

THEORY OF THE PHOTOELECTRIC AND PHOTOMAGNETIC EFFECTS PRODUCED BY LIGHT PRESSURE

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A theory of carrier drag due to electromagnetic-wave pulses is developed. Formulas for the photo emf at $\omega\tau \gg 1$ are derived for the case when light absorption by free carriers is possible only if acoustic phonons participate. The formulas are applicable in the quantum limit $\hbar\omega \gg T$. The case when optical transitions occur between two subbands is also considered. The photo emf arising in a transverse magnetic field and constituting the monopolar photomagnetic effect (PME) is investigated.

IN the absorption of light by free carriers, the momentum of the electromagnetic wave is absorbed together with its energy. Consequently, the electron system can acquire a translational motion that is manifest in the form of a current or a voltage. Since the process of photon absorption by the electron is always accompanied explicitly or implicitly by an interaction with the lattice, the resultant photon momentum $\hbar\kappa$ is equal in each elementary absorption act to the sum of the momenta acquired by the electron and by the lattice. Therefore, in general form, nothing definite can be said concerning not only the magnitude of the momentum acquired by the electron system, but also concerning its sign. A situation wherein the sign of this momentum was opposite to the sign of the photon momentum was first observed in^[1], and the experimental data therein serve as the basis for the theory presented below.

The phenomenological theory of the photo emf due to light pressure was developed by Barlow^[2] and by Gurevich and Romyantsev^[3]. An essential limitation of such a theory is the classical condition $\hbar\omega \ll T$ (T —lattice temperature, $\hbar\omega$ —photon energy). In this approximation, the photo emf is interpreted as a Hall effect due to the electric and magnetic fields of the light wave. The limitation $\hbar\omega \ll T$ is connected with the process of photon absorption, and not with the formation of a direct current or of an electric field. In the present paper we consider a quantum situation that is important in the optical band. We consider also the photomagnetic effect (PME) due to the pressure of the light.

1. PHOTO EMF AND PME IN THE CASE OF LIGHT ABSORPTION DUE TO THE INTERACTION WITH ACOUSTIC PHONONS

The kinetic equation of electrons interacting with phonons and with a flux W of monochromatic photons, in the presence of an electric field E and a non-quantizing magnetic field H , and in the case of a quadratic dispersion law, is given by

$$\frac{eE}{\hbar} \nabla_{\mathbf{k}} f_0(E_{\mathbf{k}}) + ([\Omega \mathbf{g}] \mathbf{k}) - \frac{(\mathbf{g} \mathbf{k})}{\tau(E_{\mathbf{k}})} + \left(\frac{\partial f}{\partial t} \right)_{\text{phot}} = 0, \quad (1)^*$$

* $[\Omega \mathbf{g}] \equiv \Omega \times \mathbf{g}$.

where $\Omega = eH/m_n c$; m_n , e , and \mathbf{k} are the effective mass, the charge, and the wave vector of the electrons; c is the velocity of light; $f_0(E_{\mathbf{k}})$ and $(\mathbf{k} \cdot \mathbf{g}(E_{\mathbf{k}}))$ are the equilibrium and asymmetrical parts of the electron distribution function, which is constant in time; τ is the relaxation time; $(\partial f / \partial t)_{\text{phot}}$ determines the change of the asymmetrical part of the distribution function, a change due to processes of absorption and emission of photons.

When using Eq. (1), we confine ourselves by the same token to relatively weak light fluxes and electric fields. In this approximation, when it is assumed that the electrons interact in the absorption act only with acoustic phonons, $(\partial f / \partial t)_{\text{phot}}$ takes the following form:

$$\left(\frac{\partial f}{\partial t} \right)_{\text{phot}} = G(E_{\mathbf{k}})(\mathbf{k} \boldsymbol{\kappa}) = \frac{3e^2 \hbar^3 E_{\text{ac}}^2 W (1 - e^{-\hbar\omega/T})}{32\pi^2 \rho S c m_n^2 \hbar \omega} \times \frac{(\mathbf{k} \boldsymbol{\kappa})}{\hbar \boldsymbol{\kappa}} \int [e^{\hbar\omega/T} M(-\boldsymbol{\kappa}, -\omega) \delta(E_{\mathbf{k}_1} - E_{\mathbf{k}} + \hbar\omega) - M(\boldsymbol{\kappa}, \omega) \delta(E_{\mathbf{k}_1} - E_{\mathbf{k}} - \hbar\omega)] \frac{(\mathbf{k} \boldsymbol{\kappa})}{\hbar \boldsymbol{\kappa}} f_0(E_{\mathbf{k}}) (1 - f_0(E_{\mathbf{k}_1})) d^3 k_1 d\Omega_{\mathbf{k}}, \quad (2)$$

where E_{ac} is the deformation-potential constant, ρ is the density, S is the speed of sound, \bar{n} is the refractive index, and $\hat{n}_{\mathbf{q}}$ is the filling function of phonons with wave number \mathbf{q} ,

$$M(\boldsymbol{\kappa}, \omega) = \sum_{\lambda=1}^2 \left(\frac{(\mathbf{e}_{\lambda} \mathbf{k})}{E_{\mathbf{k}} - E_{\mathbf{k}+\boldsymbol{\kappa}} + \hbar\omega} + \frac{(\mathbf{e}_{\lambda} \mathbf{k}_1)}{E_{\mathbf{k}_1} - E_{\mathbf{k}_1-\boldsymbol{\kappa}} - \hbar\omega} \right)^2 (2\hat{n}_{|\mathbf{k}+\boldsymbol{\kappa}-\mathbf{k}_1|} + 1) |\mathbf{k} - \mathbf{k}_1 + \boldsymbol{\kappa}|,$$

\mathbf{e}_{λ} is the photon polarization vector and $d\Omega_{\mathbf{k}}$ is the solid angle in \mathbf{k} -space.

In (2) we assumed the inequality $\omega\tau \gg 1$ and neglected the phonon energy compared with the average electron energy $\langle E_{\mathbf{k}} \rangle$, an assumption valid if

$$\sqrt{\frac{2m_n S^2}{\langle E_{\mathbf{k}} \rangle}} \left(1 + \sqrt{1 + \frac{\hbar\omega}{\langle E_{\mathbf{k}} \rangle}} \right) \ll 1.$$

If the last inequality is satisfied when $\langle E_{\mathbf{k}} \rangle = T$, then $n|\mathbf{k} - \mathbf{k}_{\pm\boldsymbol{\kappa}}| \gg 1$. In this case we obtain from (2), accurate to terms linear in $\boldsymbol{\kappa}$

$$G(E_{\mathbf{k}}) = \frac{2^{7/2} e^2 W E_{\text{ac}}^2 T (1 - e^{-\hbar\omega/T})}{5\rho S^2 c m_n^{7/2} \bar{n} (\hbar\omega)^4} E_{\mathbf{k}} f_0(E_{\mathbf{k}}) \times \{ e^{\hbar\omega/T} (1 - f_0(E_{\mathbf{k}} - \hbar\omega)) \sqrt{E_{\mathbf{k}} - \hbar\omega} - (1 - f_0(E_{\mathbf{k}} + \hbar\omega)) \sqrt{E_{\mathbf{k}} + \hbar\omega} \}. \quad (3)$$

Substituting (3) in (1), we get

$$g(E_k) = \tau^* \{L + \tau[\Omega L] + \tau^2 \Omega(\Omega L)\}, \quad (4)$$

where

$$L = \frac{e}{\hbar k} \frac{\partial E_k}{\partial k} \frac{\partial f_0}{\partial E_k} E + G(E_k) \boldsymbol{\kappa}, \quad \tau^* = \tau(E_k) [1 + (\Omega \tau)^2]^{-1}.$$

From (4) follows the formula for the current density

$$\mathbf{j} = \frac{e^2 n_0}{m_n} \{ \langle \tau \rangle \mathbf{E} + \langle \tau \tau^* \rangle [\Omega \mathbf{E}] + \langle \tau^2 \tau^* \rangle \Omega(\Omega \mathbf{E}) \} - \frac{\alpha(\omega) e \hbar \bar{n} \omega}{m_n c} \{ \langle \tau^* \rangle \mathbf{W} + \langle \tau \tau^* \rangle [\Omega \mathbf{W}] + \langle \tau^2 \tau^* \rangle \Omega(\Omega \mathbf{W}) \}, \quad (5)$$

where n_0 is the electron concentration, and $\alpha(\omega)$ is the coefficient of light absorption by the electrons, equal to

$$\alpha(\omega) = \frac{2^{1/2} e^2 m_n^{1/2} (1 - e^{-\hbar\omega/T}) Q(\omega)}{3\pi \bar{n} c T^{1/2} (\hbar\omega)^3 \tau(T) \hbar},$$

$$\langle \chi(E_k) \rangle = - \frac{(2m_n)^{3/2}}{3\pi^2 \hbar^3 n_0} \int_0^\infty \chi(E_k) E_k^{3/2} \frac{\partial f_0}{\partial E_k} dE_k,$$

$$\tau^{-1}(E_k) = \frac{2^{3/2} E_{aR}^2 m_n^{3/2} T E_k^{1/2}}{\pi \rho S^2 \hbar^4}$$

$$\langle \chi(E_k) \rangle = \frac{4}{5\hbar\omega Q(\omega)} \int_0^\infty \sqrt{E_k(E_k + \hbar\omega)^5} \chi(E_k + \hbar\omega)$$

$$- \sqrt{E_k^5(E_k + \hbar\omega)} \chi(E_k) f_0(E_k) (1 - f_0(E_k + \hbar\omega)) dE_k,$$

$$Q(\omega) = \int_0^\infty (2E_k + \hbar\omega) \sqrt{E_k(E_k + \hbar\omega)} f_0(E_k) (1 - f_0(E_k + \hbar\omega)) dE_k.$$

When $\Omega \perp \mathbf{W}$, $\mathbf{E} \perp \Omega$, $\langle \Omega \tau \rangle \ll 1$, and $\mathbf{j} = 0$, the electric field intensity is

$$\mathbf{E} = \frac{\alpha(\omega) e \bar{n} \hbar \omega}{\sigma_n m_n c} \left\{ \mathbf{W} \langle \tau \rangle + [\nu \mathbf{W}] \left(\Omega \langle \tau^2 \rangle - \frac{\bar{\mu}_n H}{c} \langle \tau \rangle \right) \right\}, \quad (6)$$

where $\nu = H/H$, $\mu_n = e \langle \tau^2 \rangle / m_n \langle \tau \rangle$ is the Hall mobility, and σ_n is the electric conductivity. The first term determines the photo emf in the direction of light propagation. In spite of the fact that the current in this direction is equal to zero, the magnetic field perpendicular to it induces an electric field in the direction of $\nu \times \mathbf{W}$. This is due to the fact that the energy composition of the electron flux dragged by the light differs from the composition of the oppositely directed electron flux produced by the longitudinal electric field. Thus, the transverse photoelectric effect is a stationary monopolar photomagnetic effect that differs in principle from the effects known under the same name^[4,5], including the PME considered in^[3].

In the case of a nondegenerate distribution of the electrons, the explicit form of formula (6) is

$$\mathbf{E} = \frac{16\pi e^3 n_0 T^2 (1 - e^{-\hbar\omega/T})}{15c^2 m_n^2 \hbar \omega^3 \sigma_n} \left\{ \mathbf{W} F\left(\frac{\hbar\omega}{T}\right) + \frac{\bar{\mu}_n H}{c} [\nu \mathbf{W}] \left[\frac{2}{\pi} \left(\frac{\hbar\omega}{T}\right)^2 e^{\hbar\omega/2T} K_1\left(\frac{\hbar\omega}{2T}\right) - F\left(\frac{\hbar\omega}{T}\right) \right] \right\}, \quad (7)$$

where $K_1(z)$ is the Hankel function^[6],

$$F(z) = \left\{ \frac{15}{4} + 3z + z^2 + \sqrt{\frac{z}{\pi}} \left(z - \frac{15}{2} \right) - \left(\frac{15}{4} - 3z + z^2 \right) e^z \operatorname{erfc} \sqrt{z} \right\}.$$

In the limit of $\hbar\omega \gg T$ it follows from (7) that

$$\mathbf{E} = \frac{16\pi e^3 \hbar n_0}{15c^2 m_n^2 \omega \sigma_n} \left\{ \mathbf{W} - \frac{\bar{\mu}_n H}{c} [\nu \mathbf{W}] \left(1 - \sqrt{\frac{4T}{\pi \hbar\omega}} \right) \right\}. \quad (8)$$

In the case of a degenerate distribution we get from (6)

$$\mathbf{E} = \frac{2^{1/2} \cdot 10^2 e^3 (\hbar\omega)^{1/2} (1 - e^{-\hbar\omega/T})}{63\sigma_n \pi c^2 m_n^{1/2} \hbar} \left\{ \mathbf{W} \left[P\left(\frac{E_F}{\hbar\omega}\right) - \theta\left(\frac{E_F}{\hbar\omega} - 1\right) P\left(\frac{E_F}{\hbar\omega} - 1\right) \right] + \frac{\bar{\mu}_n H}{c} [\nu \mathbf{W}] \left[\frac{10^5}{8} \left(\frac{E_F}{\hbar\omega}\right)^{1/2} \left(R\left(\frac{E_F}{\hbar\omega}\right) - \theta\left(\frac{E_F}{\hbar\omega} - 1\right) R\left(\frac{E_F}{\hbar\omega} - 1\right) \right) - \left(P\left(\frac{E_F}{\hbar\omega}\right) - \theta\left(\frac{E_F}{\hbar\omega} - 1\right) P\left(\frac{E_F}{\hbar\omega} - 1\right) \right) \right] \right\}, \quad (9)$$

where E_F is the Fermi energy, $\theta(z) = 0$ when $z < 0$ and $\theta(z) = 1$ when $z > 0$,

$$P(z) = z^{1/2} (15z^2 + 42z + 35) - (1+z)^{1/2} (15z^2 - 12z + 8) + 8,$$

$$R(z) = (1+2z)\sqrt{z(1+z)} - {}^{1/2}\ln(1+2z+2\sqrt{z(1+z)}).$$

When $\hbar\omega \gg E_F$, formula (9) takes the form

$$\mathbf{E} = \frac{16}{15} \frac{\pi e^3 \hbar n_0}{\sigma_n c^2 m_n^2 \omega} \left\{ \mathbf{W} - \frac{\bar{\mu}_n H}{c} [\nu \mathbf{W}] \left(1 - \left(\frac{E_F}{\hbar\omega}\right)^{1/2} \right) \right\}. \quad (10)$$

2. PHOTO EMF AND PME IN INTERBAND OPTICAL TRANSITIONS

The light pressure can become manifest also in interband optical transitions. A particularly interesting situation is one in which these transitions are not accompanied by change of the total number of carriers of given sign, for in the opposite case the effect can be masked by different photo emf's due to the occurrence of minority carriers. Favorable conditions of this kind are realized in the hole bands of many semiconductors, including Ge, in which the effect of light pressure was first observed in the optical band. As applied to the situation holding in^[1], we shall calculate the photo emf, generalizing it to the case when an external magnetic field is present. In this case it is possible to dispense with a definite specification of the dispersion of the energy bands, but these must be assumed to be spherically symmetrical.

In the approximation linear in the light intensity, the nonequilibrium parts of the distribution functions of the heavy and light holes are determined by independent kinetic equations obtained from (1) by means of the substitutions

$$f_0(E_k) \rightarrow f_0(E_k^{(\alpha)}), \quad \mathbf{g} \rightarrow \mathbf{g}_\alpha, \quad \tau \rightarrow \tau_\alpha, \quad \Omega \rightarrow \Omega_\alpha = \frac{e\mathbf{H}}{\hbar^2 c k} \frac{\partial E_k^{(\alpha)}}{\partial k},$$

$$e \rightarrow -e, \quad \left(\frac{\partial f}{\partial t} \right)_{\text{phot}} \rightarrow \left(\frac{\partial f^{(\alpha)}}{\partial t} \right)_{\text{phot}},$$

where $\alpha = 1$ corresponds to the band of heavy holes and $\alpha = 2$ to the band of light holes. In writing

$(\partial f^{(\alpha)})/\partial t$ phot we used the simplified version of the theory of optical transitions between bands 1 and 2, proposed by Kahn^[7]. In accordance with this theory, $G_1(\mathbf{k})$ and $G_2(\mathbf{k})$ are determined by the following relations:

$$G_1(k) = - \frac{3\Gamma}{4\pi k \kappa} \int \frac{(\mathbf{k}\boldsymbol{\kappa})}{k\kappa} k^2 [f_0(E_{\mathbf{k}^{(1)}}) - f_0(E_{\mathbf{k}^{(2)}})] \delta(E_{\mathbf{k}^{(2)}} - E_{\mathbf{k}^{(1)}} - \hbar\omega) d\Omega, \quad (11)$$

$$G_2(k) = \frac{3\Gamma}{4\pi k \kappa} \int \frac{(\mathbf{k}\boldsymbol{\kappa})}{k\kappa} (\mathbf{k} - \boldsymbol{\kappa})^2 [f_0(E_{\mathbf{k}^{(1)}}) - f_0(E_{\mathbf{k}^{(2)}})] \delta(E_{\mathbf{k}^{(2)}} - E_{\mathbf{k}^{(1)}} - \hbar\omega) d\Omega,$$

where $\Gamma = 4\pi^2 e^2 \hbar^2 A_{12}^2 \mathbf{W} / \text{cm}^2 \bar{n} \omega$, m is the mass of the free electron, A_{12} is a coefficient determining the magnitude of the interband dipole moment.

The solution of the kinetic equations takes the form of relation (4), in which it is necessary to make the aforementioned substitutions; this leads to a summary current density of the heavy and light holes:

$$\mathbf{j} = \sum_{\alpha=1}^2 \left\{ \frac{e^2 p_0}{m} [\langle \tau_{\alpha} \rangle_{\alpha} \mathbf{E} - \langle \tau_{\alpha} \tau_{\alpha}^* \rangle_{\alpha} [\Omega_{\alpha} \mathbf{E}] + \langle \tau_{\alpha}^2 \tau_{\alpha}^* \rangle_{\alpha} \Omega_{\alpha} (\Omega_{\alpha} \mathbf{E})] \right. \\ \left. + \frac{e \kappa \alpha (\omega)}{\hbar k_0} \frac{\partial}{\partial k_0} (E_{k_0}^{(2)} - E_{k_0}^{(1)}) [\langle \tau_{\alpha}^* \rangle_{\alpha} \mathbf{W} - [\tau_{\alpha} \tau_{\alpha}^*]_{\alpha} [\Omega_{\alpha} \mathbf{W}] + [\tau_{\alpha}^2 \tau_{\alpha}^*]_{\alpha} \Omega_{\alpha} (\Omega_{\alpha} \mathbf{W})] \right\}, \quad (12)$$

where

$$\langle \chi(k) \rangle_{\alpha} = -\frac{m}{3\hbar} \int_0^{\infty} k^3 \frac{\partial E_k^{(\alpha)}}{\partial k} \frac{\partial f_0}{\partial E_k^{(\alpha)}} \chi(k) dE_k^{(\alpha)} \left\{ \int_0^{\infty} [f_0(E_k^{(1)}) + f_0(E_k^{(2)})] k^2 dk \right\} \\ \tau_{\alpha}^* = \tau_{\alpha} (1 + (\Omega_{\alpha} \tau_{\alpha})^2)^{-1}, \\ [\chi(k)]_1 = -\frac{1}{4\pi k_0^3} \int \chi(k) k (\mathbf{k} \kappa) [f_0(E_{\mathbf{k}\kappa}^{(1)}) - f_0(E_{\mathbf{k}\kappa}^{(2)})] \frac{\partial E_k^{(1)}}{\partial k} \cdot \\ \cdot \delta(E_{\mathbf{k}\kappa}^{(2)} - E_{\mathbf{k}\kappa}^{(1)} - \hbar\omega) d^3k, \\ [\chi(k)]_2 = \frac{1}{4\pi k_0^3} \int \chi(k) (\mathbf{k} - \kappa)^2 (\mathbf{k} \kappa) [f_0(E_{\mathbf{k}-\kappa}^{(1)}) - f_0(E_{\mathbf{k}}^{(2)})] \cdot \\ \cdot \frac{\partial E_k^{(2)}}{\partial k} \delta(E_{\mathbf{k}}^{(2)} - E_{\mathbf{k}-\kappa}^{(1)} - \hbar\omega) d^3k, \\ \alpha(\omega) = \frac{\Gamma k_0^4}{2\pi^2 W} \frac{[f_0(E_{k_0}^{(1)}) - f_0'(E_{k_0}^{(1)} + \hbar\omega)]}{(\partial/\partial k_0)(E_{k_0}^{(2)} - E_{k_0}^{(1)})},$$

$\alpha(\omega)$ is the absorption coefficient, p_0 is the total hole concentration, and k_0 is determined by the equation

$$E_{k_0}^{(2)} = E_{k_0}^{(1)} + \hbar\omega.$$

The integrals $[\chi(k)]_{\alpha}$, accurate to the zeroth power of κ , are given by

$$[\chi(k)]_1 = \frac{N_1 N_2 k_0 \chi(k_0)}{3(N_1 - N_2)^2} \left\{ \gamma_1 + \frac{\partial}{\partial k_0} \ln \left[\chi(k_0) k_0^4 \frac{\partial E_{k_0}^{(1)}}{\partial k_0} \cdot \right. \right. \\ \left. \left. \cdot (f_0(E_{k_0}^{(1)}) - f_0(E_{k_0}^{(1)} + \hbar\omega)) \right] \right\}, \\ [\chi(k)]_2 = -\frac{N_1 N_2 k_0 \chi(k_0)}{3(N_1 - N_2)^2} \left\{ \gamma_2 + \frac{2(N_1 - N_2)}{k_0 N_2} \right. \\ \left. + \frac{\partial}{\partial k_0} \ln \left[\chi(k_0) k_0^4 \frac{\partial E_{k_0}^{(2)}}{\partial k_0} (f_0(E_{k_0}^{(2)} - \hbar\omega) - f_0(E_{k_0}^{(2)})) \right] \right\}; \quad (13)$$

where

$$\gamma_1 = \frac{2\pi^2 \hbar^2}{m_2 k_0^2} N_2 \left(1 - \frac{(m_1 - m_2) N_1}{m_1 (N_1 - N_2)} \right), \\ \gamma_2 = \frac{2\pi^2 \hbar^2}{m_1 k_0^2} N_1 \left(1 - \frac{(m_1 - m_2) N_2}{m_2 (N_1 - N_2)} \right),$$

$N_{\alpha} = (k_0^2/2\pi^2) (\partial E_{\mathbf{k}}^{(\alpha)}/\partial k_0)^{-1}$ is the density of states, and $m_{\alpha}^{-1} = \hbar^{-2} \partial^2 E_{\mathbf{k}}^{(\alpha)}/\partial k_0^2$ is the effective mass at $\mathbf{k} = k_0$.

The electric field at $\mathbf{H} \perp \mathbf{W}$, $\mathbf{H} \perp \mathbf{E}$, $\langle \Omega_{\alpha} \tau_{\alpha} \rangle \ll 1$, and $\mathbf{j} = 0$ is determined by the following formula:

$$\mathbf{E} = -\frac{\alpha(\omega) e \bar{\mu}_p k_0 (N_1 - N_2)}{\sigma_p \cdot 2\pi^2 \hbar c N_1 N_2} \sum_{\alpha=1}^2 [\tau_{\alpha}]_{\alpha} \mathbf{W} \\ - [\mathbf{vW}] \left([\tau_{\alpha}^2 \Omega_{\alpha}]_{\alpha} - \frac{\bar{\mu}_p H}{c^2} [\tau_{\alpha}]_{\alpha} \right), \quad (14)$$

where

$$\bar{\mu}_p H/c = \sum_{\alpha=1}^2 \langle \tau_{\alpha}^2 \Omega_{\alpha} \rangle_{\alpha} \left| \sum_{\alpha=1}^2 \langle \tau_{\alpha} \rangle_{\alpha} \right|,$$

$\bar{\mu}_p$ is the Hall mobility of the holes, and σ_p is their electric conductivity. It follows from (14) that the PME field can have a direction opposite to that following from the direction of the photo emf current at $\mathbf{H} = 0$.

The presence of logarithmic derivatives in (13) indicates that the photo emf and the PME are determined essentially by the form of the dependence on k_0 of all the quantities under the logarithm sign. The reason for such a dependence is that the non-equilibrium asymmetric part of the distribution function is

formed as a small difference, proportional to the photon momentum, between two large quantities, one of which determines the change of the distribution function for the holes with a wave vector satisfying the condition $(\mathbf{k} \cdot \kappa) < 0$, and the other $(\mathbf{k} \cdot \kappa) > 0$.

In the case of a quadratic dispersion law ($E_{\mathbf{k}}^{(\alpha)} = \hbar^2 k^2/2m_{\alpha}$), we get from (14)

$$\mathbf{E} = -\frac{\alpha(\omega) e \bar{\mu}_p \hbar \omega}{3\sigma_p c (m_1 - m_2)} \left\{ F(k_0) \mathbf{W} - [\mathbf{vW}] \frac{\bar{\mu}_p H}{c} (\mathcal{F}(k_0) - F(k_0)) \right\}, \quad (15)$$

where

$$F(k_0) = \left\{ k_0 \frac{\partial}{\partial k_0} (\tau_1(k_0) - \tau_2(k_0)) + \tau_1(k_0) \left(5 - 2\eta \frac{E_{k_0}^{(1)}}{T} \right) \right. \\ \left. - \tau_2(k_0) \left(3 + 2 \frac{m_1}{m_2} - 2\eta \frac{E_{k_0}^{(2)}}{T} \right) \right\},$$

$$\mathcal{F}(k_0) = \frac{e}{\bar{\mu}_p} \left\{ \frac{\tau_1^{(2)}(k_0)}{m_1} \left(2k_0 \frac{\partial \ln \tau_1(k_0)}{\partial k_0} + 5 - 2\eta \frac{E_{k_0}^{(1)}}{T} \right) \right. \\ \left. - \frac{\tau_2^{(2)}(k_0)}{m_2} \left(2k_0 \frac{\partial \ln \tau_2(k_0)}{\partial k_0} + 3 + 2 \frac{m_1}{m_2} - 2\eta \frac{E_{k_0}^{(2)}}{T} \right) \right\},$$

$$E_{k_0}^{(1)} = \frac{m_2 \hbar \omega}{m_1 - m_2}, \quad E_{k_0}^{(2)} = \frac{m_1 \hbar \omega}{m_1 - m_2}, \quad k_0^2 = \frac{2\omega m_1 m_2}{\hbar (m_1 - m_2)},$$

$$\eta(k_0) = 1 - f_0(E_{k_0}^{(1)}) + f_0(E_{k_0}^{(2)}).$$

Formula (15) can be readily analyzed in the case of a simple power-law dependence of the relaxation time on the energy. It is seen from it that both the longitudinal and transverse fields can reverse sign with changing temperature or frequency of the light. An analysis of formula (15) at $\mathbf{H} = 0$, carried out in^[1] and applied to p-Ge, in which an important role is played by the scattering of holes by both acoustic and optical phonons, has shown that it is in good agreement with experiment.

In conclusion it should be noted that the formulas presented above for \mathbf{E} are valid if one assumes weak attenuation of the electromagnetic wave. It is easy to show that if the sample has the form of a parallelepiped, and \mathbf{H} and \mathbf{W} are directed along its generatrix ($\mathbf{H} \perp \mathbf{W}$), then it follows from the condition $\text{curl } \mathbf{E} = 0$ that the field of the PME is independent of the coordinates, whereas the field of the photo emf varies in the same way as \mathbf{W} . If we make in (6)–(10), (14), and (15) the substitution $\mathbf{W} \rightarrow W_0 (1 - \exp(-\alpha_{\text{eff}} l))/\alpha_{\text{eff}} l$, where W_0 is the intensity of the wave entering the crystal, α_{eff} is the effective coefficient of its attenuation, l is the dimension of the crystal in the direction of \mathbf{W} , then these formulas will determine the field $\langle \mathbf{E} \rangle$ averaged over the sample. Then $\langle \mathbf{E} \rangle$ in the direction of $\mathbf{W}_0 \times \mathbf{H}$ coincides with its value at any point of the sample.

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121