic potential binding energy of the electron with the dislocation, and a is the effective radius of the potential, assumed to have the order of magnitude of the interatomic distance. Positive values of β correspond to attraction of the electron by the dislocation, and negative to repulsion.

Thus, we have the perturbed equation

$$\mathscr{H}_0\psi - \beta u(\psi, u) = E\psi. \tag{2}$$

We change over first to the α representation, in which

$$c_\alpha - \frac{\beta u_\alpha(\psi, u)}{E_\alpha - E} = 0, \qquad c_\alpha = (\psi, \psi_\alpha).$$

We then multiply both sides of this equation in scalar fashion by u_α and use the invariance of the scalar product. As a result we obtain the following equation for the determination of the energy levels of the perturbed problem:

$$1 - \beta\sum_n \int dp_x \frac{|u_n(p_x)|^2}{E_n(p_x, p_z) - E} = 0. \tag{3}$$

Here

$$u_n(p_x) = \frac{1}{\sqrt{2\pi\hbar 2^n n!}}\left(\frac{eH}{\pi c\hbar}\right)^{1/4}\int \exp\left[\frac{i}{\hbar}xp_x - \frac{(y-y_0)^2}{2r_H^2}\right] \times H_n\left(\frac{y-y_0}{r_H}\right)u(x, y)\,dx\,dy. \tag{4}$$

From (4) and from an examination of the asymptotic form of the Hermite polynomials at large values of n, it follows that if the function u(x, y) is "smeared out" over distances on the order of a, then the functions $u_n(p_x)$ differ noticeably from zero only if the condition $n < N \approx (r_H/a)^2$ is satisfied. For metals we have $(r_H/a)^2 \approx \epsilon_F/\hbar\Omega$, amounting to about 10^4–10^5 in ordinary magnetic fields. Thus, we are actually summing in (3) up to numbers on the order of 1000–10 000. In addition, in the calculation of $u_n(p_x)$ for numbers smaller than N it is possible, with sufficient accuracy, to replace u(x, y) by a δ-function. Then the integration in (4) can be carried out in elementary fashion and substitution of the result in (3) leads to the equation

$$1 - \frac{\beta}{2\pi^{3/2}\hbar r_H}\sum_{n=0}^{N}\frac{1}{2^n n!} \times \int_{-\infty}^{\infty}\frac{\exp(-y_0^2/r_H^2)H_n^2(y_0/r_H)\,dp_x}{\hbar\Omega(n+1/2)+(p_x\sin\theta+p_z\cos\theta)^2/2m - E} = 0. \tag{5}$$

In Eq. (5) it is convenient to introduce the dimensionless energy $\epsilon = 2E/\hbar\Omega$ and to change over to integration with respect to the dimensionless variable $\xi = y_0/r_H$. As a result we obtain the following dispersion equation:

$$1 - \frac{\beta m}{\pi^{3/2}\hbar^2\sin\theta}\sum_{n=0}^{N}\frac{1}{2^n n!\sqrt{\varepsilon-2n-1}}\int_{-\infty}^{\infty}\left(\frac{e^{-\xi^2}H_n^2(\xi)}{\xi-\xi_1} - \frac{e^{-\xi^2}H_n^2(\xi)}{\xi-\xi_2}\right)d\xi = 0. \tag{6}$$

In Eq. (6) we have

$$\xi_{1,2} = \frac{1}{\sin\theta}(\gamma \pm \cos\theta\sqrt{\varepsilon-2n-1}), \quad \gamma = \frac{p_z}{\sqrt{m\hbar\Omega}}.$$

If $\epsilon < 2n + 1$, then ξ_1 and ξ_2 are complex quantities, so that the corresponding integrands have no singularities on the real ξ axis, and the integrals converge. On the other hand if $\epsilon > 2n + 1$, then ξ_1 and ξ_2 are real, and the integral are interpreted as the limits of the corresponding expressions when ξ_1 goes over to the real axis from the upper half plane, and ξ_2 from the lower half plane. The integrals in (6) are of the Cauchy type; their behavior on the integration contour has been thoroughly investigated and is given by the formulas of Sokutskiĭ–Plemel.[3]

Equation (6) determines the energy level as functions of the parameter γ, and these levels are real only when $\epsilon < 1$. When $\epsilon > 1$, Eq. (6) has only a complex solution $\epsilon = \epsilon_n(\gamma) + i\Gamma_n(\gamma)$, and the density of states $\nu_n(\epsilon)$ is determined, as is well known, from the relation

$$\nu_n(\varepsilon) = A\int d\gamma\frac{\Gamma_n}{(\varepsilon-\varepsilon_n)^2+\Gamma_n^2} \tag{7}$$

(A is a constant).

At small Γ and fixed γ we can speak of quasi-discrete levels whose width is determined by the value of Γ. Integration with respect to γ causes the singularity with respect to the state density to change from a δ-type to a root type.

3. We begin the investigation of Eq. (6) with the case when the magnetic field is perpendicular to the dislocation. In this case integration with respect to ξ is elementary, and as a result we arrive at the equation

$$1 - \frac{2i\beta m}{\pi^{1/2}\hbar^2}\sum_{n=0}^{N}\frac{e^{-\gamma^2}H_n^2(\gamma)}{2^n n!\sqrt{\varepsilon-2n-1}} = 0. \tag{8}$$

We assume that the dimensional parameter $\bar\beta = \beta m/\pi\hbar^2$ is small (this statement will be made more precise later). Then the principal role in the sum (8) is played by a single term with energy close to the Landau level 2n + 1. Separating this term and replacing the remaining sum over n by an integral, we can approximately determine the corresponding quasi-stationary energy levels. Such levels obviously exist only when $\epsilon < 2n + 1$ and $\beta > 0$, corresponding to an attraction potential. Levels with energy larger than 2n + 1 are absent because of the strong damping.

Let us consider first a level close to the zeroth Landau level, i.e., $\epsilon_0 \sim 1$. Separating the zeroth term in the sum, we obtain

$$\frac{1}{2\sqrt{\pi}\,\bar\beta} = \frac{e^{-\gamma^2}}{\sqrt{1-\varepsilon_0}} + \int_1^N\frac{e^{-\gamma^2}H_n^2(\gamma)\,dn}{2^n n!\sqrt{2n}}. \tag{9}$$

In order to carry out the integration correctly, we note the following. At large N the integral in (9) behaves like ln N, and therefore a large contribution to the integration is made by terms with large numbers. On the other hand, at large numbers an important role is played by the relation between the number and the argument in the Hermite polynomial. Namely, (see, for example, [4]), if $\gamma^2 < 2n + 1 - O(n^{1/3})$, then the quasi-classical asymptotic form of the integrand at large values of n oscillates:

$$\frac{e^{-\xi^2}H_n^2(\xi)}{2^n n!} \sim \frac{2\sin^2\varphi}{\sqrt{\pi}\sqrt{2n+1-\xi^2}}. \tag{10}$$

In (10), $\sin^2\varphi$ is a rapidly oscillating factor, which yields ½ upon integration. On the other hand, if $\gamma^2 > 2n + 1 + O(n^{1/3})$, then the integrand is exponentially small ($\sim \exp -2n$). Therefore the last term in (9) can be written in the form

$$\int_1^N \frac{e^{-\gamma^2}H_n^2(\gamma)\,dn}{2^n n!\sqrt{2n}} \approx \int_{n_0}^N \frac{dn}{\sqrt{2\pi n}(2n+1-\gamma^2)}$$

$$= \frac{1}{2\sqrt{\pi}}\ln\frac{N + {}^1/_4(1-\gamma^2) + \sqrt{N(N+(1-\gamma^2)/2)}}{n_0 + {}^1/_4(1-\gamma^2) + \sqrt{n_0(n_0+(1-\gamma^2)/2)}},$$

where n_0 differs from $\gamma^2/2$ by an amount of the order of $O(n^{1/3})$. A rough estimate of the logarithm at $\gamma^2 < N$ yields a quantity of the order of $(1/2\sqrt{\pi}) \times \ln[4N/(2\gamma^2 + 1)]$. Substituting the obtained estimate in (9), we get

$$\left(\frac{1}{\beta} - \ln\frac{4N}{2\gamma^2+1}\right) = \frac{2\sqrt{\pi}\,e^{-\gamma^2}}{\sqrt{1-\varepsilon_0}}. \tag{11}$$

We see therefore that the smallness of $\bar{\beta}$ is understood in the sense that

$$\beta < \left[\ln\frac{4N}{2\gamma^2+1}\right]^{-1}. \tag{12}$$

Relation (12) is satisfied at all values of γ only if $\bar{\beta} < (\ln 4N)^{-1}$ or

$$\frac{4\hbar^2}{ma^2}\exp\left(-\frac{1}{\beta}\right) \ll \hbar\Omega. \tag{13}$$

In the last inequality on the left side is the energy of the bound state of the electron in a two-dimensional potential well produced by the dislocation (see, for example, [5]). Thus, the natural limitation on the value of β is that the electron-dislocation binding energy must be much smaller than the distance between the Landau levels.

Taking into account the weak dependence of the logarithm on γ^2, we get from (11) the energy of the lowest dislocation level

$$\varepsilon_0 = -\frac{4\pi\beta^2 e^{-2\gamma^2}}{(1-\beta\ln 4N)^2} + 1. \tag{14}$$

It should be noted that the damping in this state is rigorously equal to zero. The density of states at a known dependence of the energy ϵ on the parameter γ is given by the formula

$$\nu(\varepsilon) = \frac{2mLr_H}{\hbar^2}\int\delta[\varepsilon - \varepsilon(\gamma)]\,d\gamma, \tag{15}$$

where L is the total length of the dislocation lines in the sample. Substituting (14) in (15) and integrating, we readily find that at the point $\epsilon^* = 1 - 4\pi\beta^2/(1 - \beta\ln 4N)^2$ the density of states has a root singularity $\nu(\epsilon) \sim 1/\sqrt{\epsilon - \epsilon_*}$.

We now present a qualitative investigation of the density of states near the first Landau level. Putting $\epsilon \lesssim 3$, we get from (8)

$$1 - 2i\beta\pi^{1/2}\left[\frac{e^{-\gamma^2}}{\sqrt{2}} + \frac{4\gamma^2 e^{-\gamma^2}}{2i\sqrt{3-\varepsilon}} + \sum_{n=2}^{N}\frac{e^{-\gamma^2}H_n^2(\gamma)}{2^n n! i\sqrt{2n+1-\varepsilon}}\right] = 0.$$

The sum over n in the square brackets has the order of magnitude ln 4N. Therefore, assuming the condition (13) to be satisfied, we can neglect this sum in a qualitative investigation. Solving the resultant equation, we get

$$\varepsilon_1 = 3 - \frac{16\pi\beta^2\gamma^4 e^{-2\gamma^2}}{(1 - i\beta e^{-\gamma^2}\sqrt{2\pi})^2}. \tag{16}$$

We see therefore that the damping of the obtained states is at most of the order of $\Gamma \approx 16\pi\bar{\beta}^2 \cdot \bar{\beta}$. This damping should be small compared with the distance from the obtained level to the first Landau level, for otherwise one cannot speak of a singularity in the density of states. Thus, the condition $\bar{\beta} \ll 1$ must be satisfied. The latter inequality, however, is certainly satisfied if the condition (13) holds.

Substitution of (16) in (15) and integration (the damping is neglected) shows that the density of states again has a root singularity at a point located $16\pi\bar{\beta}^2/e^2$ from the Landau level.

In considering the terms of the sum (8) with large numbers, we can use the asymptotic expression (10) for the Hermite polynomials. In the case of small $\bar{\beta}$ we can neglect the damping, and moreover we can retain only one term in the sum over n. We then obtain the following expression for the energy near the n-th Landau level:

$$\varepsilon_n(\gamma) = 2n + 1 - 4\beta^2/(2n+1-\gamma^2). \tag{17}$$

To find the density of states it is necessary to substitute (17) in (15) and integrate with respect to γ between the limits 0 and $\sqrt{2n+1} - O(n^{-1/6})$. After integration it turns out that the density of states has a root singularity at the point $\epsilon_n = 2n + 1 - 4\bar{\beta}^2/(2n + 1)$. Actually, of course, the density of states does not become infinite at these points because of the finite albeit small damping. These singularities become manifest in the form of sharp peaks of the density of states as functions of the energy, located at distances $\Delta\epsilon_n = 4\bar{\beta}^2/(2n + 1)$ from the closest Landau levels.

We now proceed to study the density of states in the case when the dimensionless coupling constant is not a small quantity. It is now impossible to retain only one term in the sum (8). Therefore when $\epsilon > 1$ the damping becomes large (compared with the corresponding energy difference) and the aforementioned singularities in the density of states vanish with increasing $\bar{\beta}$. When $\epsilon < 1$ there is no imaginary part in the left side of (8). The character of the spectrum can be clarified by replacing the sum with an integral and using the asymptotic form given above for the Hermite polynomials. Integrating in analogy with the procedure used earlier, we obtain

$$\frac{1}{\beta} = \ln\frac{2N + 1 - {}^1/_2(\varepsilon+\gamma^2) + \sqrt{(2N+1-\varepsilon)(2N+1-\gamma^2)}}{{}^1/_2(\gamma^2-\varepsilon)}. \tag{18}$$

In order to describe the spectrum roughly, it is possible to neglect the quantities γ^2 and ϵ compared with the large number N in the numerator of the fraction under the logarithm sign. We then get

$$\varepsilon = -8N\exp(-1/\beta) + \gamma^2$$

or, changing to dimensional quantities

$$E = -\frac{4\hbar^2}{ma^2}\exp\left(-\frac{1}{\beta}\right) + \frac{p_z^2}{2m}. \tag{19}$$

We thus arrive at the well known (see, for example, [5]) expression for the electron energy in a two-dimensional potential well in the absence of a magnetic field. Of course, a similar result is obtained if the magnetic field is decreased. Of great importance in this case is, of course, the violation of the condition (13).

Let us display the results on the plot of the density of states (Fig. 2). The usual root singularities of the density of states in a magnetic field (at $\epsilon = 2n + 1$), of course, do not vanish when the dislocations are taken into consideration, since the corresponding energy levels have infinite degeneracy. The dislocation leads to the appearance of additional singularities in the density of states. Figure 2 shows a plot of the density of states in the case $\bar{\beta} \ll (\ln 4N)^{-1}$. Beside the root singularity at the point ϵ_0, there appear rather high peaks of $\nu(\epsilon)$ (and not singular points as would be the case without the damping) at the point ϵ_n, where $\Delta\epsilon_n \equiv 2n + 1 - \epsilon_n \approx 4\bar{\beta}^2/(2n + 1)$. Of course, the relative height of these peaks is proportional to the small dislocation concentration, but in the case of sufficiently small damping their height can be noticeable.

With increasing $\bar{\beta}$ or with decreasing magnetic field, when $\bar{\beta}$ becomes of the order of or larger than $(\ln 4N)^{-1}$, all the peaks of the density of states due to the dislocations, besides the zeroth one, spread out, so that no real singularities remain. As to the root singularity of the point ϵ_0, it shifts to the left and the corresponding spectrum is given by (19).

4. We now proceed to investigate the case when the angles between the dislocation and the magnetic field differ from a right angle. As before, we confine ourselves to small $\bar{\beta}$. The principal role is played in this case by one of the terms of the sum over n. It can be large when the energy is close to one of the Landau levels ($|\epsilon - 2n - 1| \ll 1$) or the integral on the left side of (6) is large. In both these variants, allowance for the remaining terms of the sum leads to a renormalization of the interaction constant $\bar{\beta}$ in analogy to (11), and to the appearance of small damping at $\epsilon > 1$.

1) Let us consider one term with a fixed number n in the sum (6). Assuming ϵ to be close to $2n + 1$, we can put $\epsilon = 2n + 1$ in the expressions for ξ_1 and ξ_2.

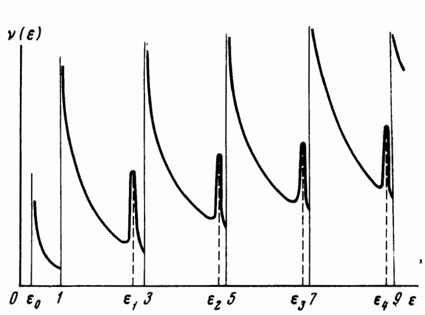

FIG. 2

Then the principal values of the integrals cancel each other and we are left only with the residue of the integrand at the point $\gamma' = \gamma/\sin\theta$, multiplied by $2\pi i$. In order for the left side of (6) to be real, it is necessary to consider energies $\epsilon < 2n + 1$. Taking the foregoing into account, we obtain

$$\frac{1}{2\sqrt{\pi}\,\beta'} = \frac{e^{-\gamma'^2}H_n^2(\gamma')}{2^n n!\sqrt{2n+1-\varepsilon}}, \quad \beta' = \frac{\beta}{\sin\theta}. \tag{20}$$

We thus obtain an equation for the determination of the spectrum; this equation coincides formally with the dispersion equation in the case when $\theta = \pi/2$, provided we make in the latter the substitutions $\gamma \to \gamma/\sin\theta$ and $\beta \to \beta/\sin\theta$. Therefore an investigation of the corresponding singularities of the spectral density entails no difficulty. Namely, when the angle θ decreases from the point $\theta = \pi/2$, the peaks of the density of states, shown in Fig. 2, shift to the left, so that now $\Delta\epsilon_n = 4\bar{\beta}^2/(2n + 1)\sin^2\theta$. In addition, a "smearing" of these peaks (except the zeroth peak) takes place, the damping Γ being proportional to $1/\sin\theta$; the latter circumstance becomes particularly lucid if attention is paid to relation (16). The angle criterion is obtained from the condition $\Delta\epsilon_n \ll 1$ and is given by

$$\sin\theta \gg 2\beta/\sqrt{2n+1}. \tag{21}$$

Since $\bar{\beta} \ll 1$, the inequality (21) is satisfied at all θ with the exception of angles quite close to the zeroth angle ($\theta \sim 2\bar{\beta}$). We see thus that the singularities obtained above in the density of states are quite "stable" and remain following an appreciable variation of the angle between the magnetic field and the dislocation. We emphasize once more that the entire described picture takes place only in an attraction potential ($\beta > 0$).

2) To study the behavior of the integral in the left side of (6), we replace the Hermite functions by their quasiclassical asymptotic forms at large numbers, in accordance with (10). Such a replacement is justified, since, first, the principal role in (6) is played by large numbers, and, second, this asymptotic form gives a good approximation at even relatively small numbers. Substituting (10) in (6) and integrating with respect to ξ from $-\sqrt{2n+1}$ to $\sqrt{2n+1}$, we obtain

$$\frac{1}{\beta} = \frac{i}{\sin\theta}\sum_{n=0}^{N}\frac{1}{\sqrt{\varepsilon-2n-1}}\left(\frac{1}{\sqrt{2n+1-\xi_1^2}} - \frac{1}{\sqrt{2n+1-\xi_2^2}}\right). \tag{22}$$

The choice of the branch of the radical is shown in Fig. 3. In the complex ξ-plane (ξ denotes either ξ_1 or ξ_2), a cut is drawn from the point $-\sqrt{2n+1}$ to $\sqrt{2n+1}$. On the upper edge of the cut, the argument of the radicand expression is assigned to zero value. Of course, the points ξ_1 and ξ_2 cannot come too close to the branch points $\pm\sqrt{2n+1}$, since the rigorously calcu-

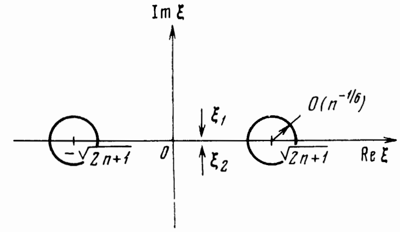

FIG. 3

lated integral has no singularities at these points. An analysis of the behavior of the Hermite polynomials $H_n(\xi)$ at $\xi^2 \sim 2n+1$ (see, for example, [4]) shows that the following inequalities should be satisfied

$$|\xi_{1,2} \pm \sqrt{2n+1}| > O(n^{-1/6}), \tag{23}$$

i.e., the points ξ_1 and ξ_2 should not fall inside circles with centers at the points $\pm\sqrt{2n+1}$ and with radii equal to $n^{-1/6}$ in order of magnitude.

We rewrite Eq. (22), substituting for ξ_1 and ξ_2 their expressions in terms of the parameters of the problem:

$$\frac{1}{\bar\beta} = \frac{i}{\sin\theta}\sum_{n=0}^{N}\frac{1}{\sqrt{x}}\left\{\left(2n+1-\frac{\Gamma_+^2}{\sin^2\theta}\right)^{-1/2} - \left(2n+1-\frac{\Gamma_-^2}{\sin^2\theta}\right)^{-1/2}\right\}, \tag{24}$$

$$x \equiv \varepsilon - 2n - 1, \qquad \Gamma_\pm = \gamma \pm \sqrt{x}\cos\theta.$$

When $\theta = 0$, Eq. (24), as can be readily verified, goes over into the exact dispersion equation for zero angle, and coincides with the equation of [1]

$$\frac{1}{2\bar\beta} = \sum_{n=0}^{N}\frac{1}{2n+1+\gamma^2-\varepsilon}. \tag{25}$$

If we retain in the right side of (25) only one term, then we obtain the following set of levels:

$$\varepsilon_n(\gamma) = 2n+1+\gamma^2-2\bar\beta. \tag{26}$$

Since parameter $\bar\beta$ is small, the exact solution of the dispersion equation differs insignificantly from the energies determined from the foregoing relation.

If $\theta \neq 0$, then retaining one term in the sum, we arrive at a rather cumbersome expression. The method we propose here for approximately solving this equation consists in the following. We obtain ϵ from the condition that the factor in the curly brackets must become infinite. Assuming that $\gamma > 0$ (replacement of γ by $-\gamma$ does not change the form of the equation), we verify that this occurs in two cases:

$$\sqrt{x} = \frac{1}{\cos\theta}(\gamma + \sqrt{2n+1}\sin\theta), \quad \sqrt{x} = \frac{1}{\cos\theta}|\gamma - \sqrt{2n+1}\sin\theta|. \tag{27}$$

We rewrite the dispersion equation in the form

$$\frac{1}{\bar\beta} = \frac{i}{\sqrt{x}}\,\frac{[(2n+1)\sin^2\theta - \Gamma_-^2]^{1/2} - [(2n+1)\sin^2\theta - \Gamma_+^2]^{1/2}}{Z} \tag{28}$$

Now it is necessary to replace $\sqrt{x}$ in the numerator and in the first factor of the denominator of the right side of Eq. (28), respectively, by the expressions given for this radical by (27), after which it is necessary to solve the obtained approximate equation with respect to $x = \epsilon - 2n - 1$.

We must therefore consider two situations. First, when x is near the value

$$x_1 = \frac{1}{\cos^2\theta}(\gamma + \sqrt{2n+1}\sin\theta)^2.$$

Then the approximate equation is

$$\frac{1}{\bar\beta} = -2\cos\theta\left(\frac{\gamma}{\gamma+\sqrt{2n+1}\sin\theta}\right)^{1/2}\frac{1}{Z}. \tag{29}$$

The second situation arises when ϵ is near the value

$$x_2 = \frac{1}{\cos^2\theta}(\gamma - \sqrt{2n+1}\sin\theta)^2.$$

It is then necessary to distinguish between two variants:

$$\frac{1}{\bar\beta} = -2\cos\theta\left(\frac{\gamma}{\gamma-\sqrt{2n+1}\sin\theta}\right)^{1/2}\frac{1}{Z}, \tag{30a}$$

if $\gamma > \sqrt{2n+1}\sin\theta$, and

$$\frac{1}{\bar\beta} = -2i\cos\theta\left(\frac{\gamma}{\sqrt{2n+1}\sin\theta-\gamma}\right)^{1/2}\frac{1}{Z}, \tag{30b}$$

if $\gamma < \sqrt{2n+1}\sin\theta$.

If we take into account the choice of the radical branch indicated in Fig. 3, then we can verify that the argument of the root Z contained in (29), (30a), and (30b), varies in the following manner when x changes from 0 to ∞: it is initially equal to π, between the first and second singular points it is equal to $-\pi/2$, and after the second singular point it vanishes. Therefore Eq. (29) can be solved when $\bar\beta < 0$ (repulsion), Eq. (30a) when $\bar\beta > 0$ (attraction), and (30b) again when $\bar\beta > 0$. In (29) it is necessary to choose that solution for which $x > x_1$, and in (30a) the solution for which $x < x_2$, while in (30b) the solution with $x > x_2$. These solutions have the following form:

$$(29) \to x = \frac{1}{\cos^2\theta}\left\{\gamma^2 + (2n+1)\sin^2\theta + \left[4(2n+1)\gamma^2\sin^2\theta + \frac{4\gamma\bar\beta^2\cos^2\theta}{\gamma+\sqrt{2n+1}\sin\theta}\right]^{1/2}\right\}, \tag{31}$$

$$(30\text{a,b}) \to x = \frac{1}{\cos^2\theta}\left\{\gamma^2 + (2n+1)\sin^2\theta - \left[4(2n+1)\gamma^2\sin^2\theta + \frac{4\gamma\bar\beta^2\cos^2\theta}{\gamma-\sqrt{2n+1}\sin\theta}\right]^{1/2}\right\}. \tag{32}$$

Formula (31) is realized in the case of a repulsion potential. Expression (32) corresponds to repulsion if $\gamma < \sqrt{2n+1}\sin\theta$, and to attraction if $\gamma > \sqrt{2n+1}\sin\theta$. We note first that in the limiting case $\theta = 0$ formulas (31) and (32) give a result that coincides with (26).

It is meaningless to find the explicit expression for the density of states in this case. It suffices to recall that the singularity in the density of states appears at those points for which $d\epsilon/d\gamma = 0$. Differentiating the energy determined from relation (31) with respect to γ, we can verify that this formula yields no singularity of the density of states (with the exception of the special case $\theta = 0$). As to formula (32), the derivative considered here vanishes when $\gamma = \sqrt{2n+1}\sin\theta$. However, it is impossible to come too close to this point because of (23). A more detailed analysis shows that the density of states near this point is quite large, if the angles θ are sufficiently small, namely if

$$\sin^2\theta < \bar\beta. \tag{33}$$

But when (33) is satisfied there is essentially no region in which formula (32) is valid in the case of a repulsion potential. An attraction potential corresponds to a peak of the density of states at the point

$$\varepsilon \approx 2n+1+\frac{2(2n+1)\sin^2\theta - \sqrt{4(2n+1)^2\sin^4\theta + 4\bar\beta^2\cos^2\theta\,(n^{2/3}+1)}}{\cos^2\theta}. \tag{34}$$

(We have replaced γ in formula (32) by $\sqrt{2n+1}\sin\theta$ everywhere with the exception of the denominator of the second term under the radical sign, where we put $\gamma - \sqrt{2n+1}\sin\theta \sim \sin\theta\, n^{-1/6}$ in accordance with (23).)

We are now able to describe in general outline those changes of the state density which occur when the angle between the magnetic field and the dislocations is varied from $\pi/2$ to 0.

a) The dislocation attracts electrons. When $\theta = \pi/2$, there are quasistationary states in the electron spectrum; their energies differ from the corresponding Landau level by amounts determined by the relation $\Delta\epsilon_n = 4\bar{\beta}^2/(2n + 1)$. These levels are not equidistant. With decreasing angle, a shift to the left and a smearing of these levels takes place. However, this smearing becomes appreciable only at very small angles, so that the described states are quite stable against changes of the angle. At very small angles it is necessary to use formula (34) to determine the quasistationary levels. Besides the described quasistationary states, the neutron spectrum contains a stationary state whose energy is given by (19). This state is always present in the spectrum, regardless of whether a magnetic field is present or not. It is likewise independent of the angle θ.

b) The dislocation repels electrons. In this case the quasistationary states are present in the spectrum essentially only at a zero angle between the magnetic field and the dislocation.

If the angle differs from zero then, as shown in [1], the damping of the corresponding states increases exponentially, and therefore these states cannot be observed.

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