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HEAT CAPACITY OF SPIN WAVES IN A BLOCH DOMAIN WALL

M. K. BUSHEV

Moscow State University

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A macroscopic derivation is given of the spectrum of spin waves in a 180° Bloch wall. The heat capacity of these waves is found and is shown to be dominant in the temperature region below a certain temperature T_0, which is proportional to the equilibrium magnetization and for uniaxial ferrites is of the order of 1°K. The possibility is suggested of obtaining information about the magnetization process in ferrites by measurements of their heat capacity as a function of temperature.

In a ferromagnetic magnetically ordered crystal, the domains, as is well known, go over smoothly from one to another. We shall consider uniaxial ferromagnets, for which a 180° neighborhood is characteristic. It is known that the transitional layer (or Bloch wall) between two such neighboring domains has a thickness of order 10^{-5} cm, whereas the thickness of the domains is usually two or three orders larger. The transitional layer, however, cannot exist at all if the specimen is too small (we shall discuss this in more detail below) or is in a strong magnetic field, in which technical saturation is attained. We shall suppose, however, that all the conditions for existence of domain walls are present.

It is known that the thermal properties of ferromagnets are basically determined by oscillations of the lattice, but that at lower temperatures contributions of the conduction electrons and of spin waves in the domains begin to show up. In ferrites, phonons and spin waves make a contribution to the heat capacity. In the transitional layers, however, there can also exist spin waves. Despite the fact that the relative volume of the transitional layers in a given specimen is much less than the volume of the domains, nevertheless conditions are possible under which the thermal properties of spin waves in these layers exhibit themselves. The purpose of the present paper is to explain the conditions under which this effect can exhibit itself.

In order to determine the heat capacity of the spin waves in a 180° Bloch wall, it is necessary to find the spectrum of these waves. This problem was solved by Winter[1] microscopically and, in our view, in a much more complicated manner than is necessary. The dispersion law of spin waves in the transition layers can be found more simply by a macroscopic method. For this purpose, we start from the concept of the effective field, introduced by Landau and Lifshitz:

$$\mathbf{H}_{\text{eff}} = \alpha\nabla^2\mathbf{M} + \beta\mathbf{n}(\mathbf{Mn}), \tag{1}$$

here α is the exchange-energy constant (for simplicity we consider the isotropic case), $\beta(>0)$ is the magnetic anisotropy constant, **M** is the density of the vector magnetization, and **n** is the unit vector in the direction of the axis of easy magnetization. The magnetic moments in the transitional layer are acted upon by this (linearized with respect to **M**) field. The expression (1) is externally very similar to the analogous expression for the effective field that acts in the interior of the domains (see, for example,[3]). In contrast with the latter, however, formula (1) does not and cannot contain a term expressing the presence of an external magnetic field. Such a field can change the relative volumes of domains with different orientations of the vector **M** or, if it is sufficiently strong, can convert the specimen to a single domain. But as long as the transitional layer exists, the distribution of magnetic moments in it practically "does not feel" the presence of the external field.

This is the first difference between the $\mathbf{H}_{\text{eff}}$ assumed by us and the analogous field in the interior of the domains. Another, more important difference is that for an equilibrium distribution of moments in the domains, the corresponding field $\mathbf{H}^0_{\text{eff}}$ is exactly equal to zero (this in fact is the condition for equilibrium), whereas $\mathbf{H}^0_{\text{eff}}$ in the transitional layers is different from zero. In fact, as Landau and Lifshitz[2] showed, the equilibrium distribution of magnetic moments in the Bloch wall has the form of a "fan" with axis normal to the wall. Thus if the origin is chosen at the center of the wall, the x axis along the normal, and the z axis along the direction of the vector **n**, the equilibrium distribution of magnetization $\mathbf{M}^0$ as a function of the coordinate x has the form

$$\mathbf{M}^0 = M^0(0, \sin\vartheta, \cos\vartheta), \tag{2}$$

where the azimuth angle θ is determined by the equation

$$\cos\vartheta = -\text{th}(\sqrt{\beta/\alpha}\,x). \tag{2a}$$

Therefore if we consider the small deviation of the vector magnetization **M** from the equilibrium position,

$$\mathbf{M}(\mathbf{r}, t) = \mathbf{M}^0(\mathbf{r}) + \mathbf{m}(\mathbf{r}, t), \tag{3}$$

then the corresponding $\mathbf{H}_{\text{eff}}$ is obviously expressed in the form

$$\mathbf{H}_{\text{eff}} = \mathbf{H}^0_{\text{eff}} + \mathbf{H}'_{\text{eff}}, \tag{3a}$$

where

$$\mathbf{H}^0_{\text{eff}} = \alpha\nabla^2\mathbf{M}^0 + \beta\mathbf{n}(\mathbf{M}^0\mathbf{n}), \quad \mathbf{H}'_{\text{eff}} = \alpha\nabla^2\mathbf{m} + \beta\mathbf{n}(\mathbf{mn}). \tag{3b}$$

We write the equation of motion of the magnetic moment in the form

$$\partial\mathbf{M}/\partial t = \gamma[\mathbf{MH}_{\text{eff}}]; \tag{4}$$*

here $h\gamma = g\mu_0$, where $g \approx 2$ is the spectroscopic splitting factor, $\mu_0 \approx 10^{-20}$ erg/G is the Bohr magneton,

*$[\mathbf{MH}_{\text{eff}}] \equiv \mathbf{M} \times \mathbf{H}_{\text{eff}}$.

and $\hbar$ is Planck's constant. In accepting formula (4) as the equation of motion, we are at the same time assuming that the magnitude of the vector **M** does not change with time, so that the system of equations (4) must degenerate to a system of two equations for two independent components of the vector **M**. Furthermore, we neglect the damping term in the Landau-Lifshitz[2] equation; but as we shall see later, the temperature range of interest to us will be that of order 1°K, in which the damping may be completely neglected[3].

Deviation of the vector **M** from the equilibrium position entails the appearance of a magnetic field **h**, which in the magnetostatic approximation is described by the equations

$$\mathrm{rot}\,\mathbf{h} = 0, \quad \mathrm{div}(\mathbf{h} + 4\pi\mathbf{m}) = 0. \tag{5}$$

We consider spin waves only within the domain walls. Therefore we may suppose that outside them, the equations m = 0 and h = 0 are satisfied. In such a case equations (5), from which follow the conditions of continuity of the tangential and normal components of the vectors **h** and **h** + 4π**m** respectively, lead to the equations

$$h_x + 4\pi m_x = 0, \quad h_y = h_z = 0. \tag{5a}$$

This field **h** must be added to $\mathbf{H}'_{\mathrm{eff}}$.

Before writing the system of equations (4), we shall express the components of the vectors $\mathbf{M}^0$ and **m** in terms of their components in an associated system of coordinates. In this system, the vector $\mathbf{M}^0$ takes the form $\mathbf{M}^0 = (0, 0, M^0)$. From the condition of conservation of the magnitude of the vector magnetic moment for small deviations from the equilibrium position, there follows, with neglect of the square of the small quantity m, the equation $\mathbf{M}^0\mathbf{m} = 0$. Thus the vector **m** has in the associated system the components $\mathbf{m} = (m_x, m_\eta, 0)$, and in the fixed, correspondingly, $\mathbf{m} = (m_x, m_\eta \cos\theta, -m_\eta \sin\theta)$, where $\cos\theta$ is given by equation (2a). The components of the vector $\mathbf{M}^0$ in the fixed system are given by equations (2).

It is easy to show that the equilibrium values of $\mathbf{M}^0$ and $\mathbf{H}^0_{\mathrm{eff}}$ are collinear, and therefore the equation of motion for the equilibrium moment reduces to an identity, as it should. The equations of motion for the components m_x and m_η will be, respectively,

$$\begin{aligned} \gamma^{-1}\partial m_x / \partial t &= -\alpha M^0 \nabla^2 m_\eta + \beta M^0 \cos 2\theta m_\eta, \\ \gamma^{-1}\partial m_\eta / \partial t &= \alpha M^0 \nabla^2 m_x - 4\pi M^0 m_x - \beta M^0 \cos 2\theta m_x. \end{aligned} \tag{6}$$

A particular solution of these equations has the form

$$m_{x,\eta} = C_{1,2} \sin\theta \exp[i(k_y y + k_z z - \omega t)], \tag{7}$$

hence it follows that these are plane waves, vanishing exponentially outside the Bloch wall, that is for $|x| \gg \delta$ (since $\sin\theta = 1/\mathrm{ch}(x/\delta)$, where $\delta = (\alpha/\beta)^{1/2}$ is the wall "thickness"). We substitute (7) in (6), and from the condition that the determinant of the system of equations for the arbitrary constants C_1 and C_2 must vanish, we get the dispersion law

$$\varepsilon(k) = \hbar\omega(k) = g\mu_0 M^0[\alpha k^2(\alpha k^2 + 4\pi)]^{1/2}, \tag{8}$$

where $k^2 = k_y^2 + k_z^2$.

Now, on the basis of the expression (8) obtained for the energy of a spin wave as a function of the wave vector k, it is easy to find an expression for the heat capacity C_B of spin waves in the Bloch wall. Let the specimen have dimensions L_x, L_y, L_z respectively, along the coordinate axes chosen earlier. If D denotes the thickness of the domains, then the total energy of the spin waves in all the Bloch walls of the specimen will be

$$\frac{L_y L_z}{(2\pi)^2} \frac{L_z}{D} \int \frac{\varepsilon(k)\, d^2k}{e^{\varepsilon/T} - 1} \tag{9}$$

(the temperature T is measured in energy units).

It is clear from expression (9) that the largest contribution to the total energy is made by waves with energy

$$\varepsilon(k) \approx T. \tag{10}$$

From this we can determine the temperature intervals in which there is a quadratic or a linear dispersion law. For $\alpha k^2 \gg 4\pi$ we have a quadratic law:

$$\varepsilon = g\mu_0 M^0 \alpha k^2. \tag{11}$$

From equations (10) and (11) we express αk^2 in terms of the temperature T and, by substituting in the indicated inequality, obtain the condition for existence of a quadratic spectrum,

$$T \gg T_0, \tag{12}$$

where

$$T_0 = 4\pi g\mu_0 M^0. \tag{12a}$$

In the inverse case $T \ll T_0$, there will be a linear spectrum:

$$\varepsilon = 2g\mu_0 M^0 (\pi\alpha)^{1/2} k. \tag{13}$$

We substitute formula (11) in (9) and, remembering that the temperature lies in the interval $T_0 \ll T \ll \Theta$, where Θ is the Curie temperature of the ferromagnetic specimen, get the expression for the energy of unit volume

$$E_B = \frac{1}{2\pi D} \frac{1}{2g\mu_0 M^0 \alpha} \int_0^\Theta \frac{\varepsilon\, d\varepsilon}{e^{\varepsilon/T} - 1} = \frac{1}{2\pi D} \frac{T^2}{2g\mu_0 M^0 \alpha} \int_0^\infty \frac{x\, dx}{e^x - 1}$$

(here we have set $x \equiv \epsilon/T$). We finally have

$$E_B = {}^1/_{24}\pi (g\mu_0 M^0 \alpha D)^{-1} T^2$$

and the corresponding heat capacity of unit volume, with use of (12a), will be

$$C_B = \frac{\pi^2}{3\alpha D} \frac{T}{T_0}. \tag{14}$$

The thickness D of the domains is related to the length L_z of the crystal by the formula[2]

$$D = 2(2L_z)^{1/2}(\alpha/\beta)^{1/4}. \tag{15}$$

We substitute this expression in (14); and noting that the constant α is expressed in terms of the Curie temperature by the formula (see, for example,[3], p. 64)

$$\alpha = 4\pi a^2 \Theta / T_0$$

(a is the lattice constant; for simplicity we suppose that the lattice is cubic), we find

$$C_B = \frac{\pi}{24} \frac{1}{a^2(2L_z)^{1/2}} \left(\frac{\beta}{4\pi a^2} \frac{T_0}{\Theta}\right)^{1/4} \frac{T}{\Theta} \quad \text{if } T \gg T_0. \tag{16}$$

Quite analogously, we obtain for the linear spectrum (13)

$$C_B = \frac{1{,}8\pi}{a^2(2L_z)^{1/2}} \left(\frac{\beta}{4\pi a^2} \frac{T_0}{\Theta}\right)^{1/4} \frac{T^2}{T_0\Theta} \quad \text{if } T \ll T_0. \tag{17}$$

We notice that from formulas (16) and (17) there follows an interesting peculiarity of the heat capacity of spin waves in a Bloch wall: it depends on the length of the crystal in the direction of the axis of easy magnetization. So that a decrease of this dimension can lead to a significant increase of the magnitude of C_B. To decrease this length, however, has meaning only as long as the specimen still remains multidomain; the condition for this, as is well known[2], has the form $L_y L_z \gg \alpha$.

We can now estimate the contribution of the value of C_B to the total heat capacity of magnetic crystals. What basically interests us is how C_B is related to the heat capacity C_s of spin waves in the interior of the domains, since we know (see, for example,[3]) the relation of the latter to the phonon (proportional to T^3) and electron ($\propto T$) heat capacities of ferromagnets and to the phonon heat capacity of ferrites. The macroscopic derivation of the spectrum of spin waves in the domains does not differ essentially from our derivation of the dispersion law (8)—with allowance, of course, for the "tridimensionality" of these waves and for what was said above about the equilibrium values of M^0 and H^0_{eff}. The corresponding formulas for the various temperature intervals[3] have the form

$$C_s = 0.2\frac{1}{a^3}\left(\frac{T}{\Theta}\right)^{3/2} \quad \text{if} \quad T \gg \frac{T_0}{4}, \frac{\beta T_0}{8\pi}; \tag{18a}$$

$$C_s = 0.6\frac{1}{a^3}\left(\frac{T}{T_0}\right)\left(\frac{T}{\Theta}\right)^{3/2} \quad \text{if} \quad \frac{\beta T_0}{8\pi} \ll T \ll \frac{T_0}{4}; \tag{18b}$$

$$C_s = \frac{1}{2\pi^{3/2}}\frac{\beta}{(8\pi)^2 a^3}\frac{T_0^2}{\Theta T}\left(\frac{T}{\Theta}\right)^{1/2}\exp\left(-\frac{\beta T_0}{4\pi T}\right) \quad \text{if} \quad \frac{T_0}{4} \ll T \ll \frac{\beta T_0}{8\pi}; \tag{18c}$$

$$C_s = \frac{1}{2\pi^{3/2}}\frac{\beta^2}{(8\pi)^2 a^3}\frac{T_0}{\Theta}\left(\frac{T}{\Theta}\right)^{1/2}\exp\left(-\frac{\beta T_0}{4\pi T}\right) \quad \text{if} \quad T \ll \frac{T_0}{4} \ll \frac{\beta T_0}{8\pi}. \tag{18d}$$

On the basis of formulas (16)—(18), the heat capacities C_B and C_s can be compared in the indicated temperature intervals. For concreteness, we turn to the illustrative table[1)] of heat capacities of certain materials (the numerical coefficients were determined to an order of magnitude). In this table, the first temperature interval ($T \ll T_0$) is absent, since in it, under any conditions, the value of C_B is negligibly small in comparison with C_s. The second temperature interval (formula (18b)) is possible only for ferromagnetic specimens (with anisotropy constant $\beta \lesssim 1$), in which, however, the electronic heat capacity overwhelms the thermal effect of spin waves. Conversely, the third and fourth intervals are possible only in ferrites (β larger). As is seen from the table, in these intervals, even for specimen

Ferrite	T_0, °K	Θ, °K	β	$C_s a^2$	$10 C_B a^2$	$10^{-2} C_s a^2$	$10^{-1} C_B a^2$
				$T_0/4 \ll T \ll \beta T_0/8\pi$		$T \ll T_0/4 \ll \beta T_0/8\pi$	
$BaFe_{12}O_{19}$	0.7	723	40	$T^{-1/2}e^{-2,3/T}$	$TL_z^{-1/2}$	$T^{1/2}e^{-2,3/T}$	$T^2L_z^{-1/2}$
$BaFe_{18}O_{27}$	0.5	728	60	$T^{-1/2}e^{-2,5/T}$	$TL_z^{-1/2}$	$T^{1/2}e^{-2,5/T}$	$T^2L_z^{-1/2}$
$BaZnFe_{17}O_{27}$	0.7	703	40	$T^{-1/2}e^{-2,3/T}$	$TL_z^{-1/2}$	$T^{1/2}e^{-2,3/T}$	$T^2L_z^{-1/2}$

dimensions L_z below 10^{-1} cm, the value of C_B everywhere exceeds C_s.

Thus at temperatures below $\beta T_0/8\pi$, the value C_B of the heat capacity of spin waves in a Bloch wall is determinative for ferrites. The presence of an external magnetic field increases, as it were, the constant β[3], and thereby a given upper limit of temperature is raised. It is necessary, however, to note at once that, although it is usually so assumed, it is not at all clear whether the familiar equilibrium distribution of magnetic moments of the type (2a) is retained in the presence of a magnetic field. On the other hand, there are experimental indications that in certain ferrites, even a field of order 10^3 Oe does not lead to disappearance of the domain boundaries[5].

If, however, the domain walls disappear in a strong magnetic field, then the heat capacity of a ferrite specimen obviously becomes of phonon type; that is, there is a cubic dependence on temperature. Thus measurements of the heat capacity of a specimen as a function of temperature can give information about the fields at which the displacement process is completed and the domain-rotation process begins in magnetization.

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1)The data are taken from the book of Smit and Wijn [4].